

One-Dimensional Spin Glass with Oscillating Long-Range Interaction

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and Chapter 3.2 in

SPIN GLASSES AND RELATED PROBLEMS

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$$V(\mathbf{p}) = \frac{V_0}{\left(\frac{p^2 - Q^2}{2Q\kappa}\right)^2 + 1}; \quad \kappa \ll Q.$$

$$V_1(x) = \kappa V_0 e^{-\kappa|x|} \cos Qx$$

$$V_3(x) = \frac{Q}{2\pi} \kappa V_0 e^{-\kappa x} \frac{\sin Qx}{x}$$

$$\kappa \ll c \ll Q.$$

chain of Ising spins $\sigma_i = \pm 1$ with the oscillating interaction:

$$H = \frac{1}{2} \sum_{ij} \sigma_i V_{ij} \sigma_j - \sum_i \sigma_i h_i$$

$$V_{ij} = \kappa e^{-\kappa|x_i - x_j|} \cos Q(x_i - x_j). \quad (1)$$

$$H = \frac{1}{2} \sum_{ij} \sigma_i V_{ij} \sigma_j - \sum_i \sigma_i h_i$$

$$V_{ij} = \kappa e^{-\kappa|x_i - x_j|} \cos Q(x_i - x_j).$$

$$\begin{aligned} Z &= \sum_{\{\sigma_i = \pm 1\}} \int \mathcal{D}\psi \mathcal{D}\psi^* \exp \left\{ -\frac{1}{T} \int \left[\frac{1}{\kappa^2} \left| \left(i \frac{d}{dx} + Q \right) \psi \right|^2 \right. \right. \\ &\quad \left. \left. + |\psi|^2 \right] dx + \frac{1}{T} \sum_i \sigma_i (\psi(x_i) + \psi^*(x_i) + h) \right\} \\ &= \int \mathcal{D}\psi \mathcal{D}\psi^* \exp \left\{ -\frac{1}{T} H_{\text{eff}}[\psi] \right\} \end{aligned} \quad (2)$$

where

$$\begin{aligned} H_{\text{eff}}[\psi] &= \int \left[\kappa^{-2} \left| \left(i \frac{d}{dx} + Q \right) \psi \right|^2 + |\psi|^2 \right. \\ &\quad \left. - T \sum_i \delta(x - x_j) \ln \text{ch} \frac{\psi + \psi^* + h}{T} \right] dx. \end{aligned} \quad (3)$$

Expand in powers of ψ

$$\begin{aligned}
 H_{\text{eff}}[\psi] = \int & \left\{ \kappa^{-2} \left| \left(i \frac{d}{dx} + Q \right) \psi \right|^2 \right. \\
 & + \left(1 - \frac{1}{T} \sum_j \delta(x - x_j) \right) |\psi|^2 \\
 & \left. + \sum_j \delta(x - x_j) \left[-\frac{1}{2T} (\psi^2 + \psi^{*2}) + \frac{1}{2T^3} (\psi + \psi^*)^4 \right] \right\} dx.
 \end{aligned}
 \quad \gamma = \kappa/c \ll 1$$

At $T < T_c = c$ the averaged value of the coefficient at $|\psi|^2$, which is equal to $\tau = 1 - c/T$, becomes negative, which implies the instability of the state with $\langle \psi \rangle = 0$.

$$t \ll t_m \sim \exp(\gamma^{-1} |\tau|^{3/2}). \quad \text{Phase slips are absent}$$

At $|\tau| \gg \gamma^{2/3}$ it is convenient to pass over to the new variables: the amplitude and phase of the field ψ :

$$\psi = \rho \exp(i\varphi + iQx). \quad (5)$$

The free energy $H_{\text{eff}}[\psi]$ from (3) can be minimized over ρ , which leads to the phase-dependent energy

$$\begin{aligned}
 H[\varphi] = & \int \left\{ \frac{\rho^2}{\kappa^2} \left(\frac{d\varphi}{dx} \right)^2 - T \sum_i \delta(x - x_i) \right. \\
 & \cdot \left[\ln \text{ch} \left(\frac{2\rho}{T} \cos(Qx + \varphi) \right) \right. \\
 & \left. \left. - \left\langle \ln \text{ch} \left(\frac{2\rho}{T} \cos \alpha \right) \right\rangle_\alpha \right] \right\} dx \quad (6)
 \end{aligned}$$

where $\langle F(\alpha) \rangle_\alpha = \int_0^{2\pi} F(\alpha) d\alpha / 2\pi$, and ρ is approximately determined by the equation

$$\rho = c \left\langle \cos \alpha \cdot \text{th} \left(\frac{2\rho}{T} \cos \alpha \right) \right\rangle. \quad (7)$$

