Dynamic description of spin glass models with stable 1-step RSB

p-spin-interaction spin-glass models: Connections with the structural glass problem

T. R. Kirkpatrick D. Thirumalai Phys Rev **B** 36, 5388 (1987)

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A. Crisanti[1], H. Horner[2], H.-J. Sommers[3] Z. Phys. B 92, 257–271 (1993)

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1. Comparison of the dynamic and static approaches to p-spin model
2. "Tri-critical point" for glassy transition
3. Dynamic model of structural glass
4. Exact equations for "spherical p-spin" model
5. Frustrated regular Josephson array

p-spin-interaction spin-glass models: Connections with the structural glass problem

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$$H = -\sum_{i_1 < i_2 \cdots < i_p} J_{i_1} \cdots {}_{i_p} \sigma_{i_1} \cdots \sigma_{i_p} - \sum_{i=1}^{N} h_i \sigma_i$$

$$P(J_{i_1} \cdots {}_{i_p}) = \left[\frac{N^{p-1}}{\pi p! J^2} \right]^{1/2} \times \exp\left[\frac{-(J_{i_1 \cdots i_p})^2 N^{p-1}}{J^2 p!} \right]$$

$$\beta H = \sum_i \left[\frac{r_0}{2} \alpha_i^2 + u \sigma_i^4 \right] - \beta \sum J_{i_1} \cdots {}_{i_p} \sigma_{i_1} \cdots \sigma_{i_p} \quad -\beta \sum_{i=1}^{N} h_i \sigma_i \ ,$$

$$\Gamma_0^{-1} \partial_t \sigma_i(t) = -\frac{\delta(\beta H)}{\delta \sigma_i(t)} + \xi_i(t) \ , \qquad \langle \xi_i(t) \xi_j(t') \rangle = \frac{2}{\Gamma_0} \delta_{ij} \delta(t-t') \ .$$

$$C_{ij}(t-t')=\langle\sigma_i(t)\sigma_j(t')\rangle \qquad G_{ij}(t-t')=\frac{\partial\langle\sigma_i(t)\rangle}{\partial h_j(t')},\ t>t'$$

$$G(t)=-\Theta(t)\partial_t C(t)$$ FDT

$$Z\{J_{i_1\cdots i_p}, l_i, \hat{l}_i\}=\int D\sigma\, D\hat{\sigma}\, \exp\left[\int dt[l_i(t)\sigma_i(t)+i\hat{l}_i(t)\hat{\sigma}_i(t)]+L(\sigma,\hat{\sigma})\right]$$

$$L(\sigma,\hat{\sigma})=\int dt\sum_i i\hat{\sigma}_i(t)\left[-\Gamma_0^{-1}\partial_t\sigma_i(t)-\frac{\delta(\beta H)}{\delta\sigma_i(t)}+\Gamma_0^{-1}i\hat{\sigma}_i(t)\right]+V\{\sigma\}\ .$$

The functional Jacobian,

$$V=-\tfrac{1}{2}\int dt\sum_i\frac{\delta^2(\beta H)}{\delta\sigma_i^2}$$

ensures the normalization,

$$Z\{J_{i_1}\cdots i_p,\ l_i=\hat{l}_i=0\}=1\ .$$

$$[Z]_J=\int\prod dJ_{i_1\cdots i_p}P(J_{i_1\cdots i_p})Z\{J_{i_1\cdots i_p}\}=\int D[\sigma]D[\hat{\sigma}]\exp[L_0(\sigma,\hat{\sigma})+\Delta(\sigma,\hat{\sigma})]\ ,$$

$$L_0(\sigma,\hat{\sigma})=\int dt\sum_i[i\hat{\sigma}_i(-\Gamma_0^{-1}\partial_t\sigma_i-r_0\sigma_i-4u\sigma_i^3-h_i+i\Gamma_0^{-1}\hat{\sigma}_i)+i\hat{l}_i\hat{\sigma}_i+l_i\sigma_i]+V\{\sigma\}$$

$$\Delta(\sigma,\hat{\sigma})=\frac{p\beta^2J^2}{4N^{p-1}}\sum_{i,j,i_3\cdots i_p}[i\hat{\sigma}_i(t)\sigma_j(t)i\hat{\sigma}_i(t')\sigma_j(t')\sigma_{i_3}(t)\sigma_{i_3}(t')\cdots\sigma_{i^p}(t)\sigma_{i_p}(t')$$
$$+(p-1)i\hat{\sigma}_i(t)\sigma_j(t)i\hat{\sigma}_j(t')\sigma_i(t')\sigma_{i_3}(t)\sigma_{i_3}(t')\cdots\sigma_{i_p}(t)\sigma_{i_p}(t')]$$

$$Q_1(\hat{\sigma}\,\hat{\sigma}) = \frac{1}{N}\sum_{i=1}^{N} i\hat{\sigma}_i(t)i\hat{\sigma}_i(t'), \qquad Q_3(\hat{\sigma}\sigma) = \frac{1}{N}\sum_{i=1}^{N} i\hat{\sigma}_i(t)\sigma_i(t'),$$

$$Q_2(\sigma\sigma) = \frac{1}{N}\sum_{i=1}^{N} \sigma_i(t)\sigma_i(t'), \qquad Q_4(\sigma\hat{\sigma}) = \frac{1}{N}\sum_{i=1}^{N} \sigma_i(t)i\hat{\sigma}_i(t')$$

