

Spin glass dynamics at $T > T_g$

1. Edwards-Anderson Spin Glass model
 - a) Energy functional and dynamic equations with noise
 - b) Martin-Siggia-Rose generating functional
 - c) Mean-field approximation and equations for the response and correlation functions
 - d) Solution near T_g : singularity in $dG/d\omega$
2. Dynamics of 3D spin glasses:
real experiments and Monte-Carlo studies
3. Model of diffusion on a percolation network and on diluted hypercube

Relaxational dynamics of the Edwards-Anderson model and the mean-field theory of spin-glasses

H. Sompolinsky & Annette Zippelius

Phys. Rev. B 25, 6860 (1982)

Order parameter: $q_{\text{EA}} = \lim_{t \rightarrow \infty} [\langle S_i(0)S_i(t) \rangle]_J$.

The EA Hamiltonian is $H = - \sum_{\langle ij \rangle} J_{ij} S_i S_j$, spin variables S_i take the values ± 1

Distribution of random J_{ij} $P(J_{ij}) = (2\pi z / \tilde{J}^2)^{-1/2} \times \exp[-z (J_{ij} - J_0/z)^2 / 2\tilde{J}^2]$

Z is the number of nearest neighbours

We consider here a soft-spin version of the EA model defined by

$$\beta H = \frac{1}{2} \sum_{\langle ij \rangle} (r_0 \delta_{ij} - 2\beta J_{ij}) \sigma_i \sigma_j + u \sum_i \sigma_i^4 + \sum_i h_i \sigma_i, \quad \beta = 1/T .$$

To study the relaxational dynamics of spin glasses, we propose a simple phenomenological Langevin equation,

$$\Gamma_0^{-1} \partial_t \sigma_i(t) = - \frac{\delta(\beta H)}{\delta \sigma_i(t)} + \xi_i(t) \quad \langle \xi_i(t) \xi_j(t') \rangle = \frac{2}{\Gamma_0} \delta_{ij} \delta(t - t') ,$$

$$= \sum_j (r_0 \delta_{ij} - \beta J_{ij}) \sigma_j(t) + 4u \sigma_i^3(t) + h_i(t) + \xi_i(t) .$$

Objects of interest: pair spin correlation function

$$C_{ij}(t - t') = \langle \sigma_i(t) \sigma_j(t') \rangle$$

and the linear-response function

$$G_{ij}(t - t') = \frac{\partial \langle \sigma_i(t) \rangle}{\partial h_j(t')}, \quad t > t'$$

The FDT reads in the present context

$$C_{ij}(\omega) = \frac{2}{\omega} \text{Im} G_{ij}(\omega)$$

and

$$C_{ij}(t=0) = G_{ij}(\omega=0)$$

$$\text{Re} G_{ij}(\omega) = - \int \frac{d\omega'}{\pi} \frac{\text{Im} G_{ij}(\omega')}{\omega - \omega'}$$

Dynamic generating functional:

P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1978).

C. De Dominicis, J. Phys. (Paris) C **1**, 247 (1976); C. De Dominicis and L. Peliti, Phys. Rev. B **18**, 353 (1978).

$$\mathbf{Z}\{J_{ij}, l_i, \hat{l}_i\} = \int D\sigma D\hat{\sigma} \exp \left[\int dt l_i(t)\sigma_i(t) + i\hat{l}_i(t)\hat{\sigma}_i(t) + L\{\sigma, \hat{\sigma}\} \right]$$

$$L\{\sigma, \hat{\sigma}\} = \int dt \sum_i i\hat{\sigma}_i(t) \left[-\Gamma_0^{-1} \partial_t \sigma_i(t) - r_0 \sigma_i(t) + \beta \sum_j J_{ij} \sigma_j(t) - 4u \sigma_i^3(t) - h_i(t) + \Gamma_0^{-1} i\hat{\sigma}_i(t) \right] + V\{\sigma\}$$

The term V , which arises from the functional and ensures the proper normalization of \mathbf{Z} , is given by^{32,33}

$$\mathbf{Z}\{J_{ij}, l_i = \hat{l}_i = 0\} = 1$$

$$V = -\frac{1}{2} \int dt \sum_i \frac{\delta^2(\beta H)}{\delta \sigma_i^2} = - \int dt \sum_i \left[\frac{1}{2} r_0 + 6u \sigma_i^2(t) \right]$$

$$\left. \frac{\delta^n \delta^m \ln \mathbf{Z}}{\delta \hat{l}_1(t_1) \cdots \delta l_m(t_m)} \right|_{l_i = \hat{l}_i = 0} = \langle i\hat{\sigma}_1(t_1) \cdots \sigma_m(t_m) \rangle_c$$

Response function: $\langle i\hat{\sigma}_j(t')\sigma_i(t) \rangle = G_{ij}(t-t') \quad (t > t')$

Averaging over J_{ij} is possible since $Z = 1$

$$[Z]_J \equiv \int \prod dJ_{ij} P(J_{ij}) Z\{J_{ij}\} = \int D\sigma D\hat{\sigma} \exp \left[L_0\{\sigma, \hat{\sigma}\} + \frac{\beta J_0}{z} \sum_{\langle ij \rangle} \int dt i\hat{\sigma}_i(t)\sigma_j(t) \right. \\ \left. + 2\frac{\beta^2 \tilde{J}^2}{z} \sum_{\langle ij \rangle} \int dt dt' [i\hat{\sigma}_i(t)\sigma_j(t')i\hat{\sigma}_i(t')\sigma_j(t) + i\hat{\sigma}_i(t)\sigma_j(t)i\hat{\sigma}_j(t')\sigma_i(t')] \right]$$

here

we use the property $J_{ij} = J_{ji}$.

$$L_0\{\sigma, \hat{\sigma}\} = \int dt \sum_i [i\hat{\sigma}_i(-\Gamma_0^{-1}\partial_t\sigma_i - r_0\sigma_i - 4u\sigma_i^3 - h_i + i\Gamma_0^{-1}\hat{\sigma}_i) + V\{\sigma\} + i\hat{l}_i\hat{\sigma}_i + l_i\sigma_i]$$