Equation (7) is obtained if $H[\varphi]$ is neglected (the validity of it is discussed below) in comparison with

$$F_{\text{MFA}} = H_{\text{eff}}[\psi] - H[\varphi] = \rho^2 - c T \left\langle \ln \text{ch} \frac{2\rho \cos \alpha}{T} \right\rangle_\alpha. \quad (8)$$

II. The Vicinity of the Transition Point $\gamma^{2/3} \ll |\tau| \ll 1$

$$F_{\text{MFA}} = \tau \rho^2 + \frac{c h^2}{2 T^3} \rho^4 - \frac{c h^2}{2 T} + \frac{c \rho^2}{T^3} h^2. \quad (9)$$

For the specific heat $C(T)$ and magnetic susceptibility $\chi(T)$ we obtain:

$$C(T) = c \theta(-\tau) \quad (|\tau| \gg \gamma^{2/3}) \quad (10)$$

$$\chi(T) = c/T \quad (\tau \gg \gamma^{2/3}) \quad (11a)$$

$$\chi(T) = 1 - |\tau| \quad (-\tau \gg \gamma^{2/3}). \quad (11b)$$

Formula (11b) is valid at observation times that are not too large (see below). Formula (10) and (11) are similar to those in the Mattis model [12] of spin glasses without frustrations, which is quite natural, since the averages $\langle \sigma_i \rangle$ are also expressed through slow variables in our model. The difference is that we have two variables, ρ and φ :

$$\langle \sigma_i \rangle = -\frac{\partial F}{\partial h_i} = \text{th} \left[\frac{2\rho}{T} \cos(Q x_i + \varphi_i) \right]$$

Phase-dependent Hamiltonian with “pinning”

$$H[\varphi(x), Q] = |\tau| \int dx \left[\gamma^{-2} \left(\frac{d\varphi}{dx} \right)^2 - c \sum_j \delta(x - x_j) \cdot \cos 2(Qx + \varphi) \right].$$

$$\langle \cos(\varphi(x) - \varphi(0)) \rangle \sim \exp(-x L_\varphi^{-1}),$$

$$L_\varphi = A c^{-1} \gamma^{-4/3} \gg c^{-1} \quad (A \sim 1)$$

Hamiltonian (13) coincides with the one studied in [9-11] in connection with the problem of charge-density wave pinning by impurities.

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Stochastic Transfer Matrix method [10]

$$\Psi_N(\varphi_N) = \text{const} \cdot \exp \left[-\frac{\varepsilon_N(\varphi_N)}{T} \right]. \quad \varepsilon_{N+1}(\varphi) - \varepsilon_N(\varphi) = -\frac{1}{2} \frac{l_{N+1}}{v_F} \left(\frac{\partial \varepsilon_N}{\partial \varphi} \right)^2 + \frac{T}{2} \frac{l_{N+1}}{v_F} \frac{\partial^2 \varepsilon_N}{\partial \varphi^2} - V \cos(\varphi + Qx_{N+1}).$$

a Hamilton-Jacobi equation (with a discrete imaginary “time” N) corresponding to the “equation of motion”

$$v_F \varphi'' + V \delta(x - x_n) \sin(\varphi + Qx) = 0. \quad (1)$$

$$\varepsilon_N(\varphi) = -\beta_N \cos(\varphi - \gamma_N)$$

$$\beta_0 = (c v_F V^2)^{1/2}.$$

Moderately Low Temperature Range $\kappa \ll T \ll T_c = c$

$$\varepsilon_{n+1}(\varphi) - \varepsilon_n(\varphi) = -\frac{\kappa^2}{4\rho_0^2} l_{n+1} \left(\frac{\partial \varepsilon_{n+1}}{\partial \varphi} \right)^2 + \frac{T}{4} \frac{l_{n+1}}{\rho_0^2} \kappa^2 \frac{\partial^2 \varepsilon_n}{\partial \varphi^2} + V(\varphi + Qx_n)$$

At $T \ll c$ the phase-pinning potential

$$V(\varphi + Qx) = -T \ln \text{ch} \left[\frac{2\rho}{T} \cos(\varphi + Qx) \right]$$

replace l_n by $\langle l_n \rangle = c^{-1}$, (at $T \gg \kappa$).

