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ФЛУКТУАЦИОННЫЕ ЯВЛЕНИЯ В СВЕРХПРОВОДНИКАХ

Андрей Варламов

Istituto Nazionale per la Fisica della Materia del CNR, Rome, Italy

и

Московский институт стали и сплавов

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Outline

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Флуктуации: качественная картина

$$\tau_{\mathrm{GL}} = \frac{\pi\hbar}{8k_B(T - T_{\mathrm{c}})}.$$

$$\xi_d(T) = \sqrt{\mathcal{D}\tau_{\mathrm{GL}}} \quad \Rightarrow \quad \xi(T) = \frac{\xi}{\sqrt{\epsilon}}.$$

$$\epsilon = \ln\frac{T}{T_{\mathrm{c}}} \approx \frac{T - T_{\mathrm{c}}}{T_{\mathrm{c}}} \ll 1$$

$$n(p) = \frac{1}{\exp\left(\frac{\mathcal{E}(p)}{k_B T}\right) - 1} = \frac{k_B T}{\mathcal{E}(p)}.$$

$$n_s^{(D)} = \int n(p) \frac{d^D p}{(2\pi\hbar)^D}.$$

$$\mathcal{E}(p) = \alpha k_B (T - T_c) + \frac{\mathbf{p}^2}{2m^*} = \frac{1}{2m^*}\left[\frac{\hbar^2}{\xi^2(T)} + \mathbf{p}^2\right].$$

$$p_0 \sim \hbar/\xi(T).$$ → $$m^* \mathcal{E}(p_0) \sim p_0^2 \sim \hbar^2/\xi^2(T).$$

$$\frac{n_s^{(D)}}{m^*} = \frac{k_B T}{m^* \mathcal{E}(p_0)} \left(\frac{p_0}{\hbar}\right)^D \sim \frac{k_B T}{\hbar^2} \xi^{2-D}(T).$$

$$\sigma = \frac{n_s^{(D)} e^2 \tau}{m^*} \Rightarrow$$

$$\chi = -\frac{e^2}{c^2} \frac{n_s^{(D)}}{m^*} \langle R^2 \rangle \Rightarrow$$

$$\sim \epsilon^{D/2-2}.$$

Флуктуации в ГЛ формализме

$$\mathcal{F}[\Psi(\mathbf{r})] = F_N + \int dV \left\{ a|\Psi(\mathbf{r})|^2 + \frac{b}{2}|\Psi(\mathbf{r})|^4 + \frac{1}{4m}|\nabla\Psi(\mathbf{r})|^2 \right\}.$$

$$Z = \mathrm{tr}\left\{ \exp\left(-\frac{\widehat{\mathcal{H}}}{T} \right) \right\}.$$

Быстрые (фермионные) и медленные (бозонные) переменные

$$Z = \int \mathfrak{D}^2\Psi(\mathbf{r})\mathcal{Z}[\Psi(\mathbf{r})],$$

$$a = \alpha T_c \epsilon.$$

$$\mathcal{Z}[\Psi(\mathbf{r})] = \exp\left(-\frac{\mathcal{F}[\Psi(\mathbf{r})]}{T} \right)$$

$$\alpha_{(D)} = \frac{2D\pi^2}{7\zeta(3)} \frac{T_c}{E_F}.$$

Квадратичное ГЛ приближение

Гинзбург,1960

$$F[\Psi_{\mathbf{k}}] = F_N + \sum_{\mathbf{k}} [a + \frac{\mathbf{k}^2}{4m}] |\Psi_{\mathbf{k}}|^2$$

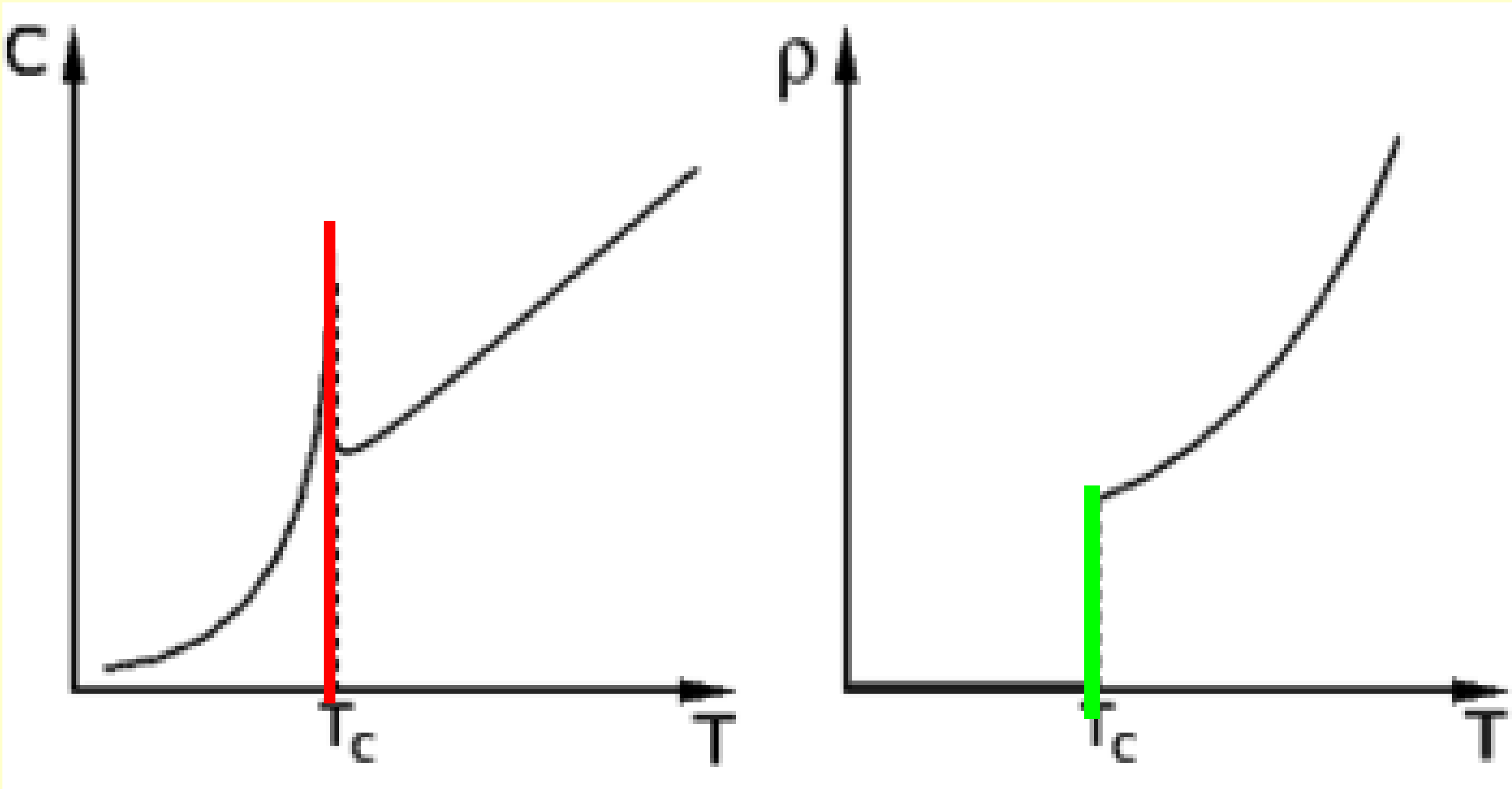
$$Z = \prod_{\mathbf{k}} \int d^2\Psi_{\mathbf{k}} \exp\left\{-\alpha\left(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c}\right)|\Psi_{\mathbf{k}}|^2\right\}$$

$$\delta C_+ = -\frac{1}{VT_c}\left(\frac{\partial^2 F}{\partial \epsilon^2}\right) = \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{\left(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c}\right)^2}$$

δC(Gi)≈ΔC

$$\frac{\delta T}{T_c} \sim Gi_{(3)} \sim \left(\frac{T_c}{E_F}\right)^4 \sim 10^{-12} \div 10^{-14},$$