Decoupling of the 4-th order terms:

$$[Z]_J = \int \prod_{\alpha}^4 DQ_{\alpha}^i(t, t') \exp \left[-\frac{z}{\beta^2 \tilde{J}^2} \int dt dt' \sum_{i,j} (K^{-1})_{ij} [Q_1^i(t, t')Q_2^j(t, t') + Q_3^i(t, t')Q_4^j(t, t')] \right. \\ \left. + \ln \int D\sigma D\hat{\sigma} \exp L\{\sigma, \hat{\sigma}, Q_{\alpha}\} \right],$$

where K is the short-range matrix ($K_{ij} = 1$ if i, j are nearest neighbors and zero otherwise), and

$$L\{\sigma, \hat{\sigma}, Q_{\alpha}\} = L_0\{\sigma, \hat{\sigma}\} + \frac{1}{2} \int dt dt' \sum_i [Q_1^i(t, t')i\hat{\sigma}_i(t)i\hat{\sigma}_i(t') + Q_2^i(t, t')\sigma_i(t)\sigma_i(t')$$

$$+ Q_3^i(t, t')i\hat{\sigma}_i(t)\sigma_i(t') + Q_4^i(t, t')i\hat{\sigma}_i(t')\sigma_i(t)].$$

(We have assumed $J_0 = 0$.)

Mean-field limit: $z = N$ (Sherrington-Kirkpatrick model)

One step back:

$$N^{-2} \sum_{i \neq j} i \hat{\sigma}_i(t) i \hat{\sigma}_i(t') \sigma_j(t) \sigma_j(t') = \frac{1}{4} N^{-2} \left[\sum_i i \hat{\sigma}_i(t) i \hat{\sigma}_i(t') + \sigma_i(t) \sigma_i(t') \right]^2 - \frac{1}{4} N^{-2} \left[\sum_i i \hat{\sigma}_i(t) i \hat{\sigma}_i(t') - \sigma_i(t) \sigma_i(t') \right]^2$$

$$[Z]_J = \int \prod_{\alpha=1}^4 DQ_{\alpha}(t, t') \exp \left[-\frac{N}{\beta^2 \tilde{J}^2} \int dt dt' [Q_1(t, t') Q_2(t, t') + Q_3(t, t') Q_4(t, t')] \right. \\ \left. + \ln \int D\sigma D\hat{\sigma} \exp L\{\sigma, \hat{\sigma}, Q_{\alpha}\} \right],$$

$$L\{\sigma, \hat{\sigma}, Q_{\alpha}\} = L_0\{\sigma, \hat{\sigma}\} + \frac{1}{2} \int dt dt' \sum_i [Q_1(t, t') i \hat{\sigma}_i(t) i \hat{\sigma}_i(t') + Q_2(t, t') \sigma_i(t) \sigma_i(t') \\ + Q_3(t, t') i \hat{\sigma}_i(t) \sigma_i(t') + Q_4(t, t') i \hat{\sigma}_i(t') \sigma_i(t)] + O(1).$$

Now $Q_i(t, t')$ ($i=1-4$) are global variables
(no space-dependence)

$$Q_2^0 = \langle \hat{\sigma} \hat{\sigma} \rangle = 0$$

a vertex $Q_2^0(t, t') \sigma(t) \sigma(t')$ will lead
to violation of causality, namely, will yield nonzero
contributions to $\langle i \hat{\sigma}(t) \sigma(t') \rangle$ with $t > t'$.

$$Q_1^0(t, t') = \frac{\beta^2 \tilde{J}^2}{2N} \sum_i \langle \sigma_i(t) \sigma_i(t') \rangle,$$

$$Q_2^0(t, t') = \frac{\beta^2 \tilde{J}^2}{2N} \sum_i \langle i \hat{\sigma}_i(t) i \hat{\sigma}_i(t') \rangle$$

$$Q_3^0(t, t') = \frac{\beta^2 \tilde{J}^2}{2N} \sum_i \langle i \hat{\sigma}_i(t') \sigma_i(t) \rangle,$$

$$Q_4^0(t, t') = \frac{\beta^2 \tilde{J}^2}{2N} \sum_i \langle i \hat{\sigma}_i(t) \sigma_i(t') \rangle.$$

$$L\{\sigma_i \hat{\sigma}_i\} = L_0\{\sigma_i, \hat{\sigma}_i\} + \frac{\beta^2 \tilde{J}^2}{2} \int dt dt' [C(t-t') i \hat{\sigma}_i(t) i \hat{\sigma}_i(t') + 2G(t-t') i \hat{\sigma}_i(t) \sigma_i(t')],$$

$$C(t-t') \equiv [\langle \sigma_i(t) \sigma_i(t') \rangle]_J,$$

$$G(t-t') \equiv [\langle i \hat{\sigma}_i(t') \sigma_i(t) \rangle]_J$$

$$L_0\{\sigma, \hat{\sigma}\} = \int dt \sum_i [i \hat{\sigma}_i (-\Gamma_0^{-1} \partial_t \sigma_i - r_0 \sigma_i - 4u \sigma_i^3 - h_i + i \Gamma_0^{-1} \hat{\sigma}_i) + V\{\sigma\} + i l_i \hat{\sigma}_i + l_i \sigma_i]$$

The new effective bare propagator is

$$G_0^{-1}(\omega) = r_0 - i\omega \Gamma_0^{-1} - \beta^2 \tilde{J}^2 G(\omega)$$

and the effective noise ϕ is a Gaussian random variable with width

$$\langle \phi_i(\omega) \phi_i(\omega') \rangle = [2\Gamma_0^{-1} + \beta^2 \tilde{J}^2 C(\omega)] \delta(\omega + \omega')$$

$$\sigma_i(\omega) = G_0(\omega) [\phi_i(\omega) + h_i(\omega)]$$

$$-4u G_0(\omega) \int d\omega_1 d\omega_2 \sigma_i(\omega_1) \sigma_i(\omega_2)$$

$$\times \sigma_i(\omega - \omega_1 - \omega_2).$$

DYNAMICS FOR $T \geq T_c$

Key quantity: $\Gamma^{-1}(\omega) = i \frac{\partial G^{-1}(\omega)}{\partial \omega}$ Effective kinetic coefficient