Lagrange multiplier λ_μ.

with this $\Delta(\sigma,\hat{\sigma})$ becomes, where $\mu = p\beta^2 J^2/2$

$$\Delta(\sigma,\hat{\sigma}) = \frac{\mu N}{2}\int dt\,dt'[Q_1 Q_2^{p-1} + (p-1)Q_3 Q_4 Q_2^{p-2}]$$

$$[Z]_J = \int \prod_{\mu=1}^{4} D[Q_\mu] \int \prod_{\mu=1}^{4}\left(\frac{N}{2\pi i}\right) D[\lambda_\mu]\exp\left(-NG(Q_\mu,\lambda_\mu) + \ln\int D[\sigma]D[\hat{\sigma}]\exp L(\sigma,\hat{\sigma},\lambda_\mu,Q_\mu)\right)$$

$$G(Q_\mu,\lambda_\mu) = \int dt\,dt' \sum_{\mu=1}^{4}\lambda_\mu Q_\mu - \frac{\mu}{2}\int dt\,dt'[Q_1(t,t')Q_2^{p-1}(t,t') + (p-1)Q_3(t,t')Q_4(t,t')Q_2^{p-2}(t,t')]$$

$$L(\sigma,\hat{\sigma},\lambda_\mu,Q_\mu) = L_0(\sigma,\hat{\sigma}) + \int dt\,dt' \sum_i [\lambda_1(t,t')i\hat{\sigma}_i(t)i\hat{\sigma}_i(t') + \lambda_2(t,t')\sigma_i(t)\sigma_i(t')$$
$$+ \lambda_3(t,t')i\hat{\sigma}_i(t)\sigma_i(t') + \lambda_4(t,t')\sigma_i(t)i\hat{\sigma}_i(t')]$$

The stationary values $\lambda_1^0 = \frac{\mu}{2}(Q_2^0)^{p-1}$, $\lambda_4^0 = \frac{\mu}{2}(p-1)Q_3^0(Q_2^0)^{p-2}$

$\lambda_3^0 = \frac{\mu}{2}(p-1)Q_4^0(Q_2^0)^{p-2}$ $\lambda_2^0 = \frac{\mu}{2}(p-1)[Q_1^0(Q_2^0)^{p-2} + (p-2)Q_3^0 Q_4^0 Q_2^{p-3}]$

$\boxed{Q_1^0 \equiv 0}$

$$Q_2^0 = C(t-t') ,$$

$$Q_3^0 = Q_4^0 = G(t-t')$$

Effective equations:

$$\sigma_i(\omega) = G_0(\omega)[f_i(\omega) + h_i(\omega)] - 4uG_0(\omega)\int \frac{d\omega_1 d\omega_2}{(2\pi)^2}\sigma_i(\omega_i)\sigma_i(\omega_2)\sigma_i(\omega-\omega_1-\omega_2)$$

$$G_0^{-1}(\omega) = r_0 - i\omega\Gamma_0^{-1} - \mu(p-1)\int_0^\infty dt\, e^{i\omega t}G(t)C^{p-2}(t)$$

$$\langle f_i(\omega)f_j(\omega')\rangle = 2\pi\delta(\omega+\omega')\delta_{i_j}\left[\frac{2}{\Gamma_0} + \mu\int_{-\infty}^{+\infty}dt\, e^{i\omega t}C^{p-1}(t)\right]$$

Equation for dynamic correlation functions

$$\hat{C}(\omega)=\int_0^\infty dt\, e^{i\omega t}C(t)$$

Im ω >0

$$G(t)=-\Theta(t)\partial_t C(t)\ ,$$

In the ergodic phase

$$\hat{C}(\omega)=\frac{C(t=0)}{-i\omega+\bar{r}_0\Gamma(\omega)}$$

$\Gamma(\omega)$ is a renormalized kinetic coefficient,

$$\Gamma^{-1}(\omega)=\Gamma_0^{-1}+\mu\int_0^\infty dt\, e^{i\omega t}C^{p-1}(t)\ .$$

$$C(t=0)=\bar{r}_0^{-1}=[r_0-\mu C^{p-1}(t=0)+12uC(t=0)]^{-1}$$

For use below we note that in the domain, $\phi(t)=C(t)/C(0)$ satisfies the equation

$$\nu_0^{-1}\dot{\phi}(t)+\phi(t)+\lambda\int_0^t dt_1\,\phi^{p-1}(t-t_1)\dot{\phi}(t_1)=0$$

with $\phi(t=0)=1$, $\nu_0=\bar{r}_0\Gamma_0$, and the nonlinear coupling given by $\lambda=\mu\bar{r}_0^{2-p}$.