Размытие перехода в 3D сверхпроводнике



$$\frac{\delta T}{T_c} \sim Gi_{(3)} \sim \left(\frac{T_c}{E_F}\right)^4 \sim 10^{-12} \div 10^{-14},$$

$$\epsilon > \sqrt{Gi_{(3)}} \approx 10^{-6}$$

Критерий Гинзбурга-Леванюка

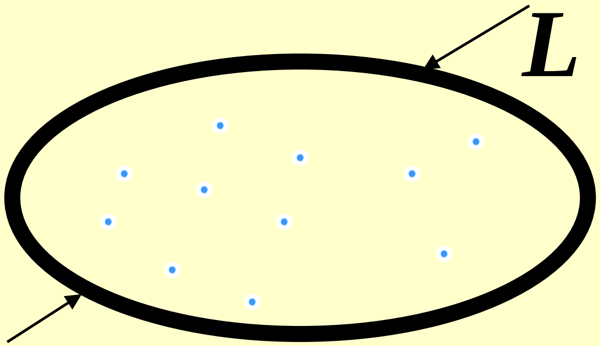
δC(Gi)≈ΔC

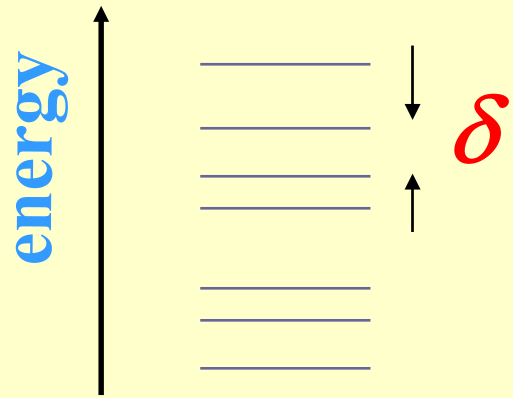
$$Gi_{(D)} = \left[\frac{7\zeta(3)\vartheta_D}{8\pi^2}\left(\frac{V_D}{V}\right)\frac{1}{\nu_D T_c \xi^D}\right]^{\frac{2}{4-D}}.$$

$$Gi_{(3)} = \left[\frac{7\zeta(3)}{64\pi^3}\,\frac{1}{\nu_3 T_c \xi^3}\right]^2$$

$$Gi_{(2)} = \frac{7\zeta(3)}{32\pi^3}\,\frac{1}{\nu_2 T_c \xi^2}.$$

Точное решение ГЛ модели для 0D сверхпроводника (Шмидт, 1965)





L *is the system size;*

d *is the number of dimensions*

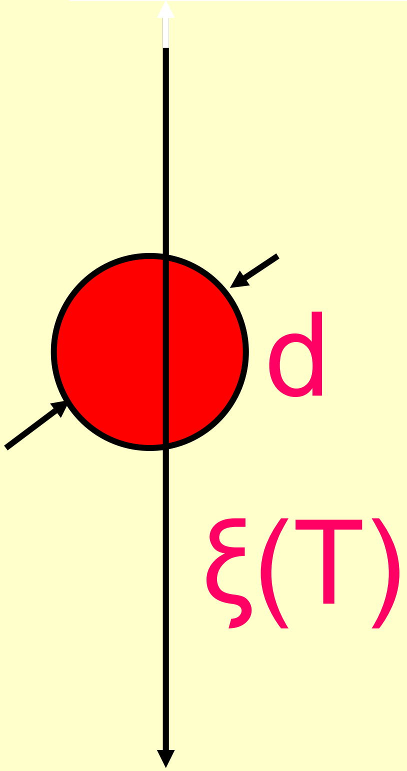
D *is the diffusion const*

Mean level spacing $\delta = 1/\nu \times L^d$

$$Gi_{(0)} = \frac{\sqrt{7\zeta(3)}}{2\pi}\frac{1}{\sqrt{\nu T_c V}} \sim \sqrt{\frac{\delta}{\Delta\ (0)}}.$$

$$\mathcal{F}[\Psi(\mathbf{r})] = F_N + \int dV \left\{ a|\Psi(\mathbf{r})|^2 + \frac{b}{2}|\Psi(\mathbf{r})|^4 + \frac{1}{4m}|\nabla\Psi(\mathbf{r})|^2 \right\}.$$

0D

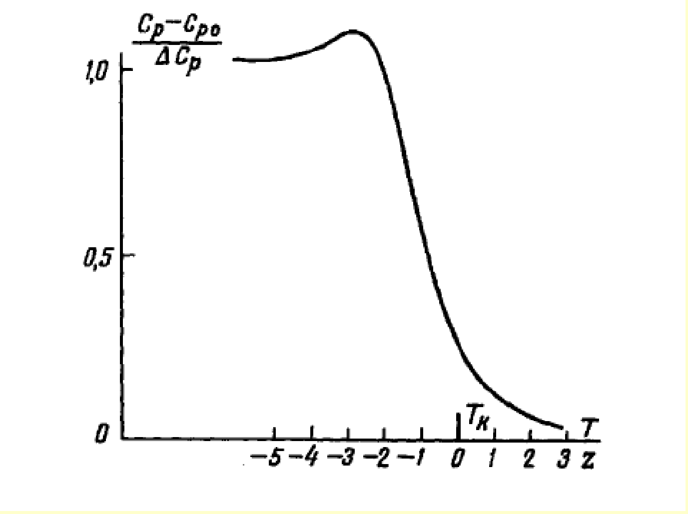


$$Z = \int \mathfrak{D}^2\Psi(\mathbf{r}) \mathcal{Z}[\Psi(\mathbf{r})],$$

$$\mathcal{Z}[\Psi(\mathbf{r})] = \exp\left(-\frac{\mathcal{F}[\Psi(\mathbf{r})]}{T}\right)$$

$$Z_{(0)} = \int d^2\Psi_0 \exp\left(-\frac{\mathcal{F}[\Psi_0]}{T}\right) = \pi \int d|\Psi_0|^2 \exp\left(-\frac{\left(a|\Psi_0|^2 + \frac{b}{2V}|\Psi_0|^4\right)}{T}\right)$$
$$= \sqrt{\frac{\pi^3 VT}{2b}} \exp(x^2)(1 - \operatorname{erf}(x))\Big|_{x = a\sqrt{\frac{V}{2bT}}}. \qquad (2.25)$$

Размытие перехода в 0D сверхпроводнике



$$\epsilon_{cr} = Gi_{(0)} = \frac{\sqrt{7\zeta(3)}}{2\pi} \frac{1}{\sqrt{\nu T_{c0} V}} \approx 13.3 \left(\frac{T_{c0}}{E_F}\right) \sqrt{\frac{\xi_0^3}{V}}.$$

Нелинейная 0D намагниченность

$$\mathcal{F}[\Psi(\mathbf{r})] = F_N + \int dV \left\{ a|\Psi(\mathbf{r})|^2 + \frac{b}{2}|\Psi(\mathbf{r})|^4 + \frac{1}{4m}|\nabla\Psi(\mathbf{r})|^2 \right\}.$$

$$T_c(H) = T_c(0)\left(1 - \frac{4\pi^2\xi^2}{\Phi_0^2}\langle \mathbf{A}^2 \rangle\right),$$

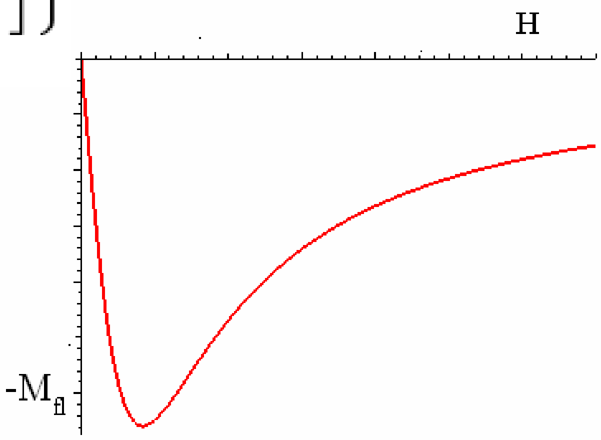
0D

$$Z_{(0)}(H) = \pi \int d|\Psi_0|^2 \exp\left(- \frac{\left[a + \frac{e^2}{m}\langle \mathbf{A}^2 \rangle\right]|\Psi_0|^2 + \frac{b}{2V}|\Psi_0|^4}{T} \right) =$$

$$= \sqrt{\frac{\pi^3 VT}{2b}} \exp\left[\frac{a^2(H)V}{2bT}\right]\left\{1 - \mathrm{erf}\left[a(H)\sqrt{\frac{V}{2bT}}\right]\right\}$$

$$F_{(0)}(\epsilon, H) = -T \ln \frac{\pi}{\alpha(\epsilon + \frac{4\pi^2\xi^2}{\Phi_0^2}\langle \mathbf{A}^2 \rangle)}.$$

$$M_{(0)}(\epsilon, H) = -\frac{1}{V}\frac{\partial F_{(0)}(\epsilon, H)}{\partial H} = -\frac{24\pi T\xi^2}{5\Phi_0^2 d}\frac{H}{(\epsilon + \frac{2\pi^2\xi^2}{5\Phi_0^2}H^2 d^2)}.$$