The dynamic-response function obeys a Dyson equation

$$G^{-1}(\omega) = G_0^{-1}(\omega) + \Sigma(\omega),$$

$$G_0^{-1}(\omega) = r_0 - i\omega\Gamma_0^{-1} - \beta^2 \tilde{J}^2 G(\omega)$$

the nonlinear coupling u .

$$\Gamma^{-1}(\omega) = \frac{\Gamma_0^{-1} + i \frac{\partial \Sigma}{\partial \omega}}{1 - \beta^2 \tilde{J}^2 G^2(\omega)}$$

random exchange

assume that

$$\frac{\partial}{\partial \omega} \text{Im} \Sigma(0) \equiv \frac{\partial}{\partial \omega} \text{Im} \Sigma \Big|_{\omega=0}$$

is finite

(it will be proven later)

Then
$$\Gamma^{-1}(\omega=0) = \frac{\Gamma_0^{-1} + \text{Im} \frac{\partial \Sigma}{\partial \omega}(0)}{1 - \beta^2 \tilde{J}^2 G^2(0)}$$

diverges at $T_c = \tilde{J}G(0)$

$$\Gamma^{-1}(\omega=0) = \frac{\Gamma_0^{-1} + \text{Im} \frac{\partial \Sigma}{\partial \omega}(0)}{1 - \beta^2 \tilde{J}^2 G^2(0)}$$

As T approaches T_c , $\Gamma^{-1}(\omega=0)$ shows critical slowing down,

$$\Gamma^{-1}(0) \propto \tau^{-1} \text{ where } \tau = \frac{T}{T_c} - 1$$

Now recall FDT: $C_{ij}(t=0) = G_{ij}(\omega=0)$

For $i=j$ we use obvious relation for Ising spins: $C(t=0) = 1$
and conclude that $G(\omega=0)=1$ leading to

$$T_c = \tilde{J}$$

Now we can solve for the small difference $g(\omega) = G(\omega) - 1 \ll 1$

$$g^2(\omega) + 2\tau g(\omega) + i\omega \tilde{\Gamma}_0^{-1} = 0$$

$$g(\omega) = -\tau + \sqrt{\tau^2 - i\omega \tilde{\Gamma}_0^{-1}}$$

$$\frac{1}{\tilde{\Gamma}_0} = \frac{1}{\Gamma_0} + \Im \frac{\partial \Sigma(\omega)}{\partial \omega}(0)$$

$$G(t) = \frac{1}{2} \left(\frac{\Gamma}{\pi} \right)^{1/2} \frac{1}{t^{3/2}} \exp\left(-\frac{t}{t_0}\right) \theta(t)$$

where $t_0 = \Gamma \tau^{-2}$

Self-energy $\Sigma(\omega)$

- 1) We do not need calculation of $\Sigma(0)$ in order to find T_c - rather, we can use FDT and the condition $S^2=1$ to find $\Sigma(0)$:

$$1 + \beta^2 J^2 = r_0 + \Sigma(0)$$

- 2) We should check the assumption of finite $\frac{\partial}{\partial \omega} \text{Im } \Sigma(0) \equiv \frac{\partial}{\partial \omega} \text{Im } \Sigma \Big|_{\omega=0}$

$$\frac{\partial \Sigma(0)}{\partial \omega} = 2(12u)^2 \int \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} C(\omega_1) C(\omega_1 - \omega_2) \frac{\partial}{\partial \omega_2} \text{Im } G(\omega_2)$$

At T_c , $G(\omega) \sim \omega^{1/2}$ and $C(\omega) \sim \omega^{-1/2}$

Thus the above integral is indeed finite

Dynamics of 3D Spin Glass

Dynamics of three-dimensional Ising spin glasses in thermal equilibrium Andrew T. Ogielski

Phys Rev B 32, 7384 (1985)

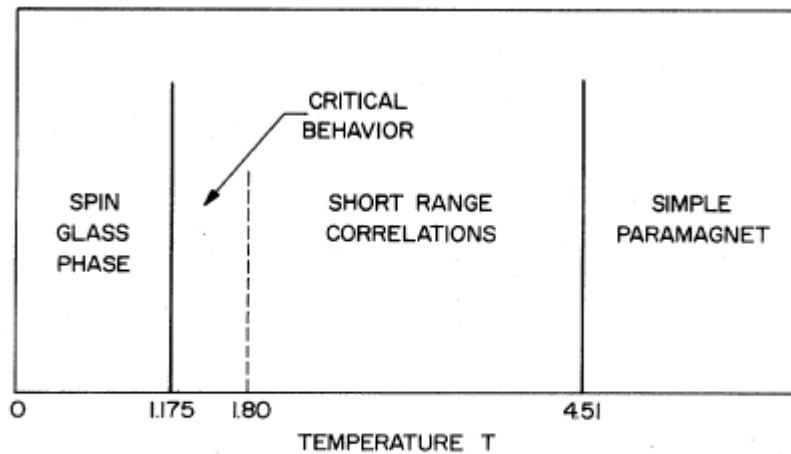


FIG. 1. Graphical representation of distinct temperature regimes observed in the three-dimensional Ising spin glass.