Solution near the glass transition point T_g

Assume that C(0) is continuous across the glass transition

$$q_{\mathrm{EA}} = q = \lim_{t \to \infty} C(t) \,.$$

Assuming that $C(\omega)$ has a time-persistent part with a nonzero value of q and a decaying part, Eqs. (3.1) yield

$$q = \frac{C(t=0)\mu q^{p-1}}{(\bar{r}_0 + \mu q^{p-1})} \qquad (*)$$

This equation leads to a physical q at a critical temperature given by Eq. (3.1c) and $\mu \to \mu_c$,

$$\mu_c = \bar{r}_{0c}^{\,p} \left[\frac{p-1}{p-2} \right]^{p-2} (p-1)$$

where $\mu = p\beta^2 J^2/2$

The critical EA order parameter is

$$q_c = q \Big|_{T=T_g} = \frac{p-2}{\bar{r}_{0c}(p-1)} \,.$$

For $T = T_g$ the approach to Eq. (3.3b) is given by

$$C(t \to \infty) = \frac{p-2}{\bar{r}_{0c}(p-1)} + \frac{A}{t^{1-\alpha}} \,.$$

Equation for the exponent α is: $2\Gamma^2(1-\alpha) = \Gamma(1-2\alpha)$ $\alpha \simeq 0.395$.

Critical slowing down:

$$\Gamma(T \to T_g^+) \sim |T - T_g|^{(1+\alpha)/2\alpha} \simeq |T - T_g|^{1.765}$$

Any p > 2 !

For small $\varepsilon = p - 2 \ll 1$

$$\mu_c \simeq \overline{r}_{0c}^{2+\epsilon}(1-\epsilon\ln\epsilon+\epsilon) ,$$

$$q_c \simeq \frac{\epsilon}{\overline{r}_{0c}} .$$

Finally, we discuss the assumption that $C(t=0)$ is continuous at T_g. This assumption seems physically well motivated for any glass transition.[23] Our discussion here, however, will lead to an interesting paradox which will be clarified in Sec. IV of this paper. We first argue that Eqs. (3.1) and (2.4) naively lead to a complete specification of the problem and that no assumption on $C(t=0)$ is needed. Assuming $C(t\to\infty)=q$, Eq. (2.4) yields

$$C(t=0)=\frac{1}{\overline{r}_0+\mu q^{p-1}}+q$$

Effective dynamic equations lead to

$$q=\frac{\mu q^{p-1}}{(\overline{r}_0+\mu q^{p-1})^2} .$$

The above 2 Eqs. are inconsistent with continuous C(0) at T_g and with Eq. (*)

Resolution of the paradox: C(0) is continuous, but FDT is broken at T_g

Static RSB solution

$$\frac{\beta F}{N} = -\frac{\beta^2 C^p}{4} + \lambda_s C - \frac{\beta^2}{4n} \sum_{a \neq b} Q_{ab}^p + \frac{1}{2n} \sum_{a \neq b} \lambda_{ab} Q_{ab} \quad - \ln A_0 - \frac{C_0^2}{4n} \sum_{a \neq b} \lambda_{ab}^2$$

$$- \frac{C_0^3}{6n} \sum_{\substack{a \neq b \neq c \\ c \neq a}} \lambda_{ab} \lambda_{bc} \lambda_{ca} + O(\lambda^4)$$

where $C_0 = A_2 / A_0$, with

$$A_n = \int d\sigma \, \sigma^n \exp[-\tfrac{1}{2}(r_0 - 2\lambda_s)\sigma^2 - u\sigma^4]$$

We will see that solution is provided by 1-step RSB –
as we know it from E.Gardner paper of 1985
Here we present the solution in term of Parisi functions
$q(x)$ and $\lambda(x)$ and will see that solution is of the form

$$q(x) = q \Theta(x - \bar{x})$$

$$\frac{\beta F}{N} = -\frac{\beta^2 C^p}{4} + \lambda_s C - \ln A_0 + \int_0^1 dx \left[\frac{\beta^2 q^p(x)}{4} - \frac{q(x)\lambda(x)}{2} + \frac{C_0^2 \lambda^2(x)}{4} \right]$$

$$- \frac{C_0^3}{6} \int_0^1 dx \left[x\lambda^3(x) + 3\lambda(x) \int_0^x dy \, \lambda^2(y) \right] + O(\lambda^4)$$

$$\frac{\beta F}{N}=-\frac{\beta^2 C^p}{4}+\lambda_s C-\ln A_0+\int_0^1 dx\left[\frac{\beta^2 q^p(x)}{4}-\frac{q(x)\lambda(x)}{2}+\frac{C_0^2\lambda^2(x)}{4}\right]$$

$$-\frac{C_0^3}{6}\int_0^1 dx\left[x\lambda^3(x)+3\lambda(x)\int_0^x dy\,\lambda^2(y)\right]+O(\lambda^4)$$