Cusp singularity
at low T

$$\varepsilon_{n+1}(\varphi) - \varepsilon_n(\varphi) = -\frac{\kappa}{4\rho_0^2 c} \left(\frac{\partial \varepsilon_n}{\partial \varphi} \right)^2 + V(\varphi + \alpha_n)$$

Generating functional

$$P(j) = \int \mathcal{D}\mu \mathcal{D}\varepsilon \exp \left\{ i \sum_n \int d\varphi \left[\mu_n \left(\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c} \left(\frac{\partial \varepsilon_n}{\partial \varphi} \right)^2 - V(\varphi + \alpha_n) \right) \right] \right\}$$

Let us pass from integrating over μ to integrating over M , so that $M''_{\varphi\varphi} = \mu$.

$$P(j) = \int \mathcal{D}M \mathcal{D}\varepsilon \exp(S - \sum_n \int d\varphi j_n \varepsilon_n)$$

$$S = i \sum_n \int d\varphi \left\{ M_n \frac{\partial^2}{\partial \varphi^2} \left[\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c} \left(\frac{\partial \varepsilon_n}{\partial \varphi} \right)^2 - V(\varphi + \alpha_n) \right] \right\}.$$

Average the functional over α_n

$$\begin{aligned}
 S &= \sum_n \left\{ \int i M_n \frac{\partial^2}{\partial \varphi^2} \left[\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c} \left(\frac{\partial \varepsilon_n}{\partial \varphi} \right)^2 \right] d\varphi \right. \\
 &\quad \left. + \ln \left[\int_0^\pi d\varphi_n \exp(2i\rho_0 M_n(\varphi_n)) \right] \right\} \\
 &= \sum_n \left\{ \int i M_n \frac{\partial^2}{\partial \varphi^2} \left[\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c} \left(\frac{\partial \varepsilon_n}{\partial \varphi} \right)^2 \right] d\varphi \right. \\
 &\quad \left. - 2\rho_0^2 \int M_n^2(\varphi) d\varphi + \sum_{K=3}^{\infty} a_K \rho_0^K \int M_n^K(\varphi) d\varphi \right\}
 \end{aligned}$$

small

$$S_0 = - \int \left\{ \frac{\partial^2}{\partial \varphi^2} \left[\frac{\partial \varepsilon}{\partial x} + \frac{\kappa^2}{4\rho_0^2} \left(\frac{\partial \varepsilon}{\partial \varphi} \right)^2 \right] \right\}^2 \frac{1}{8\rho_0^2 c} d\varphi dx \quad T/c \ll \Phi \ll 1,$$

$$\begin{array}{l}
 EX^{-1} \sim \kappa^2 \rho_0^{-2} E^2 \Phi^{-2} \\
 E^2 X^{-1} \Phi^{-3} \rho_0^{-2} c^{-1} \sim 1
 \end{array}
 \longrightarrow
 \begin{array}{l}
 X \sim \gamma^{-4/3} \Phi^{1/3} c^{-1} \\
 E \sim \gamma^{-2/3} \Phi^{5/3} c
 \end{array}$$

Estimate for the neglected terms

$$\Delta S = \sum_{\kappa=3}^{\infty} \tilde{a}_{\kappa} \frac{c}{(\rho_0 c)^{\kappa}} \int d\varphi dx \left\{ \frac{\partial^2}{\partial \varphi^2} \left[\frac{\partial \varepsilon}{\partial x} + \frac{\kappa^2}{4\rho_0^2} \left(\frac{\partial \varepsilon}{\partial \varphi} \right)^2 \right] \right\}^{\kappa} \quad \Delta S/S_0 \sim \left(\frac{\kappa}{T} \right)^{2/3} \ll 1$$

Correlation length

The total scale of the $\varepsilon(\varphi)$ variation is determined by fluctuations with $\Phi \sim 1$. Its value $E \sim c\gamma^{-2/3}$ is the same as at $T \sim T_c$.