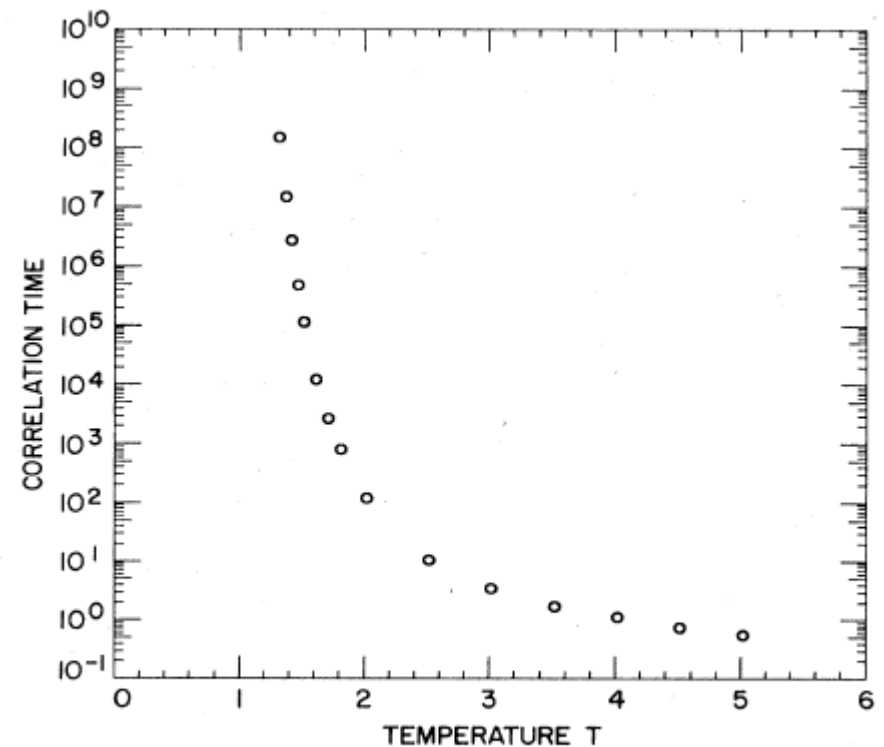


FIG. 3. Temperature dependence of the correlation time τ ,

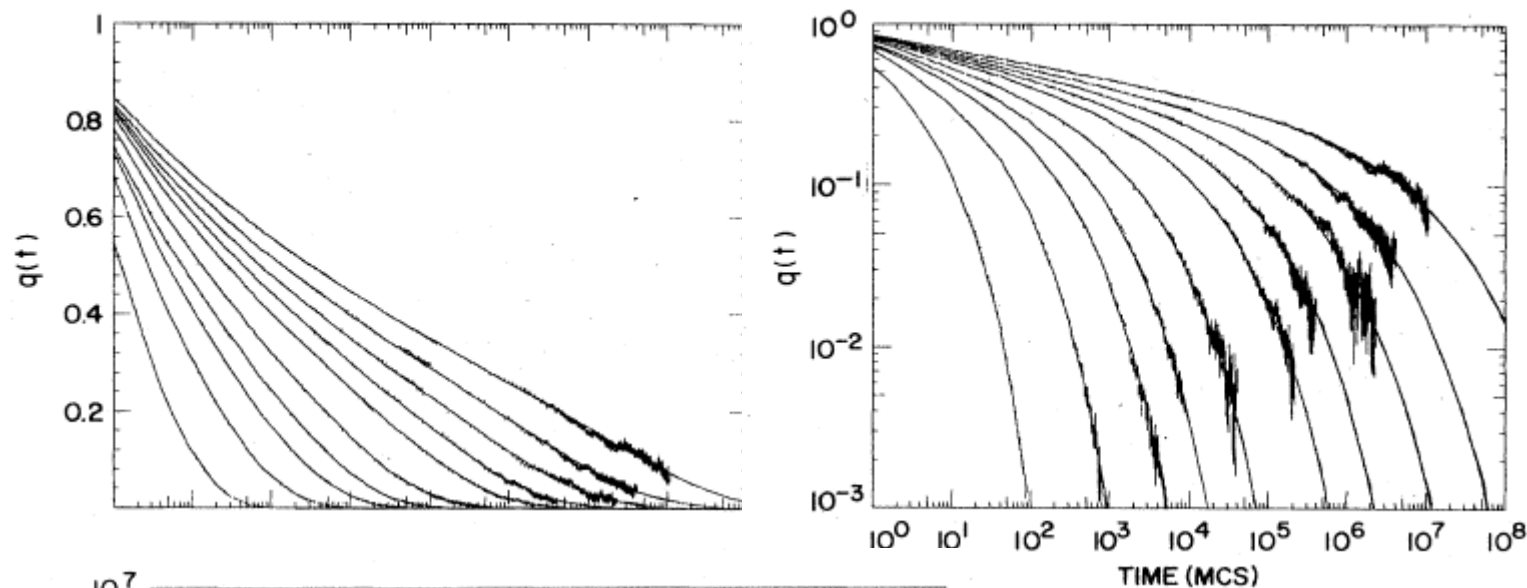


FIG. 7. Dynamic correlation function $q(t)$. Short-time behavior is well seen in (top), long-time behavior can be seen in (bottom). Data points are shown with error bars. From left to right, the temperatures are 1.70, 1.60, 1.50, 1.45, 1.40, 1.35, and 1.30.

$$q(t) = c \frac{e^{-\omega t^\beta}}{t^x}$$

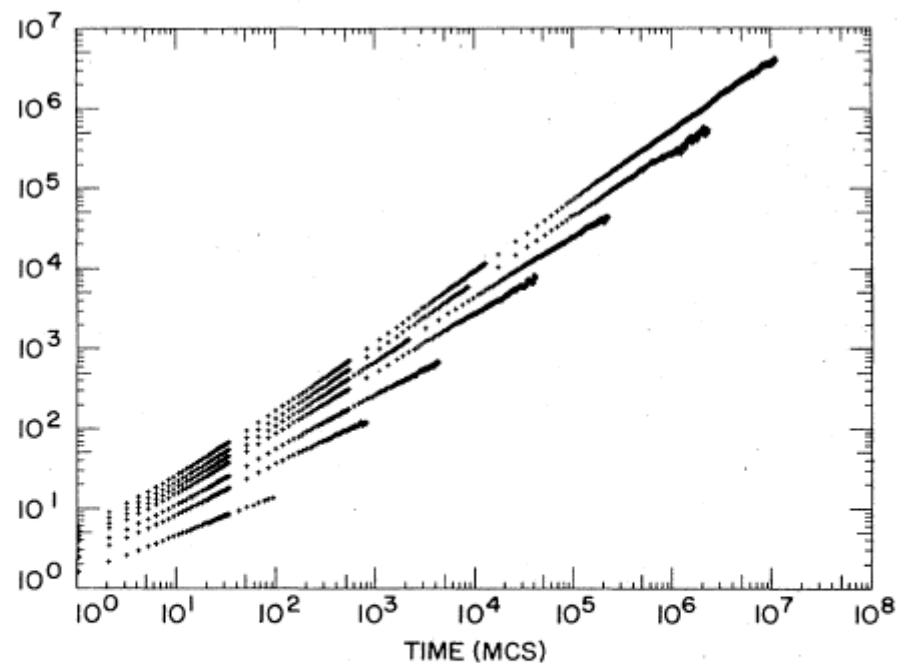


FIG. 10. Correlation functions $q(t)$ shown before in Fig. 7 are converted into a plot of $-t/\ln q(t)$ vs t on the log-log scale. Data points would appear as horizontal lines if $q(t) \sim \exp(-t/\tau)$; this is not seen here. Asymptotically straight lines seen in the graph indicate the Kohlrausch behavior $\exp(-\omega t^\beta)$ instead, with $\beta < 1$. The temperatures are $t=2.50$ (bottom), 2.00, 1.80, 1.60, 1.50, 1.40, and 1.30 (top).