The saddle-point (SP) equation $\delta F/\delta q(x)=0$ yields,

$$\lambda(x)=\mu q^{p-1}(x)\ ,$$

and $\delta F/\delta\lambda(x)=0$ gives

$$q(x)=C_0^2\lambda(x)-C_0^3 x\lambda^2(x)-C_0^3\int_0^x dy\,\lambda^2(y)$$
$$-2C_0^3\lambda(x)\int_x^1 dy\,\lambda(y)+O(\lambda^3)\ .$$

$$\frac{\beta F}{N}=-\frac{\beta^2 C^p}{4}+\lambda_s C-\ln A_0\quad+(1-\bar{x})\left[\frac{\beta^2 q^p}{4}-\frac{q\lambda}{2}+\frac{C_0^2\lambda^2}{4}\right]\quad-\frac{C_0^3}{6}(1-\bar{x})(2-\bar{x})\lambda^3$$

Saddle-point solution:

$$1=C_0^2\mu q^{p-2}-C_0^3\mu^2 q^{2p-3}(2-\bar{x})\ ,$$

$$1-\bar{x}=\frac{3}{C_0^3\mu^2}q^{3-2p}\left[\frac{1-p}{2p}+\frac{\mu C_0^2}{4}q^{p-2}\frac{\mu^2 C_0^3}{6}q^{2p-3}\right]$$

$$\lambda_s=\frac{\mu}{2}C^{p-1}$$

An equation for C follows from maximizing Eq. (4.5) with respect to λ_s. The equilibrium critical temperature T_g' is obtained when these equations first have a physical solution, $q>0$, $1-\bar{x}>0$. At critically, $\bar{x}=1$, and the self-consistent one-loop equation for $C(T=T_g')=C_c$ is given by

$$C_c \equiv C_{0c} = \frac{1}{\bar{r}_{0c}} = (r_0 - \mu_c C_c^{p-1} + 12uC_c)^{-1}$$

The critical parameters, to $O(\epsilon)$ are

$$q_c \simeq \frac{3\epsilon}{2\bar{r}_{0c}} ,$$

$$\mu_c \simeq \bar{r}_{0c}^{2+\epsilon}[1-\epsilon\ln(\tfrac{3}{2}\epsilon)+\tfrac{3}{2}\epsilon] .$$

To $O(t=1-T/T_g')$

$$q \simeq \tfrac{3}{2}(\epsilon+t) ,$$

$$\bar{x} \simeq 1-t/\epsilon ,$$

$$(T_g')^2 \approx 1+\epsilon\left(\ln\left(\frac{3}{2}\epsilon\right)-1\right)$$

E.Gardner:

The critical temperature is again at $x_0=1$ and is given by

$$T_c = \left(\tfrac{3}{2}\varepsilon\right)^{\varepsilon/2}\left(1-\tfrac{1}{2}\varepsilon\right).$$

Compare temperatures of dynamic and static transitions

Dynamic transition:

$$T_g \approx 1 + \frac{\epsilon}{2}\left(\ln\epsilon - \frac{1}{2}\right)$$

Static RSB transition:

$$T_g' \approx 1 + \frac{\epsilon}{2}\left(\ln\left(\frac{3}{2}\epsilon\right) - 1\right)$$

$$T_g - T_g' \approx \frac{\epsilon}{2}\left(\frac{1}{2} - \ln\frac{3}{2}\right) \approx 0.05\epsilon$$

Dynamic transition occurs at higher temperature !

It seems to be a general feature of 1-step RSB problems the free energy being maximized at T_g by the value $\bar{x}$ at its physical endpoint $\bar{x}=1$ in the temperature $T_g > T > T'_g$. The variational equation for $\bar{x}$ given by Eq. (4.6b) is not a relevant equation if F as a function of $\bar{x}$ is not maximized in the physical region, $0 < \bar{x} < 1$. We

Technically, the freezing predicted by the dynamical theory is easy to understand using local stability theory. For $T > T_g$ the dynamical equation for $q = C(t \to \infty)$ has the stable trivial solutions and unphysical and unstable complex solutions. At T_g two of the complex solutions become degenerate real solutions. At T_g there are three real critical points x_i $(i=1,2,3)$ of the dynamical equation that satisfy $x_1 = 0 < x_2 < x_3 = q_c < C(t=0)$. The fixed points x_1 and $x_3 = q_c$ are stable and x_2 is an unstable fixed point. We then have a situation where $C(t\to\infty)=0$ and $C(t\to\infty)=q_c$ are both stable solutions, but where it is impossible to reach $C(t\to\infty)=0$ due to the intervening unstable fixed point. The only possible conclusion is that the system freezes into a SG state because it cannot reach the equilibrium state defined by $C(t\to\infty)=0$. It is interesting to point out that in the SK model the situation is quite different. The dynamical equation has only two fixed points and at T_g there is an exchange of stability between the fixed points.