Флуктуационная намагниченность в 2D

$$F(\epsilon > 0) = -T \ln Z = -T \sum_{\mathbf{k}} \ln \frac{\pi}{\alpha\left(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c}\right)}.$$

$$k^2/4m \rightarrow \omega_c \; (n + 1/2)$$

$$F(\epsilon, H) = -\frac{SH}{\Phi_0} T \sum_{n,k_z} \ln \frac{\pi T}{\alpha T_c \epsilon + \omega_c\left(n + \frac{1}{2}\right)}$$

$$h = \frac{\omega_c}{\alpha T_c} = \frac{eH}{2m\alpha T_c} = \frac{H}{\widetilde{H}_{c2}(0)}$$

$$F_{(2)}(\epsilon,\, H) = -\frac{TS}{2\pi\xi_{xy}^2}\, h \sum_{n=0}^{n_c-1} \ln \frac{\pi}{\alpha\left[\epsilon + 2h\left(n + \frac{1}{2}\right)\right]}.$$

$$\frac{k_{\max}^2}{4m} \sim \alpha T.$$

$$n_c \sim \frac{\alpha T}{\omega_c} \sim \frac{1}{h}.$$

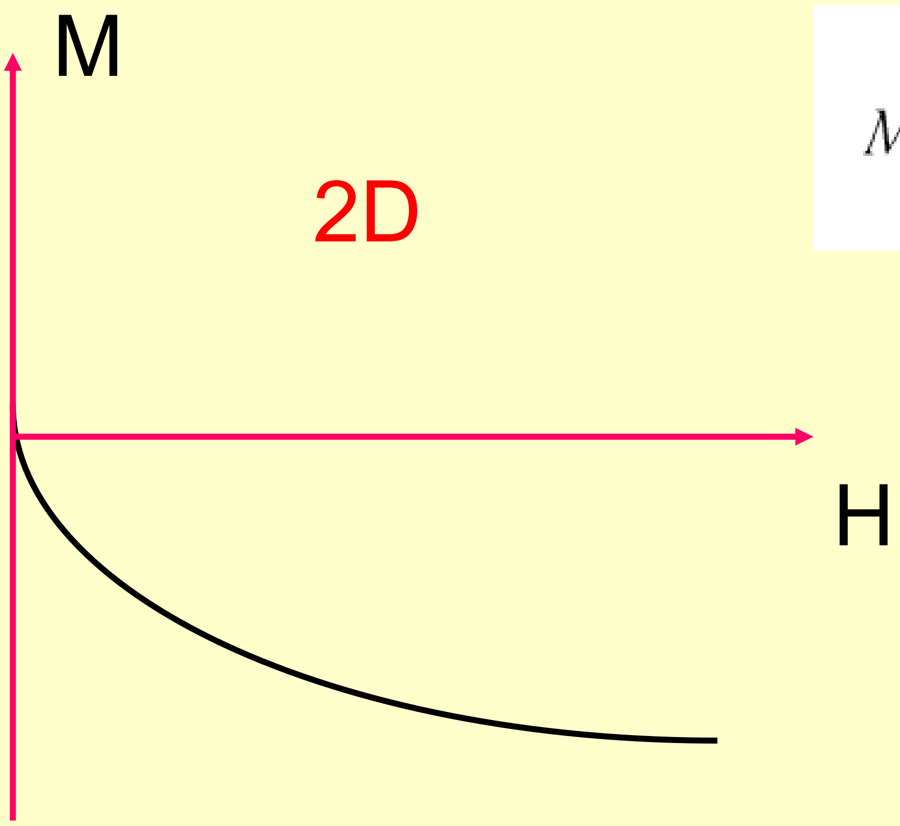
$$\delta F_{(2)}(\epsilon,\, h) = -\frac{TS}{2\pi\xi_{xy}^2} \left[h \ln \frac{\Gamma\left(\frac{1}{2} + \frac{\epsilon}{2h}\right)}{\sqrt{2\pi}} + \frac{\epsilon}{2} \ln h \right] .$$

$$\delta F_{(2)}(h \ll \epsilon) = -\frac{TS}{2\pi\xi_{xy}^2}\left(\frac{\epsilon}{2}\ \ln\,\epsilon - \frac{h^2}{12\epsilon}\right).$$

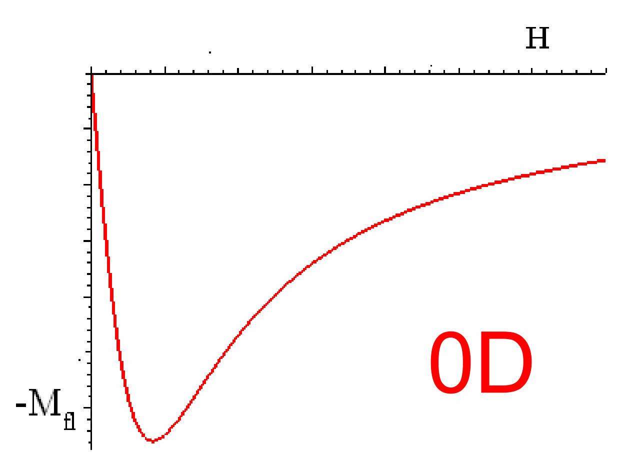
$$\delta F_{(2)}(\epsilon \ll h) = \frac{TS\ \ln 2}{4\pi\xi_{xy}^2}\,h.$$

$$M_{(2)}(\epsilon,\,h) = -\frac{1}{\widetilde{H}_{c2}(0)S}\,\frac{\partial}{\partial h}\,[\delta F_{(2)}(\epsilon,\,h)] =$$

$$= \frac{T}{\Phi_0}\left\{\ln\,\frac{\Gamma\left(\frac{1}{2}+\frac{\epsilon}{2h}\right)}{\sqrt{2\pi}} - \frac{\epsilon}{2h}\left[\psi\left(\frac{1}{2}+\frac{\epsilon}{2h}\right)-1\right]\right\}.$$



$$M_{(2)}(h \ll \epsilon) = -\frac{h}{6\epsilon}\left(\frac{T}{\Phi_0}\right),$$



$$M_{(2)}(h \gg \epsilon) \to M_\infty = -\frac{\ln 2}{2}\left(\frac{T}{\Phi_0}\right) = -0{,}346\left(\frac{T}{\Phi_0}\right).$$

Ширина критической области в магнитном поле

а. H=0

$$F(\epsilon > 0) = -T \ln Z = -T \sum_{\mathbf{k}} \ln \frac{\pi}{\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})}.$$

$$Gi_{(D)} = \left[\frac{7\zeta(3)\vartheta_D}{8\pi^2} \left(\frac{V_D}{V}\right) \frac{1}{\nu_D T_c \xi^D}\right]^{2/(4-D)}$$

$$Gi_{(2)} = \frac{7\zeta(3)}{32\pi^3} \frac{1}{\nu_2 T_c \xi^2}.$$

b. H≠0, T-T_c<< T_c. LLL приближение

$$F(\epsilon,H)=-\frac{SH}{\Phi_0}T\int_{-\pi/s}^{\pi/s}\frac{dk}{2\pi}\ln\frac{\pi T}{\alpha T_c\epsilon+\frac{H}{4m\Phi_0}+J\left(1-\cos(k_z s)\right)},$$

$$\delta C(\epsilon,H)=-\frac{1}{VT_c}\left(\frac{\partial^2 F(\epsilon,H)}{\partial\epsilon^2}\right)=\frac{H}{\Phi_0 s}\left(-\frac{\partial}{\partial\epsilon}\right)\frac{1}{\sqrt{\epsilon(H)\left[\epsilon(H)+r\right]}}.$$

$$\delta C_{(2)}(r\ll\epsilon,H)=\frac{H}{\Phi_0 s}\frac{1}{\epsilon^2(H)}.$$

$$Gi_{(2)}(H)=\epsilon_{cr}(H)=\sqrt{Gi_{(2)}(T_c,0)\frac{2H}{\widetilde{H}_{c2}(0)}}.$$

c. H - $H_{c2}(0)$ << $H_{c2}(0)$

$$\chi(T) = \frac{\partial M}{\partial H} = -\frac{1}{4\pi\beta_A\left[2\varkappa_2^2(T) - 1\right]}$$

$$\Delta\chi(0) \approx \frac{v_F^2}{c^2}(p_F l)^2 \frac{\pi^2}{63\zeta(3)\cdot 2.88\cdot\beta_A}\frac{e^2}{v_F} \approx 0.034\frac{v_F^2}{c^2}(p_F l)^2\frac{e^2}{v_F}.$$

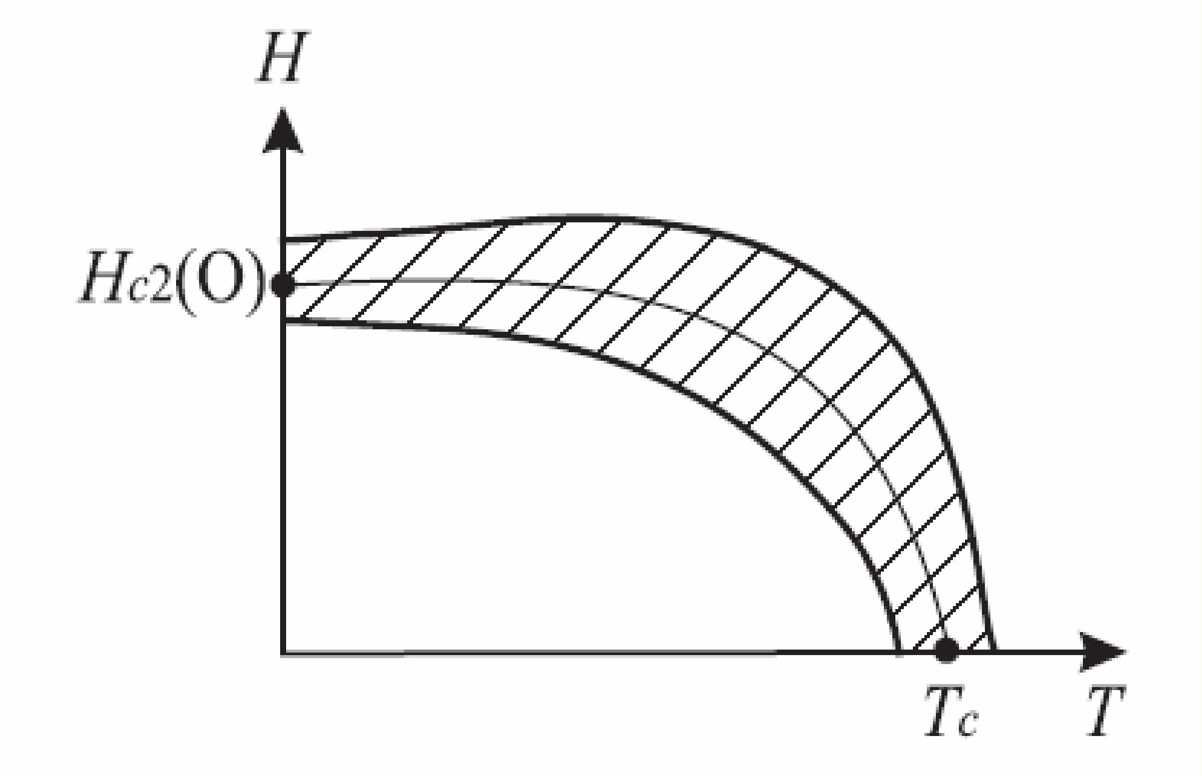
Naturally, this value is considerably less than $(4\pi)^{-1}$.