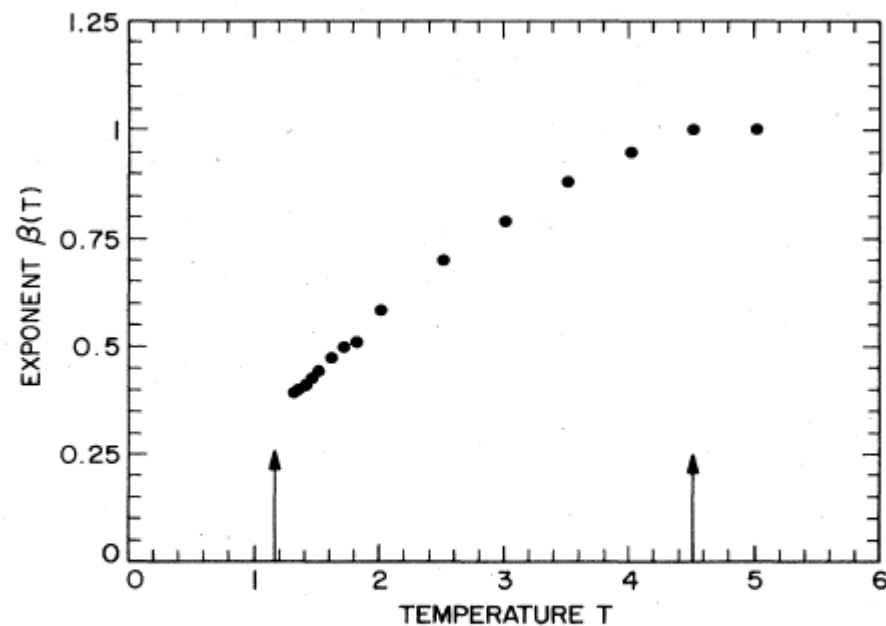


FIG. 11. Temperature dependence of the exponent β defined in Eq. (13). The arrows mark the spin-glass transition temperature T_g and the Curie point T_c of nonrandom Ising model.

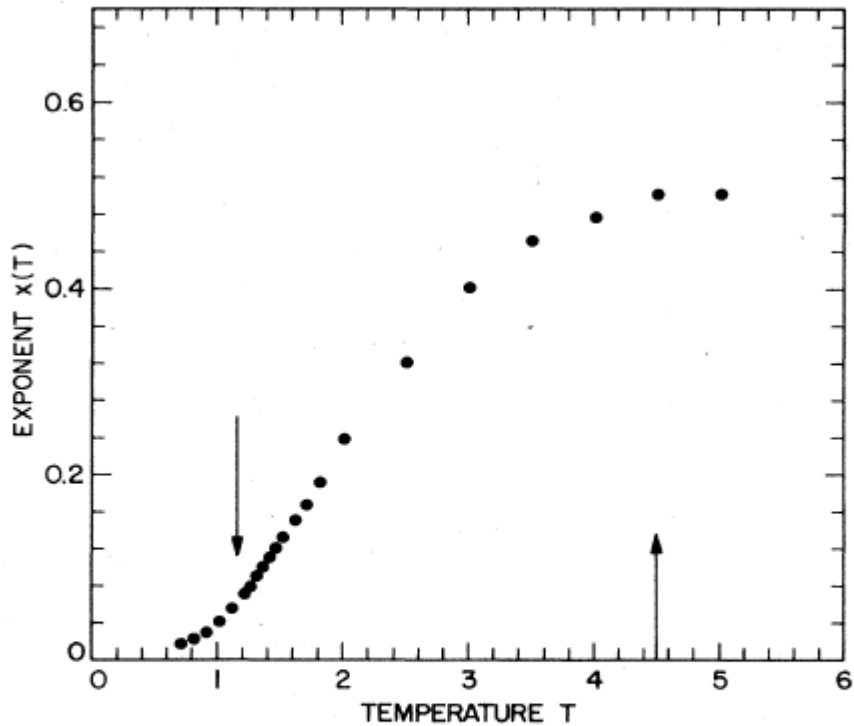


FIG. 12. Temperature dependence of the exponent x defined by Eq. (13) above T_g , and determined from the algebraic decay of $q(t)$ around and below T_g . The arrows mark T_g and T_c as in Fig. 11.