Continuous transformation of 1-step RSB to full Parisi RSB

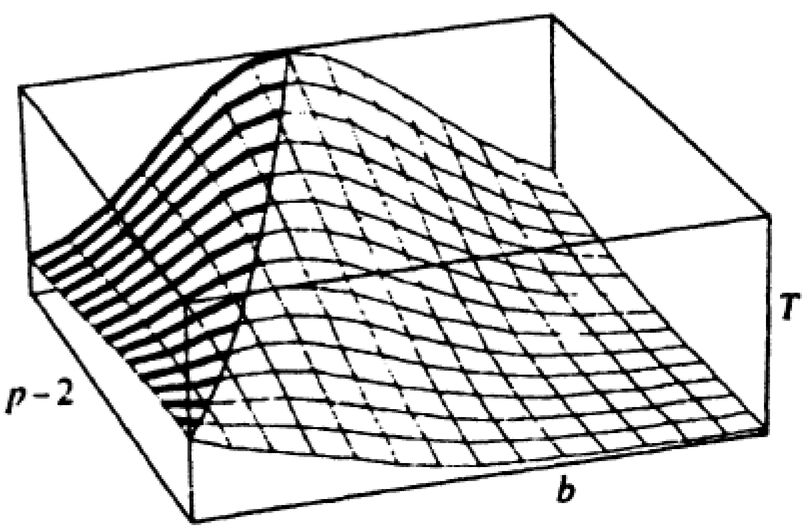
Slow cooling dynamics of the Ising *p*-spin interaction spin-glass model

D. M. Kagan and M. V. Feigelman Zh. Éksp. Teor. Fiz. **109**, 2094–2114 (June 1996)

$$H=-\sum_{1\leq i_1<\ldots<i_p\leq N} J_{i_1\ldots i_p}\sigma_{i_1}\cdots\sigma_{i_p}-b\sum_{i=1}^{N}\sigma_i\,,$$

External field b reintroduces effectively pair-wise spin-spin coupling

Phase diagram. Thin lines—continuous transition, thick lines—discontinuous transition.



We assume Glauber dynamics for σ_i: the probability for σ_i to change its sign during unit time is

$$w\{(\sigma_1,\ldots,\sigma_i,\ldots,\sigma_N)\to(\sigma_1,\ldots,-\sigma_i,\ldots,\sigma_N)\}$$

$$b_{\mathrm{tr}}\propto\varepsilon=p-2.$$

Relation to model of glass transition in a liquid

Dynamical model of the liquid-glass transition

E. Leutheusser PHYSICAL REVIEW A 29 2765 **(1984)**

Let us consider the following nonlinear equation of motion for a damped oscillator:

$$\ddot{\Phi}(t)+\gamma\dot{\Phi}(t)+\Omega_0^2\Phi(t)+4\lambda\Omega_0^2\int_0^t d\tau\,\Phi^2(\tau)\dot{\Phi}(t-\tau)=0 \quad (1)$$

with the initial condition $\Phi(t=0)=1$, $\dot{\Phi}(t=0)=0$, where

the oscillatory coordinate $\Phi(t)$ is thought to represent

the density correlation function of a classical fluid at a certain wave number.

by introducing Laplace transforms

$$\Phi(z)=\mathscr{L}\{\Phi(t)\}=i\int_0^\infty dt\, e^{izt}\Phi(t), \quad \mathrm{Im}z>0$$

$$\Phi(z)=-\cfrac{1}{z-\cfrac{\Omega_0^2}{z+D(z)}} \qquad D(z)=i\gamma+4\lambda\Omega_0^2\mathscr{L}\{\Phi^2(t)\}$$

Critical behavior at $\lambda = 1$

$$\Phi_v(z) = -\frac{1}{z - \frac{\Omega^2}{z + D_v(z)}},$$

$$D_v(z) = \{\mathscr{C}(z) + [\mathscr{C}^2(z) - 8\mathscr{C}(z)/z]^{1/2}\}/2$$

$$\mathscr{C}(z) = i\gamma + \mathscr{L}\{\Phi_v^2(t)\}.$$

$$D_v(z) \sim z^{\alpha - 1},$$

where $\alpha = 0.395$ is a solution of $\Gamma(1-2\alpha) = 2\Gamma^2(1-\alpha)$,

$$\mathscr{C}(z) \sim z^{2\alpha - 1},$$

The same exponent as for
p>2 spin glass model

Above T_g: $\lambda < 1$

where $s = \epsilon^{1/2\alpha}$ is a critical frequency, $\alpha = 0.395$,

$$D_v(z) = z^{\alpha - 1} d(z/s),$$

The relaxation rate δ vanishes as

$$\mathscr{C}(z) = z^{2\alpha - 1} c(z/s)$$

$$\delta \sim s^{1+\alpha} \sim \epsilon^{(1+\alpha)/2\alpha}.$$

the zero-frequency limit of the viscosity $D \sim \epsilon^{-\mu}$, $\mu = (\alpha + 1)/2\alpha = 1.765$

Indeed, compare equations of structure glass model and p-spin model:

$$\ddot{\Phi}(t)+\gamma\dot{\Phi}(t)+\Omega_0^2\Phi(t)+4\lambda\Omega_0^2\int_0^t d\tau\,\Phi^2(\tau)\dot{\Phi}(t-\tau)=0$$

$$\nu_0^{-1}\dot{\phi}(t)+\phi(t)+\lambda\int_0^t dt_1\,\phi^{p-1}(t-t_1)\dot{\phi}(t_1)=0$$

Coincide for p=3 !