$$\chi_{\text{fl}}\left(t \ll \widetilde{h}\right) = -\frac{e^2}{\pi^2 c^2}\frac{v_F^2\tau}{d}\frac{1}{\widetilde{h}},$$

$$\widetilde{h} = \frac{H - H_{c2}(T)}{H_{c2}(T)}$$

$$Gi_{(\text{film})}\left(t \ll \widetilde{h}\right) = \widetilde{h}_{cr}\left(t \ll \widetilde{h}\right) = \frac{2.65}{p_F^2 l d} \approx 2Gi_{(\text{film})}\left(T_c, H = 0\right).$$

Размытие линии $H_{c2}(T)$



Флуктуации ниже T_c

$$\mathcal{F}[\Psi(\mathbf{r})] = F_N + \int dV \left\{ a|\Psi(\mathbf{r})|^2 + \frac{b}{2}|\Psi(\mathbf{r})|^4 + \frac{1}{4m}|\nabla\Psi(\mathbf{r})|^2 \right\}.$$

$$|\widetilde{\Psi}|^2 = \begin{cases} |a|/b, \ \epsilon < 0 \\ 0, \ \epsilon > 0 \end{cases}.$$

$$\Psi(\mathbf{r}) = \widetilde{\Psi} + \psi(\mathbf{r}).$$

$$Z = \prod_{\mathbf{k}} \int d^2\Psi_{\mathbf{k}} \exp\left\{ -\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})|\Psi_{\mathbf{k}}|^2 \right\}.$$

$$F = -\frac{T}{2}\sum_{\mathbf{k}} \left\{ \ln\frac{\pi T_c}{3b\widetilde{\Psi}^2 + a + \frac{\mathbf{k}^2}{4m}} + \ln\frac{\pi T_c}{b\widetilde{\Psi}^2 + a + \frac{\mathbf{k}^2}{4m}} \right\}.$$

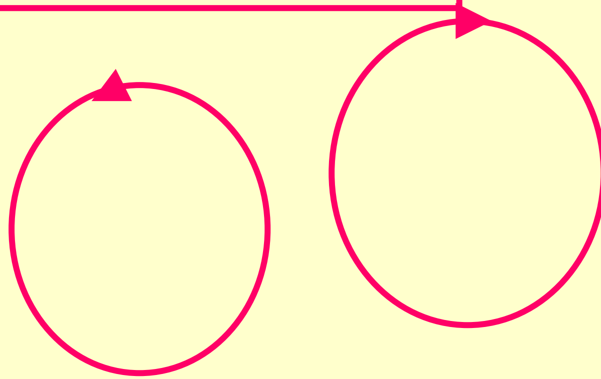
$$F(\epsilon > 0) = -T \ln Z = -T \sum_{\mathbf{k}} \ln \frac{\pi}{\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})}.$$

$$F(\epsilon < 0) = -\frac{T}{2} \sum_{\mathbf{k}} \left\{ \ln \frac{\pi T_c}{2|a| + \frac{\mathbf{k}^2}{4m}} + \ln \frac{\pi T_c}{\mathbf{k}^2/4m} \right\}.$$

Modulus fluctuations

Phase fluctuations

GL picture



BKT picture

Сверхпроводящий переход в 2D пленке

a. Mean field theory

$$\mathbf{j} = -n_s \frac{2e^2}{m}\mathbf{A}$$ and $$n_s = \widetilde{\Psi}^2 = -\frac{a}{b}$$ ⟶ T_{c0}

b. Fluctuation GL theory

$$\langle \mathbf{j}_{\mathrm{fl}} \rangle = -\frac{\partial F_{\mathrm{fl}}(\mathbf{A})}{\partial \mathbf{A}} = T\frac{\partial \ln Z(\mathbf{A})}{\partial \mathbf{A}}$$

$$= -\frac{\int \mathfrak{D}\Psi(\mathbf{r})\mathfrak{D}\Psi^*(\mathbf{r})\frac{\partial \mathcal{F}[\Psi(\mathbf{r}),\mathbf{A}])}{\partial \mathbf{A}}\exp\left(-\frac{\mathcal{F}[\Psi(\mathbf{r}),\mathbf{A}]}{T}\right)}{\int \mathfrak{D}\Psi(\mathbf{r})\mathfrak{D}\Psi^*(\mathbf{r})\exp\left(-\frac{\mathcal{F}[\Psi(\mathbf{r}),0]}{T}\right)}.$$

$$\langle \mathbf{j}_{\mathrm{fl}} \rangle = \mathbf{j}_1 + \mathbf{j}_2 = -\frac{2e^2}{m}\mathbf{A}\left\langle |\Psi(\mathbf{r})|^2 \right\rangle_{\mathbf{A}=0} - \frac{e}{m}\left\langle \mathrm{Im}[\Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r})]\right\rangle_{\mathbf{A}}$$

$$\langle \mathbf{j}_{\mathrm{fl}} \rangle = -\frac{2e^2}{m}\mathbf{A}\left[\left\langle |\Psi(\mathbf{r})|^2 \right\rangle_{\mathbf{A}=0} - \sum_{\mathbf{k}} \frac{T}{(2\alpha T_c|\epsilon| + \mathbf{k}^2/4m)}\right].$$

$$\langle|\Psi(\mathbf{r})|^2\rangle = \widetilde{\Psi}^2 + \langle\psi_r^2\rangle + \langle\psi_i^2\rangle + 2\widetilde{\Psi}\,\langle\psi_r\rangle$$

$$\langle\psi_r^2\rangle = \frac{T}{2}\sum_{\mathbf{k}}\frac{1}{2\alpha T_c|\epsilon| + \mathbf{k}^2/4m},$$

$$\langle\psi_i^2\rangle = \frac{T}{2}\sum_{\mathbf{k}}\frac{1}{\mathbf{k}^2/4m}.$$

$$\langle\psi_r\rangle = -\frac{1}{2\widetilde{\Psi}}\left(3\langle\psi_r^2\rangle + \langle\psi_i^2\rangle\right).$$

$$\langle\mathbf{j}_{\mathrm{fl}}\rangle = -\frac{2e^2}{m}\mathbf{A}\left[\widetilde{\Psi}^2 - 2\sum_{\mathbf{k}}\frac{T}{(2\alpha T_c|\epsilon| + k^2/4m)}\right].$$

$$n_s(T) = \frac{\alpha}{b}\left[T_c|\epsilon| - \frac{2b}{\alpha^2}\sum_{\mathbf{k}}\frac{1}{(2|\epsilon| + k^2/4m\alpha T_c)}\right].$$