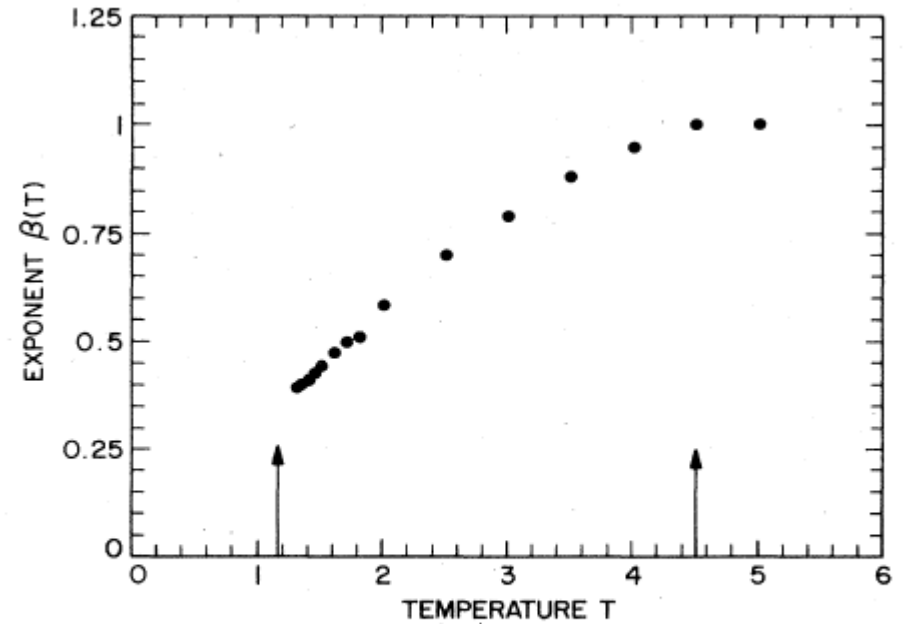


FIG. 11. Temperature dependence of the exponent β defined in Eq. (13). The arrows mark the spin-glass transition temperature T_g and the Curie point T_c of nonrandom Ising model.

$$q(t) \approx t^{-x} Q(t/\tau),$$

$$x = \frac{1}{2} \left[\frac{d-2+\eta}{z} \right],$$

$$\tau \approx (T - T_g)^{-z\nu}.$$

$$T_g = 1.175 \pm 0.025$$

$$\nu = 1.3 \pm 0.1,$$

$$z = 6.0 \pm 0.8$$

$$\tau = \int_0^\infty dt t q(t) / \int_0^\infty dt q(t)$$

Comparison with MF model

MF model

- 1) exponential relaxation above T_g
- 2) exponent $z\nu = 2$
- 3) exponent $x = 1/2$

3D Ising SG (Ogielski MC)

- 1) stretched exponential, $1/3 < \beta < 1$
- 2) exponent $z\nu \approx 8$
- 3) exponent $x < 0.1$ at T_g

Real experiments

COMBINED THREE-DIMENSIONAL POLARIZATION ANALYSIS AND SPIN ECHO STUDY OF SPIN GLASS DYNAMICS

Journal of Magnetism and Magnetic Materials 14 (1979) 211–213

F. MEZEI and A.P. MURANI

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spin correlation function $S(\kappa, t)$ for a Cu–Mn spin glass alloy a single scan in the range $10^{-12} < t < 10^{-9}$ s.
neutron spin echo and polarization analysis

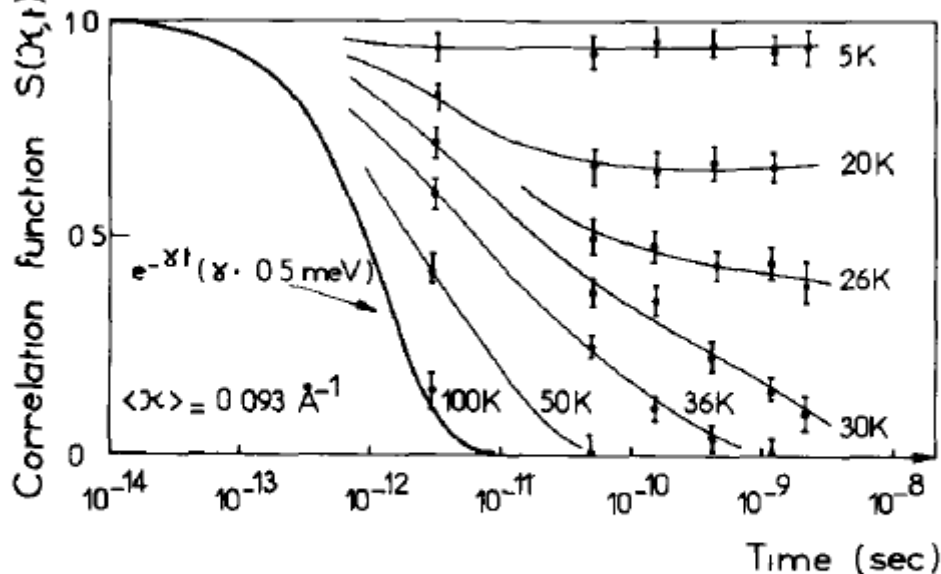


Fig. 3. The measured time dependent spin correlation function for Cu–5 at% Mn at various temperatures. The thick line corresponds to the simple exponential decay. The thin lines are guides to the eye only.

Again looks like stretched exponential in a broad range of T

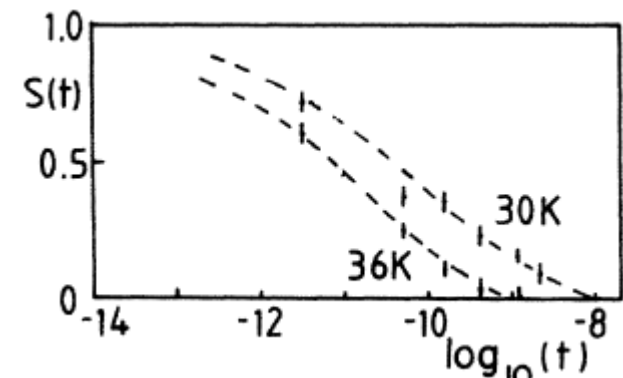


FIG. 3. Neutron spin-echo data from Ref. 11 on Cu–5 at.% Mn at two temperatures just above $T_g = 28.5$ K. The curves are stretched exponential fits with $\beta = 0.33$ and 0.37 .