The spherical p-spin interaction spin-glass model

The dynamics

Z. Phys. B 92, 257–271 (1993)

A. Crisanti[1], H. Horner[2], H.-J. Sommers[3]

Exact dynamic equations

$$H = -\sum_{1\le i_1<\cdots<i_p\le N} J_{i_1\ldots i_p}\,\sigma_{i_1}\cdots\sigma_{i_p} - h\sum_{i=1}^{N}\sigma_i$$

$$\sum_{i=1}^{N}\sigma_i^2 = N\ .$$

$$\Gamma_0^{-1}\partial_t\sigma_i(t) = -r\,\sigma_i(t) - \frac{\partial\beta H}{\partial\sigma_i(t)} + \xi_i(t)$$

$$C(t,t') = \overline{\langle\sigma_i(t)\,\sigma_i(t')\rangle}$$

$$G(t,t') = \frac{\delta\,\overline{\langle\sigma_i(t)\rangle}}{\delta\beta h_i(t')}$$

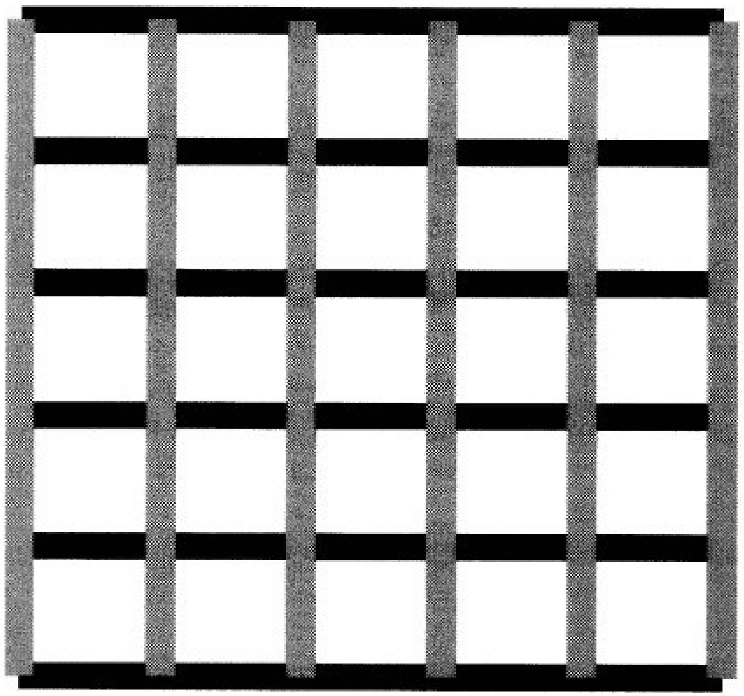
spherical constraint $\overline{\langle\sigma_i^2(t)\rangle} = C(t,t) = 1$

$$(\partial_t + r(t))\,G(t,t') - (p-1)\int_{t'}^{t} \mathrm{d}t_1\,\sqrt{\mu(t)\mu(t_1)} \times G(t,t_1)C^{p-2}(t,t_1)G(t_1,t') = \delta(t-t')$$

$$C(t,t') = \int \mathrm{d}t_1\,\mathrm{d}t_2\,G(t,t_1)\,G(t',t_2)[2\delta(t_1-t_2) + b(t_1)b(t_2) + \sqrt{\mu(t_1)\mu(t_2)}\,C^{p-1}(t_1,t_2)]\ .$$

Possible Glassiness in a Periodic Long-Range Josephson Array

P. Chandra, L. Ioffe, D. Sherrington, Phys. Rev. Lett. **75**, 713 (1995)



$$\mathcal{H} = -\sum_{m,n}^{2N} s_m^* J_{mn} s_n$$

$$s_m = e^{i\phi_m}$$

$$\hat{J} = \begin{pmatrix} 0 & \hat{J} \\ \hat{J}^\dagger & 0 \end{pmatrix}$$

$$J_{jk} = (J_0/\sqrt{N}) \exp(2\pi i \alpha jk/N) \text{ and } 1 \le (j,k) \le N$$

$$\alpha = NHl^2/\phi_0$$

$$\lim_{N\to\infty} \sum_{j_1, i_2 \ldots j_n} J_{i_1 j_1} J_{j_1 i_2} \ldots J_{j_n i_1} = J_0^{2n} \left(\frac{1}{\alpha}\right)^{n-1}$$

$$F\{m_k\} = -\sum_{kj} m_k^* J_{kj} m_j - T\sum_k S(m_k) + F_O\{m_k\}$$

$$- \sum_k (h_k m_k^* + h_k^* m_k),$$

$$S(m) = S_0 - |m|^2 - \frac{1}{4}|m|^4 + O(|m|^6),$$

$$\hat{J} = \begin{pmatrix} 0 & \hat{J} \\ \hat{J}^\dagger & 0 \end{pmatrix} \qquad J_{jk} = (J_0/\sqrt{N})\exp(2\pi i\alpha jk/N)$$