$$T_c = T_{c0}\left(1 - 2Gi_{(2d)}\ln Gi_{(2d)}^{-1}\right),$$

c. BKT theory

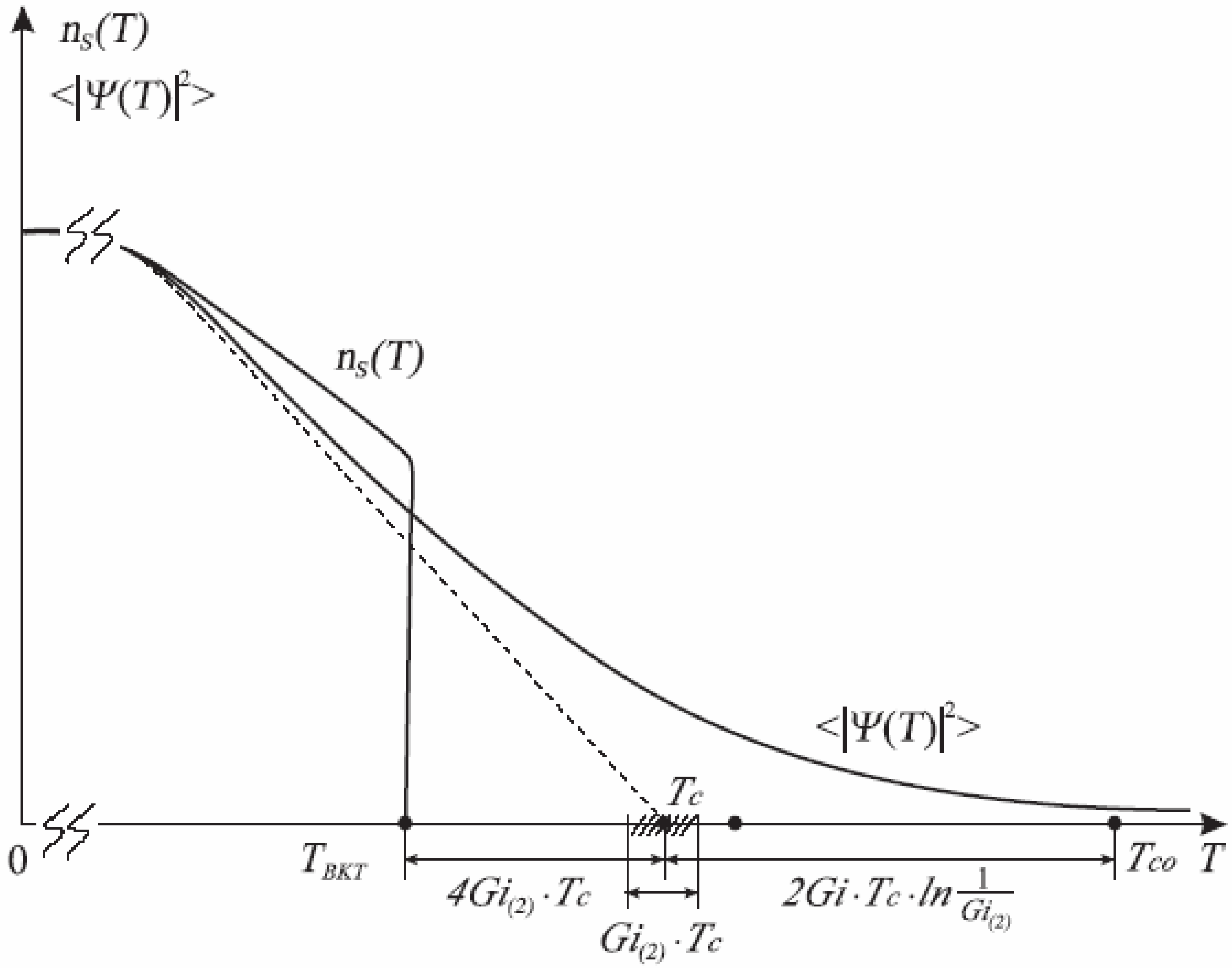
$$E_v = \frac{\pi n_{s2}(T)}{2m} \ln \frac{R}{\xi}.$$

$$F = E - TS = \left[\frac{\pi n_{s2}(T)}{2m} - 2T\right] \ln \frac{R}{\xi}.$$

$$n_{s2}(T_{\rm BKT}) = \frac{4mT_{\rm BKT}}{\pi}.$$

Interpolation formula:

$$n_{s2}(T) = \frac{mT_{\rm BKT}}{\pi}\left[\frac{|\epsilon|}{Gi_{(2)}} - 2\ln\frac{Gi_{(2)}}{|\epsilon| + Gi_{(2)}} + 4\right],$$


$n_S(T)$
$<|\Psi(T)|^2>$
$n_S(T)$
$<|\Psi(T)|^2>$
T_c
0
T_{BKT}
T_{co}
T
$4Gi_{(2)} \cdot Tc$
$Gi_{(2)} \cdot Tc$
$2Gi \cdot Tc \cdot ln \frac{1}{Gi_{(2)}}$

Проявление флуктуаций в транспортных свойствах

Временное уравнение Гинзбурга — Ландау

$$\partial\Psi/\partial t \quad \sim \quad \delta\mathcal{F}/\delta\Psi^*$$

$$\partial\Psi/\partial t \longrightarrow \partial\Psi/\partial t + 2ie\varphi\Psi$$

$$-\gamma_{\mathrm{GL}}\left(\frac{\partial}{\partial t} + 2ie\varphi\right)\Psi = \frac{\delta\mathcal{F}}{\delta\Psi^*}$$

Проявление флуктуаций в транспортных свойствах

Временное уравнение Гинзбурга — Ландау

$$\partial\Psi/\partial t \quad \sim \quad \delta\mathcal{F}/\delta\Psi^*$$

$$\partial\Psi/\partial t \longrightarrow \partial\Psi/\partial t + 2ie\varphi\Psi.$$

$$-\gamma_{\text{GL}}\left(\frac{\partial}{\partial t} + 2ie\varphi\right)\Psi = \frac{\delta\mathcal{F}}{\delta\Psi^*}$$

$$-\gamma_{\text{GL}}\left(\frac{\partial}{\partial t} + 2ie\varphi\right)\Psi = \frac{\delta\mathcal{F}}{\delta\Psi^*} + \zeta(\mathbf{r},\, t).$$

$$\gamma_{\rm GL} = \alpha T_{\rm c} \epsilon \tau_{\rm GL} = \pi\alpha/8.$$

$$[\widehat{L}^{-1} - 2ie\gamma_{\rm GL}\varphi(r,\, t)]\Psi(\mathbf{r},\, t) = \zeta(\mathbf{r},\, t).$$

$$\widehat{L} = \left[\gamma_{\rm GL}\frac{\partial}{\partial t} + \widehat{\mathcal{H}}\right]^{-1}, \quad \widehat{\mathcal{H}} = \alpha T_{\rm c}\left[\epsilon - \widehat{\xi}^2(\widehat{\nabla} - 2ie\mathbf{A})^2\right].$$

При φ=0:

$$\Psi^{(0)}(\mathbf{r},\, t) = \widehat{L}\zeta(\mathbf{r},\, t).$$

силы Ланжевена должны удовлетворять флуктуационно-диссипативной теореме

при t=t':

$$\langle \Psi_{\mathbf{p}}^{(0)*}(t')\Psi_{\mathbf{p}}^{(0)}(t)\rangle \longrightarrow \langle |\Psi_{\mathbf{p}}|^2\rangle$$

$$\langle |\Psi_{\mathbf{p}}|^2\rangle = \frac{\int \mathfrak{D}\Psi_{\mathbf{p}}\mathfrak{D}\Psi_{\mathbf{p}}^*|\Psi_{\mathbf{p}}|^2 \exp\{-\alpha(\epsilon+\xi^2\mathbf{p}^2)|\Psi_{\mathbf{p}}|^2\}}{\int \mathfrak{D}\Psi_{\mathbf{p}}\mathfrak{D}\Psi_{\mathbf{p}}^* \exp\{-\alpha(\epsilon+\xi^2\mathbf{p}^2)|\Psi_{\mathbf{p}}|^2\}} = \frac{1}{\alpha(\epsilon+\xi^2\mathbf{p}^2)}.$$

$$\downarrow$$

$$\langle \zeta^*(\mathbf{r},\, t)\zeta(\mathbf{r}',\, t')\rangle = 2T\ \mathrm{Re}\ \gamma_{\mathrm{GL}}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t').$$

Общее выражение для парапроводимости

$$[\widehat{L}^{-1} - 2ie\gamma_{\mathrm{GL}}\varphi(r,\ t)]\Psi(\mathbf{r},\ t) = \zeta(\mathbf{r},\ t).$$

$$\Psi_{k_z}(\mathbf{r},t) = \Psi^{(0)}_{\{i\}} + \Psi^{(1)}_{\{i\}},$$

$$\Psi^{(0)}(\mathbf{r},\ t) = \widehat{L}\zeta(\mathbf{r},\ t).$$

$$\Psi^{(1)}_{\{i\}} = 2ie\gamma_{\mathrm{GL}}\widehat{L}_{\{ik\}}\varphi_{\{kl\}}\widehat{L}_{\{lm\}}\zeta_{\{m\}}.$$

$$\mathbf{j} = 2e \ \mathrm{Re} \left[\Psi^{(0)*}_{\{i\}} \widehat{\mathbf{v}}_{\{ik\}} \Psi^{(1)}_{\{k\}} + \Psi^{(1)*}_{\{i\}} \widehat{\mathbf{v}}_{\{ik\}} \Psi^{(0)}_{\{k\}} \right] .$$

$$\mathbf{j} = -16Te^2 \ \mathrm{Re}(\gamma_{\mathrm{GL}}) \ \mathrm{Im}\{\gamma_{\mathrm{GL}} \widehat{\mathbf{v}}_{\{il\}} \widehat{L}_{\{lm\}} \varphi_{\{mn\}} \widehat{L}_{\{np\}} \widehat{L}^{*}_{\{pi\}}\}.$$

$$\varphi(r,t) = -E^{\beta} r^{\beta} \ \exp(-i\omega t).$$

$$\widehat{\mathbf{r}}^{\beta}_{\{li\}} = i \, \frac{\widehat{\mathbf{v}}^{\beta}_{\{li\}}}{\varepsilon_{\{i\}} - \varepsilon_{\{l\}}}$$

$$\sigma^{\alpha\alpha}(\epsilon, \, H, \, \omega) = \frac{\pi}{2} \, \alpha e^2 T \sum_{\{i,l\}=0}^{\infty} \Re\left[\frac{\widehat{\mathbf{v}}^{\alpha}_{\{il\}} \widehat{\mathbf{v}}^{\alpha}_{\{li\}}}{\varepsilon_{\{i\}} \varepsilon_{\{l\}} (\varepsilon_{\{i\}} + \varepsilon_{\{l\}} - i\gamma_{\mathrm{GL}}\omega)} \right].$$