Let's find the phase correlation length L_{φ} . For this purpose we estimate the phase variation by each step ($|\Delta \bar{\varphi}| = |\bar{\varphi}_n - \bar{\varphi}_{n-1}|$ where $\bar{\varphi}_n$ is the thermal average of φ_n). Then L_{φ} will be determined by the condition:

$$L_{\varphi} c \langle (\Delta \bar{\varphi})^2 \rangle = 1$$

Since thermal fluctuations are small, $\bar{\varphi}_n$ is determined by the position of the $\varepsilon(\varphi)$ minimum (perhaps, a local one): $\varepsilon'_n(\bar{\varphi}) = 0$. Using the recurrent equation (23) we get

$$\langle (\Delta \bar{\varphi})^2 \rangle \sim \beta^{-2} = \left\langle \left(\frac{\partial^2 \varepsilon}{\partial \varphi^2} \right)^2 \right\rangle^{-1}$$

$$\beta \sim c\gamma^{-2/3} (T/c)^{-1/3}$$

$$X_{\min} \sim c^{-1} \gamma^{-4/3} (T/c)^{1/3}, \quad (\Phi \sim T/c)$$

$$L_{\varphi} \sim c\gamma^{-4/3} (T/c)^{-2/3}$$

Observables

First of all, we find the free energy of the system in a certain metastable state. As was shown at the end of the previous section, the barrier height at $T \gg \kappa$ is much larger than T ; therefore, when calculating the characteristics of the system corresponding to short time scales (quasiequilibrium I), the phase φ_i can be considered constant at a given point i and as satisfying the condition $\varepsilon'_i(\varphi_i) = 0$. Thermal fluctuations of $\rho(x)$ can always be neglected; therefore, the free energy F_I coincides with the Hamiltonian H_{eff} for the given configuration of $\{\varphi_i\}$.

H_{eff} is given by Formula (3). For future purposes it is convenient to rewrite it as:

$$F_I = H_{\text{eff}} = \int \{ \kappa^{-2} [(V\rho)^2 + \rho^2 (V\varphi)^2] + \rho^2 \} dx - T \sum_j \ln \text{ch} \left(\frac{h + 2\rho \text{Cos}(\varphi + \alpha_j)}{T} \right) \quad (33)$$

$$\chi_{\text{eq}} = \frac{c}{T}$$

$$\langle \sigma_i \rangle = -\frac{\partial F}{\partial h_i} = \text{th} \left[\frac{2\rho}{T} \text{cos}(Q x_i + \varphi_i) \right]$$

Distribution function of phases at local minimum in nearly uniform

$$\rho(\varphi_i + \alpha_i) = \frac{1}{2\pi} + O \left(\left(\frac{\kappa}{T} \right)^{2/3} \right)$$

The behavior of the magnetic susceptibility $\chi = -\frac{\partial^2 F}{\partial h^2}$ is different at different times of observation.

Let's first consider χ in the region where $t_0 \ll t \ll t_1$ (for quasiequilibrium I, see the end of the previous section). Inserting Formula (33) for F we obtain:

$$\chi_I = \frac{c}{T} \left\langle \text{ch}^{-2} \left(\frac{2\rho_0 \text{Cos}(\varphi_i + \alpha_i)}{T} \right) \right\rangle = \frac{1}{2} \quad (42)$$

Slowly time-dependent behavior at $t_0 \ll t \ll t_1$

$$t_0 \sim \exp(E_0/T) \sim \exp[(T/\gamma)^{2/3}]$$

$$t_1 \sim \exp(\gamma^{-2/3}/T)$$

One of the major characteristics: dissipative response

$$\langle \sigma_i \rangle \approx \text{sign}[(\cos(\phi_i + Qx_i))]$$

$$\text{Im } \chi(\omega) \sim \frac{1}{T} \int dE d\Delta R(E, \Delta) \frac{\omega \tau(E)}{1 + [\omega \tau(E)]^2} N(E) \text{sech}^2 \frac{\Delta}{2T}$$