At high temperatures $F_O\{m_k\} \tilde{\sim} 1/T$ small

diagonalized by a Fourier transformation to

$$(\hat{J}\hat{J}^\dagger)_{jk} = J_0^2 e^{i\pi\alpha(j-k)} \frac{\sin[\pi\alpha(j-k)]}{\pi\alpha(j-k)}, \Rightarrow (\hat{J}\hat{J}^\dagger)_p = \frac{J_0^2}{\alpha}\theta(p)\theta(2\pi\alpha - p)$$

$$(\hat{J}\hat{J}^{\dagger})_p = \frac{J_0^2}{\alpha}\,\theta(p)\theta(2\pi\alpha - p)\,.$$

Therefore the largest eigenvalue of $\hat{J}$ is $\lambda_{\max} = J_0/\sqrt{\alpha}$; the corresponding eigenfunctions associated with one set of parallel wires are plane waves with momenta in the interval $0 \le p \le 2\pi\alpha$ so that the degeneracy of this eigenvalue is αN [8].

Therefore, in the absence of feedback effects, the

transition occurs at $T_{c0} = J_0/\sqrt{\alpha}$

We use the locator expansion [11] to determine the leading order contributions in m_i to $F_O\{m_k\}$, it is based on the expression for susceptibility

$$\hat{\chi} = \frac{1}{\hat{\mathcal{A}} - \hat{\mathcal{J}}}\,,$$

$$\frac{\partial^2 F}{\partial m_k^* \partial m_j} = (\hat{\chi}^{-1})_{kj}$$

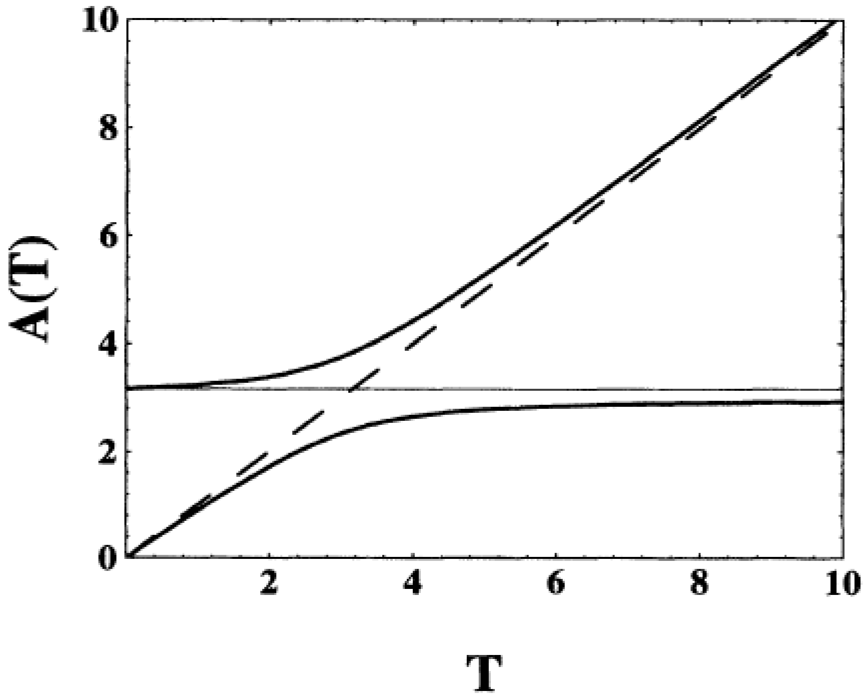
$$\chi_{jj} = (1 - |m_j|^2)/T$$

$$\left(\frac{1}{\hat{A} - \hat{J}\hat{A}^{-1}\hat{J}^{\dagger}}\right)_{jj} = \frac{1 - |m_j|^2}{T}, \quad \hat{\mathcal{A}} = \begin{pmatrix} \hat{A} & 0 \\ 0 & \hat{A}^{\dagger} \end{pmatrix}$$

Consider first m=0 limit of this Equation

Solution in 2nd order of F(m):

$$\frac{\alpha}{A - J_0^2/A\alpha} + \frac{1 - \alpha}{A} = \frac{1}{T}$$



1^{st} – order phase transition is expected and this is demonstrated for $\alpha = 1$

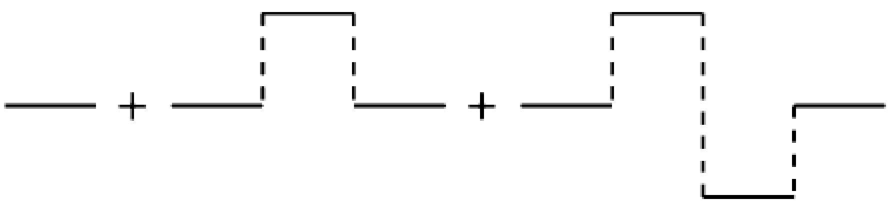
Dynamic treatment is More transparent

Glass Formation in a Periodic Long-Range Josephson Array

P. Chandra,[1] M. V. Feigelman,[2] and L. B. Ioffe[2,3] Phys. Rev. Lett. **76**, 4805 (1996)

$$\tau_b \dot{S}_i = -\frac{1}{T}\frac{\partial(\mathcal{H} + V)}{\partial S_i^*} + \zeta_i, \qquad \langle \zeta_i(t)\zeta_j(t')\rangle = 2\tau_b\delta(t-t')\delta_{ij}$$