Асламазов, Ларкин,1968

$$\widehat{\mathbf{v}}_{\{\mathbf{p}\mathbf{p}'\}} = \mathbf{v}_{\mathbf{p}}\delta_{\mathbf{p}\mathbf{p}'}, \qquad \mathbf{v}_{\mathbf{p}} = \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} = 2\alpha\ T_c\xi^2\mathbf{p}.$$

$$\sigma^{\alpha\beta}_{(D)} = 2e^2T\ \mathrm{Re}\ \gamma_{\mathrm{GL}} \sum_{\mathbf{p}} \frac{\mathbf{v}^{\alpha}_{\mathbf{p}}\mathbf{v}^{\beta}_{\mathbf{p}}}{\varepsilon^3_{\mathbf{p}}} =$$

$$= \delta^{\alpha\beta} \begin{cases} \dfrac{e^2}{32\xi}\dfrac{1}{\sqrt{\epsilon}} & \text{3D case,} \\ \dfrac{e^2}{16d}\dfrac{1}{\epsilon} & \text{2D film, thickness}: d \ll \xi, \\ \dfrac{\pi e^2\xi}{16S}\dfrac{1}{\epsilon^{3/2}} & \text{1D wire, cross-section: } S \ll \xi^2 \end{cases}$$

Парапроводимость слоистого сверхпроводника

$$\widehat{\mathcal{H}} = \alpha T_c \Big(\epsilon - \xi_{xy}^2 (\nabla_{xy} - 2ie\mathbf{A}_{xy})^2 - \frac{r}{2}(1 - \cos(k_z s) \Big)$$

$$\varepsilon_{\{n\}} = \alpha T_c \left[\epsilon + \frac{r}{2} \left(1 - \cos(k_z s)\right) + h(2n+1) \right]$$

$$\widehat{\mathbf{v}}^{x,y} = \frac{1}{2m} \left(-i\nabla - 2ie\mathbf{A}\right)^{x,y}; \quad \widehat{\mathbf{v}}^{z} = -\frac{\alpha r s}{2} T_c \sin(k_z s).$$

Парапроводимость по постоянному току

Магнитопроводимость

Парапроводимость в переменном электромагнитном поле

Холловская парапроводимость

Кинетическое уравнение Больцмана для флуктуационных пар

$$n_{\mathbf{p}}(t) = \int \langle \Psi(\mathbf{r},\, t)\Psi^*(\mathbf{r}',\, t)\rangle \, \exp(-i\,\mathbf{p}(\mathbf{r}-\mathbf{r}'))\, d(\mathbf{r}-\mathbf{r}').$$

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + 2e\mathbf{E}\,\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = -\frac{2}{\gamma_{\mathrm{GL}}}\,\varepsilon_{\mathbf{p}}(n_{\mathbf{p}} - n_{\mathbf{p}}^{(0)}) = -\frac{2}{\tau_{\mathbf{p}}}(n_{\mathbf{p}} - n_{\mathbf{p}}^{(0)}).$$

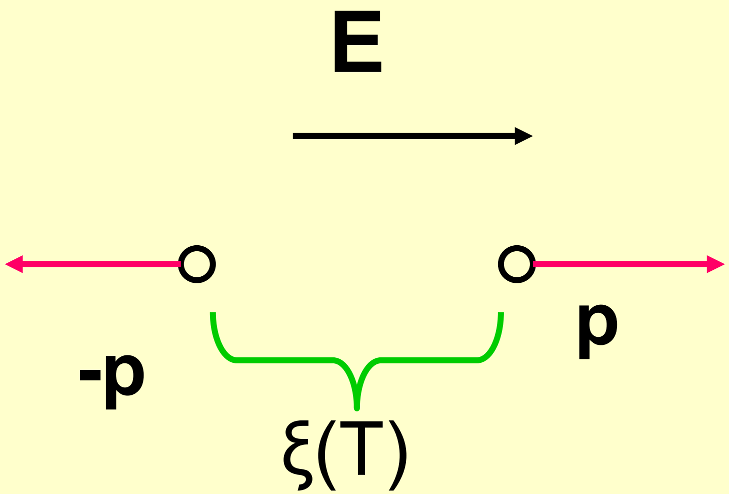
$$\tau_{\mathbf{p}} = \frac{\gamma_{\mathrm{GL}}}{\varepsilon_{\mathbf{p}}} = \frac{\tau_{\mathrm{GL}}(\epsilon)}{1+\xi^2(\epsilon)\mathbf{p}^2}.$$

Слабое поле:

$$n_{\mathbf{p}}^{(1)} = -\frac{e\mathbf{E}\gamma_{\mathrm{GL}}}{\varepsilon_{\mathbf{p}}}\,\frac{\partial n_{\mathbf{p}}^{(0)}}{\partial \mathbf{p}} = \frac{eT\gamma_{\mathrm{GL}}}{\varepsilon_{\mathbf{p}}^3}\,\mathbf{E}\cdot\frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}}.$$

→ **АЛ,1968**

Парапроводимость в сильном поле



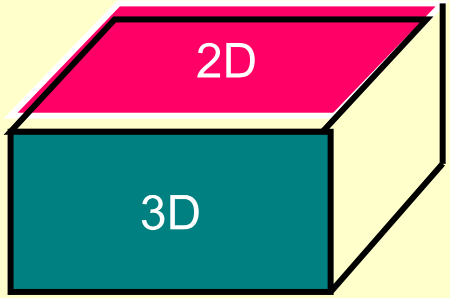
$$eE_c\,\xi(T) \sim T - T_c$$

$$E_c \sim (T-Tc)^{3/2}$$

$$\Delta j(E \gg Ec) \sim E^{2/3}, \quad D=3$$

$$\Delta j(E \gg Ec) \sim E^{1/3}, \quad D=2$$

Влияние флуктуаций на джозефсоновский ток



$$I_c = \frac{\pi \Delta^2(T)}{4eTR_n}.$$

Mean field in the vicinity of T_c gives:

$$\Delta^2(T) \to \Delta_0^2(T) = \frac{8\pi^2 T_{c0}^2}{7\zeta(3)} |\epsilon_0|,$$

$$I_c^{(0)}(|\epsilon_0|) = \frac{2\pi^3 T_{c0}}{7\zeta(3)\, eR_n} |\epsilon_0|$$

where $|\epsilon_0| = (T_{c0} - T)/T_{c0}$

GL fluctuation theory gives:

$$\Delta^2(T) \rightarrow \langle \Delta(T) \rangle^2.$$

$$\langle \Delta(T) \rangle^2_{(\mathrm{fl})} = \Delta_0^2 - \frac{1}{4mC_{(D)}} \left[3 \langle \psi_r^2 \rangle + \langle \psi_i^2 \rangle \right]$$

$$\langle \psi_r^2 \rangle = \frac{T}{2} \sum_{\mathbf{k}} \frac{1}{2\alpha T_c |\epsilon| + \mathbf{k}^2/4m},$$

$$\langle \psi_i^2 \rangle = \frac{T}{2} \sum_{\mathbf{k}} \frac{1}{\mathbf{k}^2/4m}.$$

$$I_c(T) = \frac{\pi \langle \Delta(T) \rangle^2_{(\mathrm{fl})}}{4eTR_n} = I_c^{(0)}(|\epsilon_0|) + \delta I_c^{(\mathrm{fl})}(|\epsilon_0|).$$

$$\delta I_{c(2)}(|\epsilon|) = \frac{4\pi^3 T_c}{7\zeta(3)\, eR_n} Gi_{(2)} \left[\ln \frac{|\epsilon|}{Gi_{(2)}} + \ln \left(\frac{\xi(T)}{L_J} \right) \right].$$

Экспоненциальный хвост в джозефсоновском токе вблизи T_c

$$\mathcal{F}[\varphi] = \frac{n_s}{4m}\int d^2\mathbf{r}\,[\nabla\varphi(\mathbf{r})]^2 = \frac{n_s}{4m}\sum_{\mathbf{k}}\mathbf{k}^2\varphi_{\mathbf{k}}^2.$$