Dynamic scaling in the $\text{Eu}_{0.4}\text{Sr}_{0.6}\text{S}$ spin-glass

N. Bontemps and J. Rajchenbach R. V. Chamberlin and R. Orbach

Phys Rev B 30, 6514 (1984)

frequency range (10^{-2} – 10^5 Hz)

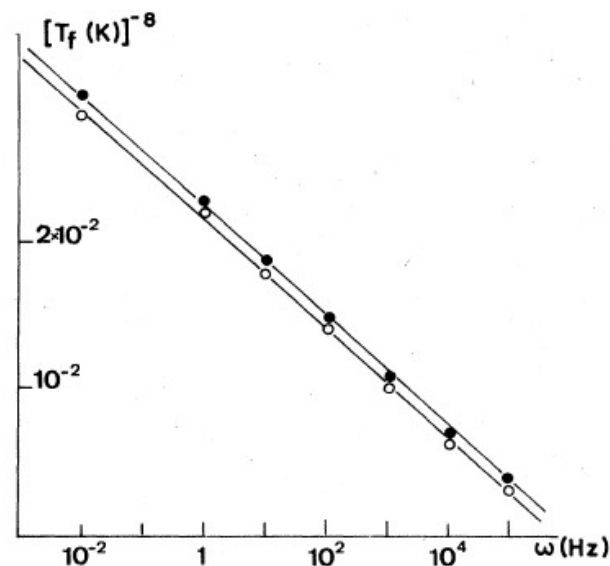
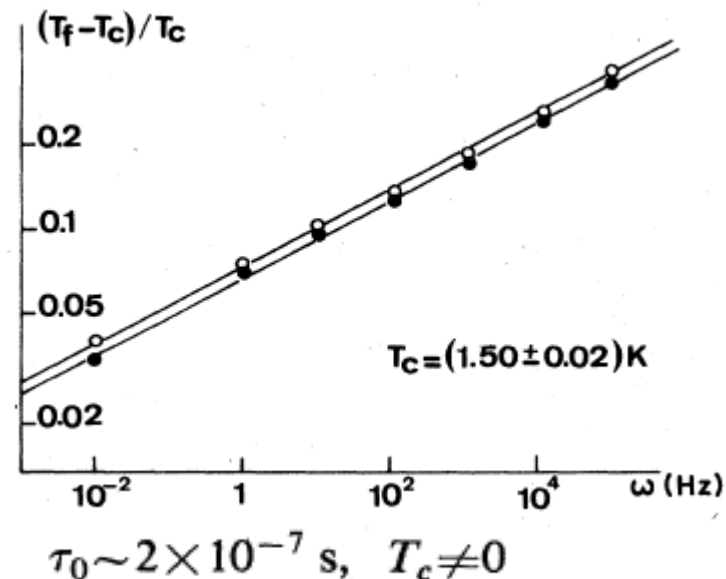
In order to cover the largest range of frequencies, we have used two different techniques for measuring χ'' and χ' on a single sample. The high-frequency regime (10 – 10^5 Hz) has been investigated at ESPCI by measuring the magnetization (or susceptibility) using a Faraday rotation method.^{7,12} The low-frequency regime (10^{-2} – 10 Hz) has been investigated at UCLA using a SQUID magnetometer.¹³ For purely technical reasons,

$$\tau/\tau_0 \propto \xi_{\text{EA}}^z \propto [(T - T_c)/T_c]^{-z\nu}. \quad (5)$$

Equation (5) defines a zero-field freezing temperature associated with the frequency ω taking $\tau \sim 1/\omega$:

$$\omega/\omega_0 \propto [(T - T_c)/T_c]^{z\nu}. \quad z\nu = 7.2 \pm 0.5 \quad (6)$$

Another fit: $T_c = 0$ $z\nu = 8 \pm 0.5$ $\tau_0 \sim 10^{-5}$ s



Model calculations:

Random walks on a closed loop and spin glass relaxation

I. A. Campbell

J. Physique Lett. **46** (1985) L-1159 - L-1162

Random walks on a hypercube and spin glass relaxation

I A Campbell†, J M Flesselles‡, R Jullien† and R Botet† *J. Phys. C: Solid State Phys.* **20** (1987) L47-L51.

The calculations were done on hypercubes of dimension 10, 14, 16 and 17

$$\langle r^2(t) \rangle = \sum_{i=1}^d (x_i(t) - x_i(0))^2 \quad q(t) = \langle r_\infty^2 \rangle - \langle r^2(t) \rangle \propto \exp[-(t/\tau)^\beta] \quad 65\,536 \text{ vertices}$$

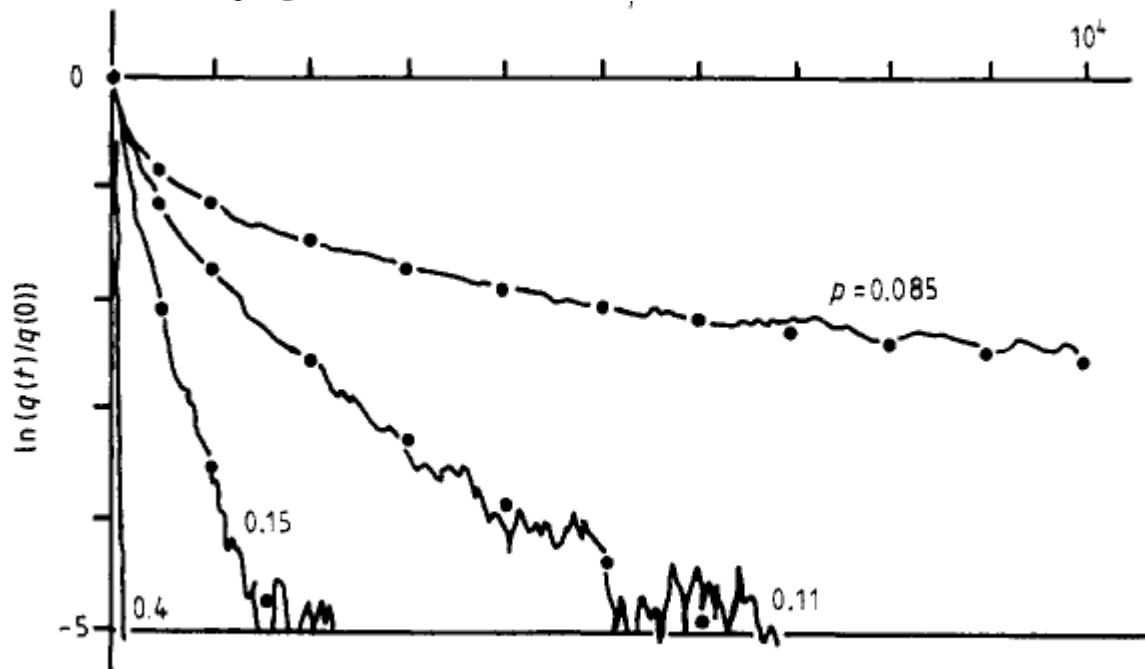


Figure 3. Selected results for $q(t)$ as a function of t at different concentrations p for $d = 16$. The points indicate best-fit curves of the form (1).