$\tau(E) \sim e^{E/T}$ E is the free-energy barrier between two states

Δ is the energy difference between these states

$R(E, \Delta)$ is the joint probability density

$N(E) \sim \Phi(E)X(E)$ is the number of spins that flip in the course of the transition between two metastable states

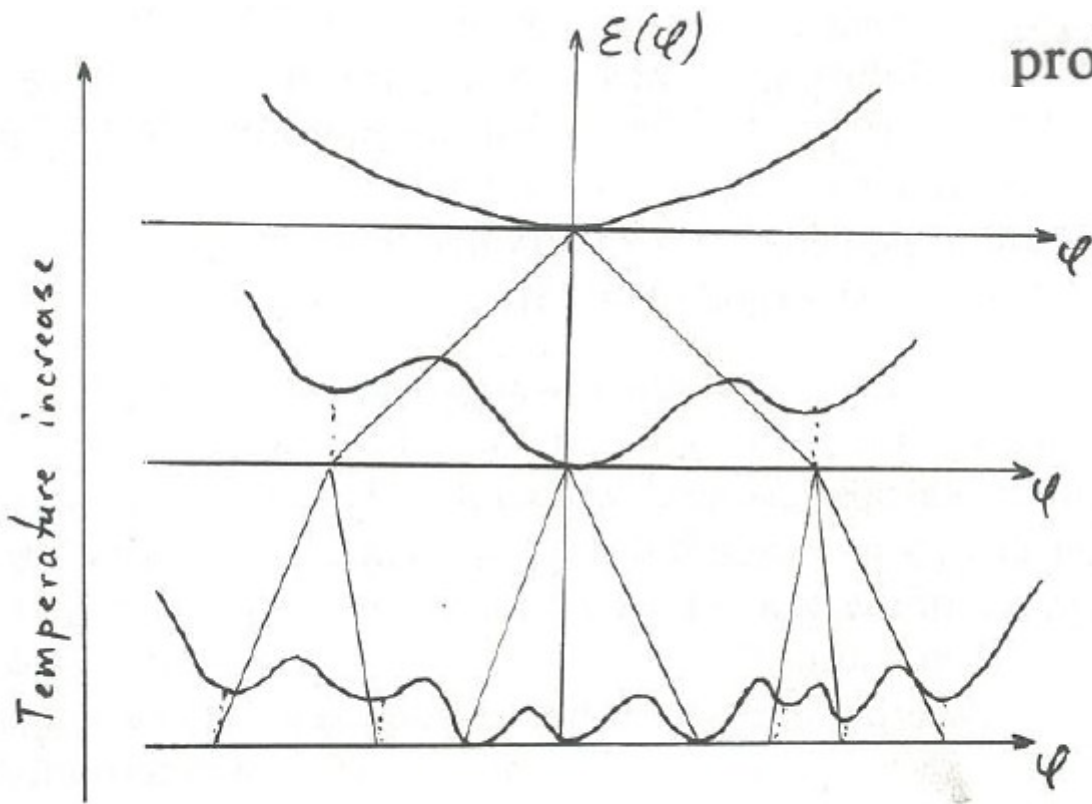


Figure 11 Hierarchical structure of metastable states.

probability of finding a zero of $f(\phi^1 + \Phi)$
within an interval of width Φ_0

$$P(\Phi) \sim \frac{\Phi_0 \beta}{F(\Phi)} = \left(\frac{\Phi_0}{\Phi} \right)^{2/3}$$

the total number of zeros

$$M_0 \sim \int_{\Phi_0}^{\pi} \frac{d\Phi}{\Phi_0} P(\Phi) \sim \left(\frac{\pi}{\Phi_0} \right)^{1/3} \sim T^{-1/3}$$