+

$$\delta E_J^{(\mathrm{fl})}(\epsilon) = \frac{E_J}{2S}\int d^2\mathbf{r}\left[\varphi^{(\mathrm{fl})}(\mathbf{r})\right]^2.$$

$$\left\{\ \Delta F\ (\phi) = E_J\ (1-\cos\ \phi)\ \right\}$$

$$F\left(\varphi^{(\mathrm{fl})}\right) = \frac{n_s}{2m}\sum_{\mathbf{k}}\left(\mathbf{k}^2 + \frac{1}{L_J^2}\right)\varphi_{\mathbf{k}}^2,$$

where the Josephson length L_J is determined by relation

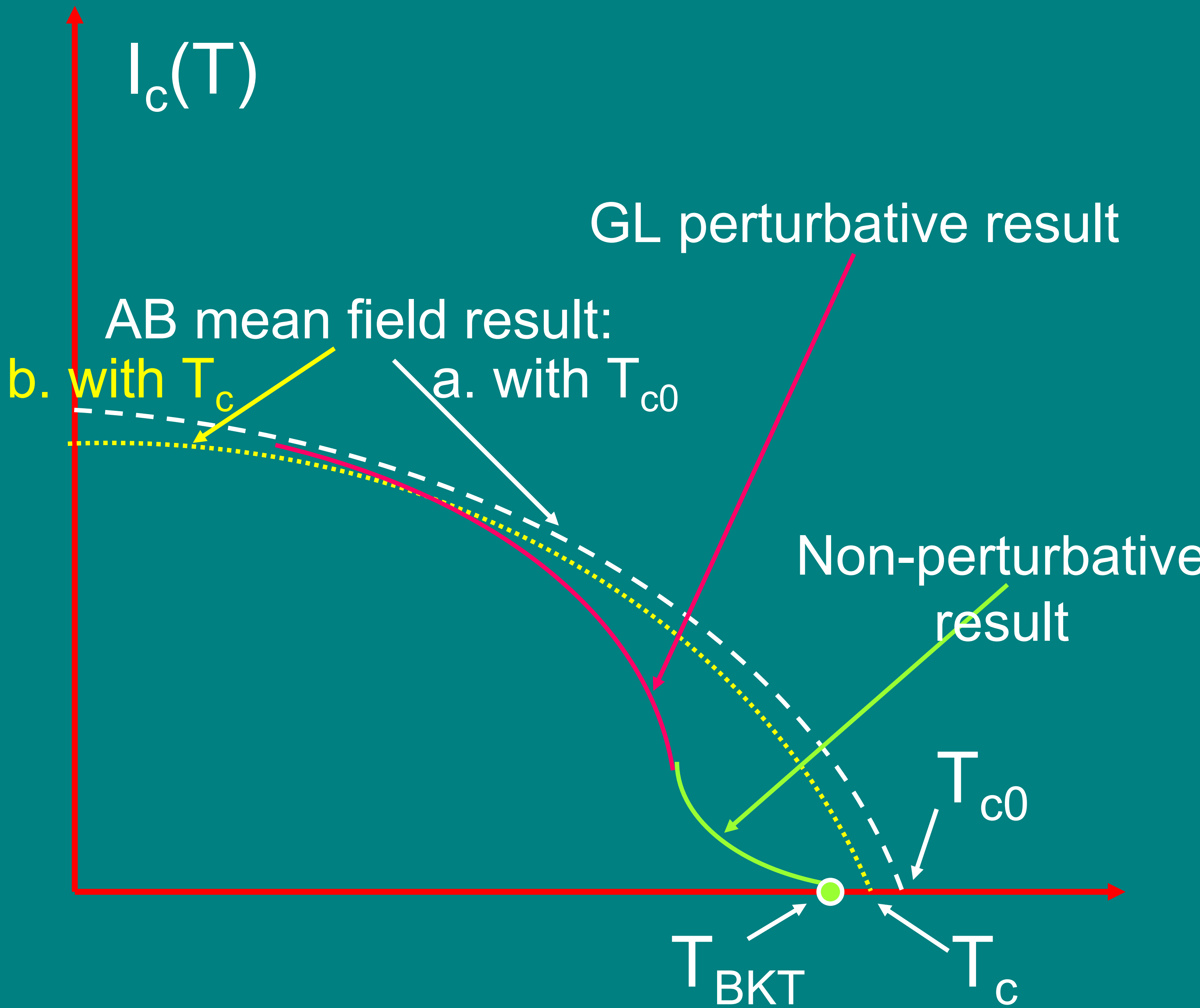
$$\frac{n_s}{2mL_J^2} = \frac{E_J}{2S}.$$

$$I_{\mathrm{c}}\left(\varphi\right) = \frac{E_J}{2S}\left\langle \sin\left(\varphi + \varphi^{(\mathrm{fl})}\right)\right\rangle$$

$$\delta E_J^{(\mathrm{fl})}\left(\epsilon\right) = E_{J0}\exp\left(-\frac{2Gi_{(2d)}}{\left|\epsilon\right|}\ln\frac{L_J}{\xi\left(T\right)}\right)$$

$$I_{\mathrm{c}}\left(\epsilon\right) = I_{\mathrm{c}}^{(AB)}\left(\epsilon\right)\exp\left(-\frac{\epsilon^*}{\left|\epsilon\right|}\right)$$

$$\epsilon^* = Gi_{(2d)}\ln\left[Gi_{(2d)}\left(\frac{L_J}{\xi}\right)^2\right]$$



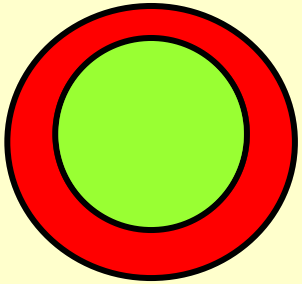
$I_c(T)$
GL perturbative result
AB mean field result:
b. with T_c
a. with T_{c0}
Non-perturbative result
T_{c0}
T_{BKT}
T_c

Микроскопическая теория

$$\mathcal{H} = \sum_{\mathbf{p},\sigma} \xi(\mathbf{p}) \widetilde{\psi}^{+}_{\mathbf{p},\sigma} \widetilde{\psi}_{\mathbf{p},\sigma} - g \sum_{\mathbf{p},\mathbf{p}',\mathbf{q},\sigma,\sigma'} \widetilde{\psi}^{+}_{\mathbf{p}+\mathbf{q},\sigma} \widetilde{\psi}^{+}_{-\mathbf{p},-\sigma} \widetilde{\psi}_{-\mathbf{p}',-\sigma'} \widetilde{\psi}_{\mathbf{p}'+\mathbf{q},\sigma'}.$$

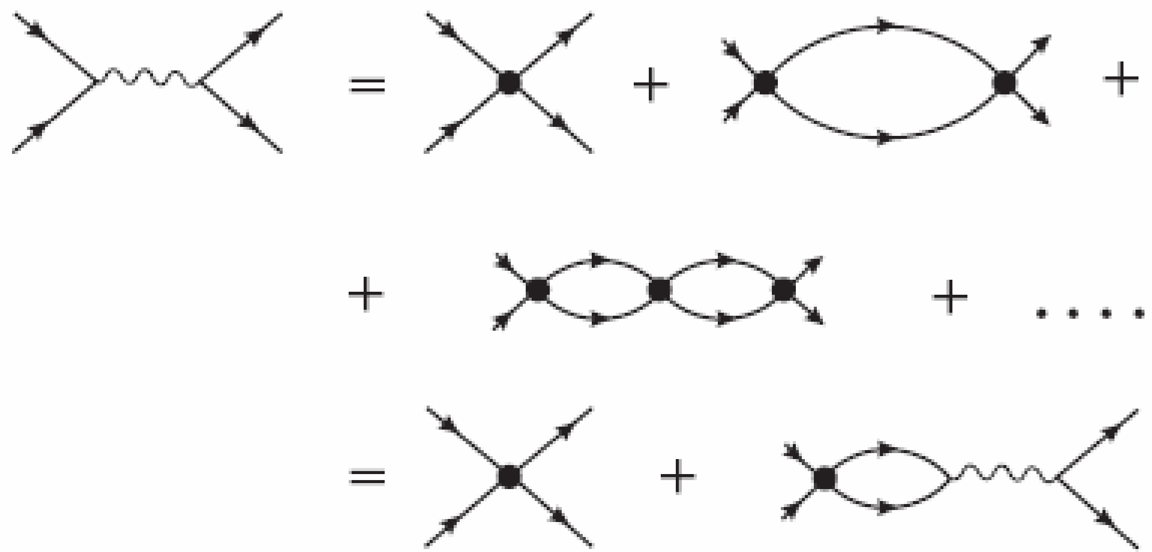
$$\xi(\mathbf{p}) = E(\mathbf{p}) - E_F$$

$$p_F - \frac{\omega_D}{v_F} < |\mathbf{p}|, \quad |\mathbf{p}'| < p_F + \frac{\omega_D}{v_F},$$



$$G(\mathbf{p},\, \varepsilon_n) = \frac{1}{i\varepsilon_n - \xi(\mathbf{p})},$$

$$\varepsilon_n = (2n+1)\pi T$$



$$L^{-1}(\mathbf{q},\,\Omega_k) = -g^{-1} + \Pi(\mathbf{q},\,\Omega_k),$$

$$\Pi(\mathbf{q},\,\Omega_k) = T\sum_{\varepsilon_n}\int \frac{d^3\mathbf{p}}{(2\pi)^3} G(\mathbf{p}+\mathbf{q},\,\varepsilon_{n+k})G(-\mathbf{p},\,\varepsilon_{-n}).$$