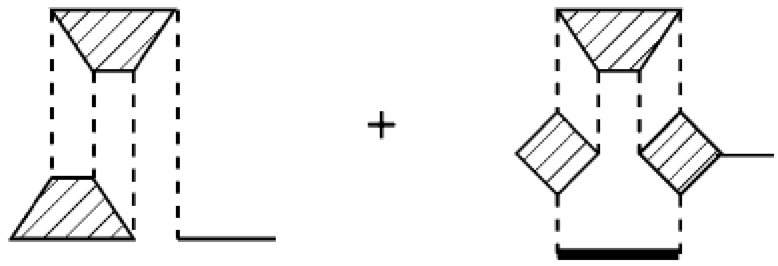
$$\hat{G}_\omega = \frac{1}{\tilde{G}_\omega^{-1} - \beta^2 (J^\dagger J)\tilde{G}_\omega}$$



Leading diagrams for G

$$G_\omega(p) = \frac{\theta(\alpha\pi - |p|)}{\tilde{G}_\omega^{-1} - (\beta J_0)^2 \tilde{G}_\omega/\alpha} + \frac{\theta(|p| - \alpha\pi)}{\tilde{G}_\omega^{-1}}.$$

The static limit ($\omega = 0$) of $T\tilde{G}_\omega^{-1}$ coincides with the locator, $A(T)$, discussed previously [3]; in the absence of Onsager feedback terms, $\tilde{G}_0^{-1} = 1$. Therefore we see in

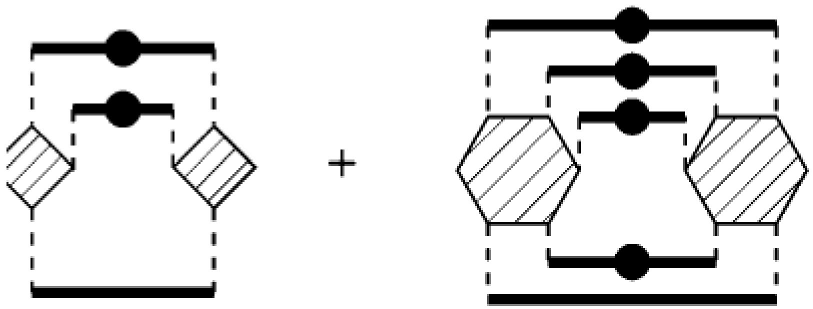


Subleading diagrams for G

$$\tilde{G}_b^{-1}(\omega) = \beta A(T) - i\omega\tau_b$$

$$a = \beta(A - J_0^2/\alpha A) \approx 2\Theta = (T - T_0)/T_0$$

Retardation in the Onsager reaction terms:



$$\tilde{G}(\omega) = \left[\tilde{G}_b(\omega)^{-1} - \Sigma(\omega)\right]^{-1}$$

$$\Sigma_\omega = \frac{3}{2}\Gamma^2 \int \hat{D}^2(t)\hat{G}(t)\,(e^{i\omega t} - 1)dt$$

Since we would like to detect a dynamical instability, we only consider the long-time behavior of the response function, i.e.,

$$\hat{G}(t) = \alpha \int \frac{e^{-i\omega t}}{a - 2(\Sigma_\omega + i\omega\tau_b)}\left(\frac{d\omega}{2\pi}\right)$$

a closed form equation

$$\int_0^\infty \hat{D}(t)e^{i\omega t}dt = \left(\frac{\alpha}{a}\right)\frac{2\tau_b + \int_0^\infty \hat{D}^3(t)e^{i\omega t}dt}{a - i\omega[2\tau_b + \int_0^\infty \hat{D}^3(t)e^{i\omega t}dt]}$$

p=4 SG model

which results in the asymptotic behavior

$$\hat{G}(t) = \frac{2\alpha}{3a\tau_R}e^{-t/\tau_R}, \qquad \hat{D}(t) = \frac{2\alpha}{3a}e^{-t/\tau_R}$$

$$\tau_R = \frac{2\mu\tau_b}{a_c}\left(\frac{a_c}{a - a_c}\right)^\nu$$

$$\nu = 1.765;$$

the long-time part of $D(t)$ shown in (15) becomes constant, $q = \sqrt{2}(\alpha/3)^{1/4}$, indicating a jump in the Edwards-Anderson order.

$$a_c = \sqrt{2}(\alpha/3)^{3/4}.$$

$$\hat{D}(t) = \frac{2\alpha}{3a} e^{-t/\tau_R}$$

Major conclusions:

1. 1-step RSB transition is preceded by dynamic transition at higher T_g
2. A number of different models lead to just the same critical dynamic behavior that is model- independent