The minima of $\epsilon(\phi)$
constitute a fractal set
with fractal dimension $D_f = \frac{1}{3}$

consider the behaviour of the function $f(\phi) = \partial\epsilon/\partial\phi$ at $\phi = \phi^1 + \Phi$,
where ϕ^1 is any zero of $f(\phi)$. The characteristic scale of $f(\phi^1 + \Phi)$ is $F(\Phi)$

$$\underline{F(\Phi) \sim E(\Phi)/\Phi \sim (\Phi/\gamma)^{2/3}}$$

$$X \sim \gamma^{-4/3} \Phi^{1/3} c^{-1}$$

$$E \sim \gamma^{-2/3} \Phi^{5/3} c$$

the scale β of $\partial f/\partial\phi$ is determined by the smallest-scale fluctuations with $\Phi \sim \Phi_0$

$$\underline{\beta \sim \gamma^{-2/3} \Phi_0^{-1/3}}$$

$$\text{Im } \chi(\omega) \sim \frac{1}{T} \int dE d\Delta R(E, \Delta) \frac{\omega\tau(E)}{1 + [\omega\tau(E)]^2} N(E) \text{sech}^2 \frac{\Delta}{2T}$$

$$N(E) \sim \Phi(E)X(E) \sim (E/\gamma)^{4/5}$$

$$M(E) \sim \frac{1}{\gamma^{2/15} E^{1/5}} \quad X(E) \equiv X(\Phi(E)) \sim \gamma^{-6/5} E^{1/5}$$

Thus the linear density of relaxation modes with barriers in the interval $(E, E + dE)$ is given by

$$W(E) dE \sim \frac{d}{dE} \left(\frac{M(E)}{X(E)} \right) dE \sim \gamma^{16/15} \frac{dE}{E^{7/5}} \quad (3.2.16)$$

characteristic scale of Δ is of order E , so that $\int d\Delta R(E, \Delta) \text{sech}^2 \frac{\Delta}{2T} \sim W(E) \frac{T}{E}$

Combining all above estimates:

$$\text{Im } \chi(\omega) \sim \int \frac{dE}{E} W(E) N(E) \frac{\omega\tau(E)}{1 + [\omega\tau(E)]^2} \sim \frac{\gamma^{4/15}}{(T \ln \omega^{-1})^{3/5}}$$

In the time domain: $c(t) \sim \frac{\gamma^{4/5}}{(T \ln t)^{3/5}}$ $\chi(t) = - (1/T) dC(t)/dt \quad (t > 0)$

Fluct-diss. relation

Two problems to solve

1. Generating functional (defined in slide 7) and corresponding action

$$S_0 = - \int \left\{ \frac{\partial^2}{\partial \varphi^2} \left[\frac{\partial \varepsilon}{\partial x} + \frac{\kappa^2}{4\rho_0^2} \left(\frac{\partial \varepsilon}{\partial \varphi} \right)^2 \right] \right\}^2 \frac{1}{8\rho_0^2 c} d\varphi dx$$

refer to the free energy $\varepsilon_n^>(\varphi)$ defined for the recursion $>$ from

the left end of the chain to the reference site n

Actually one should consider the sum $\varepsilon_n(\varphi) = \varepsilon_n^>(\varphi) + \varepsilon_n^<(\varphi)$

of two parts of free energy since this is the physical free energy in the middle of the chain

The problem: a) to produce scaling estimates similar to those presented in slide 8, but the total $\varepsilon_n(\varphi)$;

b) to check if the distribution of zeros of the function $d\varepsilon_n(\varphi)/d\varphi$ is the same as was derived for $d\varepsilon_n^>(\varphi)/d\varphi$ (slide 12); if it is not the same, to derive the correct one.

2. Imaginary part of response $\text{Im } \chi(\omega)$ was calculated (slide 12) assuming Gibbs distribution for different metastable states, each of them defined by some minimum of function $\varepsilon_n(\varphi)$. However, this assumption of full equilibrium is not valid if aging dynamics on timescales $t_a \ll t_1$ is considered (for definition of t_1 see slide 11). Namely, thermodynamic Gibbs distribution will be established for modes separated by energy barriers $E \ll T \ln(t_a/t_0)$, whereas modes with barriers $E \gg T \ln(t_a/t_0)$ will be populated just randomly. As a result, function $1/\cosh^2(\Delta/2T)$ in the integral on slide 13 should be replaced by some non-equilibrium function, dependent on the value of t_a .

The problem: to find (approximately) this non-equilibrium and non-stationary distribution function for the range of $E \sim T \ln(t_a/t_0)$ and then calculate aging part of the response function $\text{Im } \chi(\omega)$.