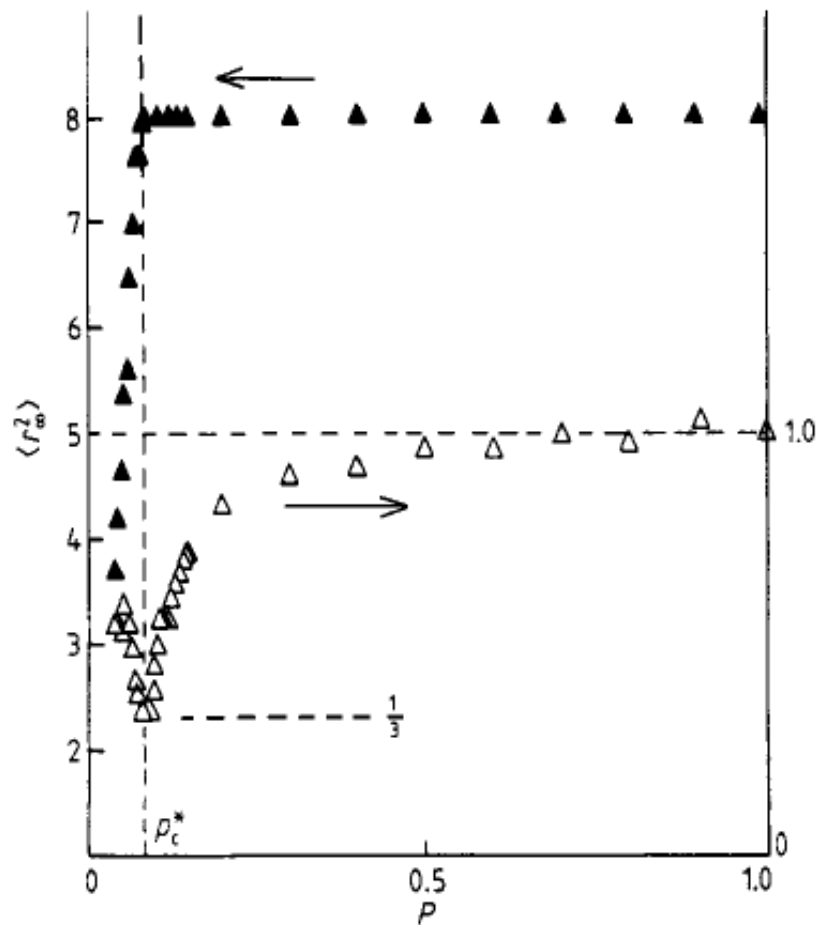


Figure 1. $\langle r_m^2 \rangle$ and β as functions of the concentration p for the 16-dimensional hypercube. The threshold concentration p_c^* is indicated by a broken line.

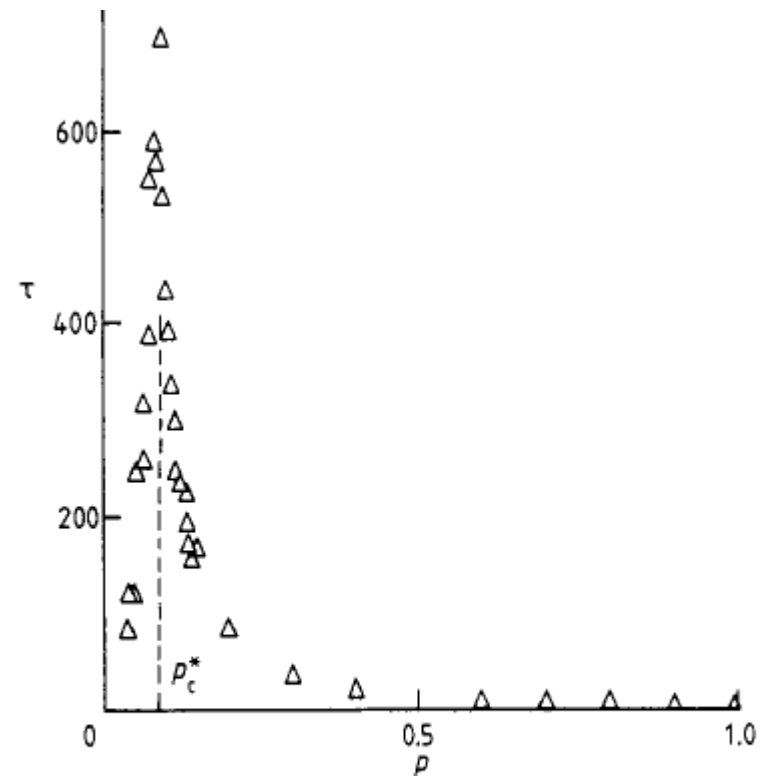


Figure 2. The relaxation rate parameter τ as a function of concentration p for the 16-dimensional hypercube. p_c^* is again indicated by a broken line.

d	p_c^*	$\beta(p_c^*)$	p_c'
10	0.16	0.44	0.148
14	0.095	0.38	0.085
16	0.085	0.34	0.073
17	0.075	0.335	0.069

Reasonable (and unsolved) theoretical model

$$H = - \sum_{\langle ij \rangle} J_{ij} S_i S_j ,$$

Sparse matrix model

Summation goes over all pairs (ij) but matrix J_{ij} is strongly diluted:
with a probability (1-p) the bond is erased, $p = Z/N \ll 1$

Static version of the same problem was studied (among others) in:

“Mean-field theory of Spin Glasses with finite Coordination number”
I.Kanter and H.Sompolinsky, Phys.Rev.Lett. **58**, 164 (1987)

Such a model contains strong statistical fluctuations (like real 3D SG)
but neglects thermodynamic fluctuations (long-wave-length modes)
Thus it can be useful for the description of the Griffiths phase in 3D
(but is not useful to study the critical exponents)