$$\mathcal{P}(\mathbf{q},\,\varepsilon_1,\,\varepsilon_2)=\int\frac{d^3\mathbf{p}}{(2\pi)^3}G(\mathbf{p}+\mathbf{q},\,\varepsilon_1)G(-\mathbf{p},\,\varepsilon_2)=$$

$$=2\pi\nu\Theta(-\varepsilon_1\varepsilon_2)\left\langle\frac{1}{|\varepsilon_1-\varepsilon_2|+i\Delta\xi(\mathbf{q},\,\mathbf{p})|_{\epsilon(\mathbf{p})=E_F}}\right\rangle_{\text{F.S.}}$$

$$\Delta\xi(\mathbf{q},\,\mathbf{p})|_{\epsilon(\mathbf{p})=E_F}=[\xi(\mathbf{q}+\mathbf{p})-\xi(-\mathbf{p})]|_{\epsilon(\mathbf{p})=E_F}\approx(\mathbf{v_p}\mathbf{q})_{\xi(\mathbf{p})=0}.$$

$$\mathcal{P}(\mathbf{q},\,\varepsilon_1,\,\varepsilon_2)=2\pi\nu\frac{\Theta(-\varepsilon_1\varepsilon_2)}{|\varepsilon_1-\varepsilon_2|}\left(1-\frac{\langle(\mathbf{v_p}\mathbf{q})^2\rangle_{\text{F.S.}}}{|\varepsilon_1-\varepsilon_2|^2}\right)$$

$$\Pi(\mathbf{q},\,\Omega_k)=T\sum_{\varepsilon_n}\mathcal{P}(\mathbf{q},\,\varepsilon_{n+k},\,\varepsilon_{-n})=$$

$$=\nu\left[\sum_{n\geqslant 0}\frac{1}{n+1/2+\frac{|\Omega_k|}{4\pi T}}-\frac{\langle(\mathbf{v_p}\mathbf{q})^2\rangle_{\text{F.S.}}}{(4\pi T)^2}\sum_{n=0}^{\infty}\frac{1}{\left(n+1/2+\frac{|\Omega_k|}{4\pi T}\right)^3}\right].$$

$$\frac{1}{\nu}\,\Pi(\mathbf{q},\,\Omega_k) = \psi\,\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}+\frac{\omega_D}{2\pi T}\right) - \psi\,\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}\right) + \\ + \frac{\langle(\mathbf{v_p q})^2\rangle_{\mathrm{F.S.}}}{2(4\pi T)^2}\psi''\,\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}\right)$$

$$L^{-1}(\mathbf{q}=0,\,\Omega_k=0,\,T_c) = -g^{-1} + \Pi\,(0,\,0,\,T_c) = 0.$$

$$T_c = \frac{2\gamma_E}{\pi}\omega_D\,\exp\,\left(-\frac{1}{\nu g}\right)\,.$$

$$L^{-1}(\mathbf{q},\,\Omega_k) = \\ = -\nu\left[\epsilon + \psi\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}\right) - \psi\left(\frac{1}{2}\right) - \frac{\langle(\mathbf{v_p q})^2\rangle_{\mathrm{F.S.}}}{2(4\pi T)^2}\,\psi''\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}\right)\right].$$

$$L(\mathbf{q},\,0) = -\frac{1}{\nu}\frac{1}{\epsilon + \xi^2\mathbf{q}^2},$$

$$\langle|\Psi_{\mathbf{q}}|^2\rangle$$

$$\xi^2_{(D)} = \frac{7\zeta(3)\mathbf{v}_{\mathbf{F}}^2}{16D\pi^2T^2}.$$

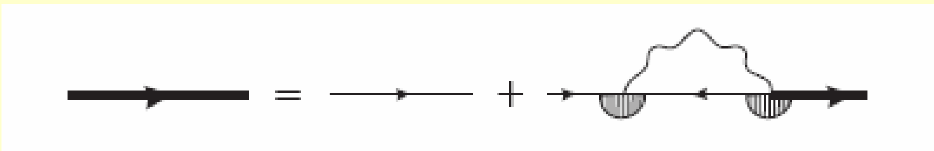
$$i\Omega_k \to \Omega$$

$$L^R(\mathbf{q},\,\Omega) = -\frac{1}{\nu}\frac{1}{-\frac{i\pi}{8T}\Omega + \epsilon + \xi^2\mathbf{q}^2} = \frac{8T}{\pi\nu}\frac{1}{i\Omega - \left(\tau_{\mathrm{GL}}^{-1} + \frac{8T}{\pi}\xi^2\mathbf{q}^2\right)}$$

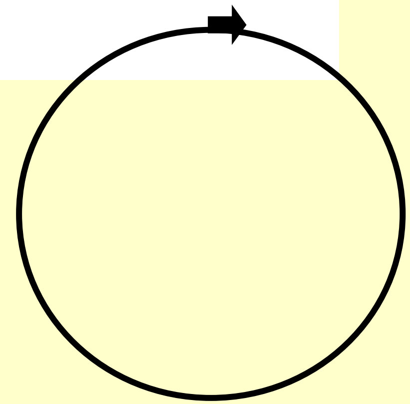
$$\tau_{\mathrm{GL}} = \pi/(8(T - T_{\mathrm{c}}))$$

$$\alpha T_{\mathrm{c}} = \nu \ \text{и}\ \gamma_{\mathrm{GL}} = \pi\alpha/8 = \pi\nu/8T_{\mathrm{c}}$$

Диаграммное представление флуктуационных поправок

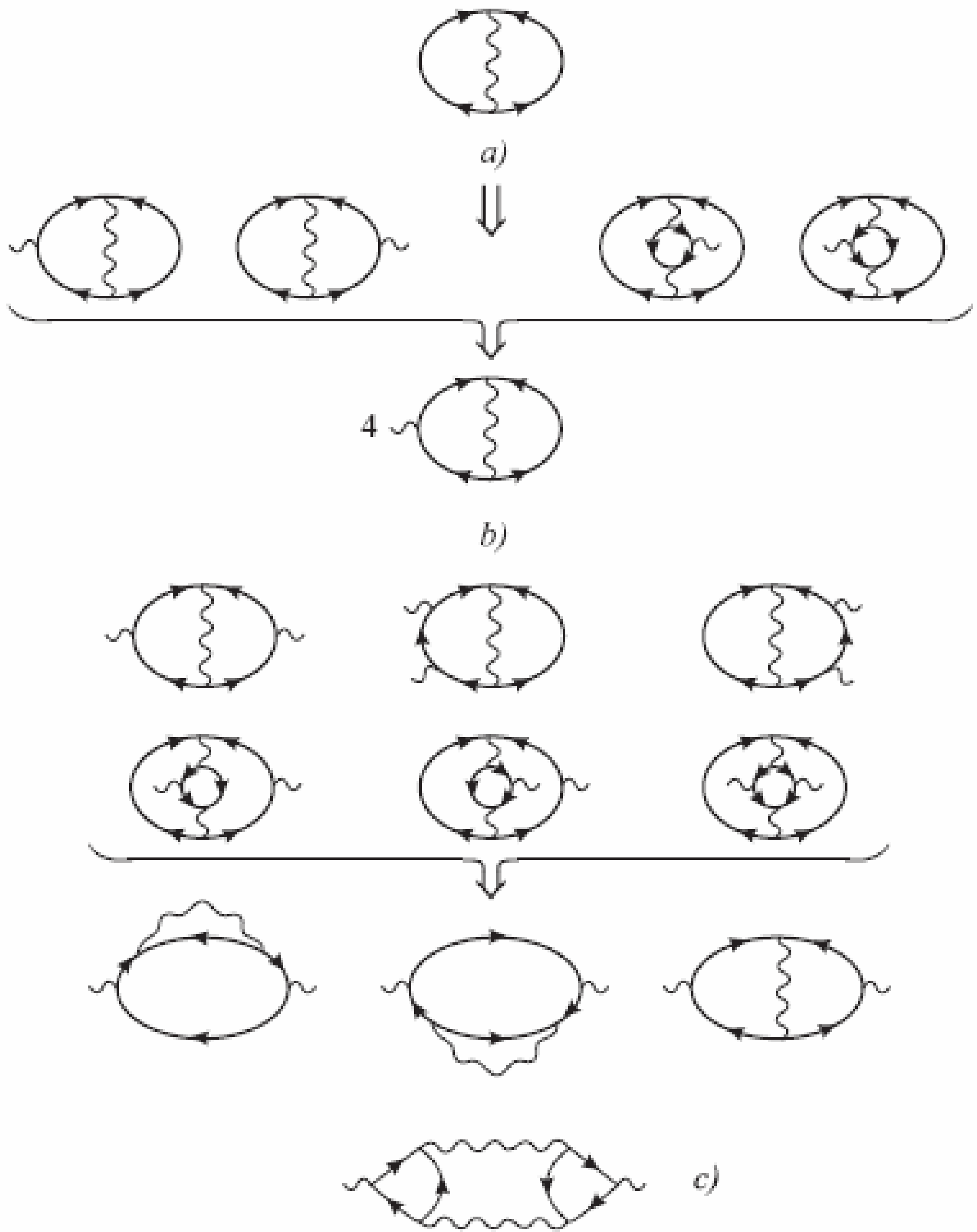


Гриновская функция



Термодинамический потенциал

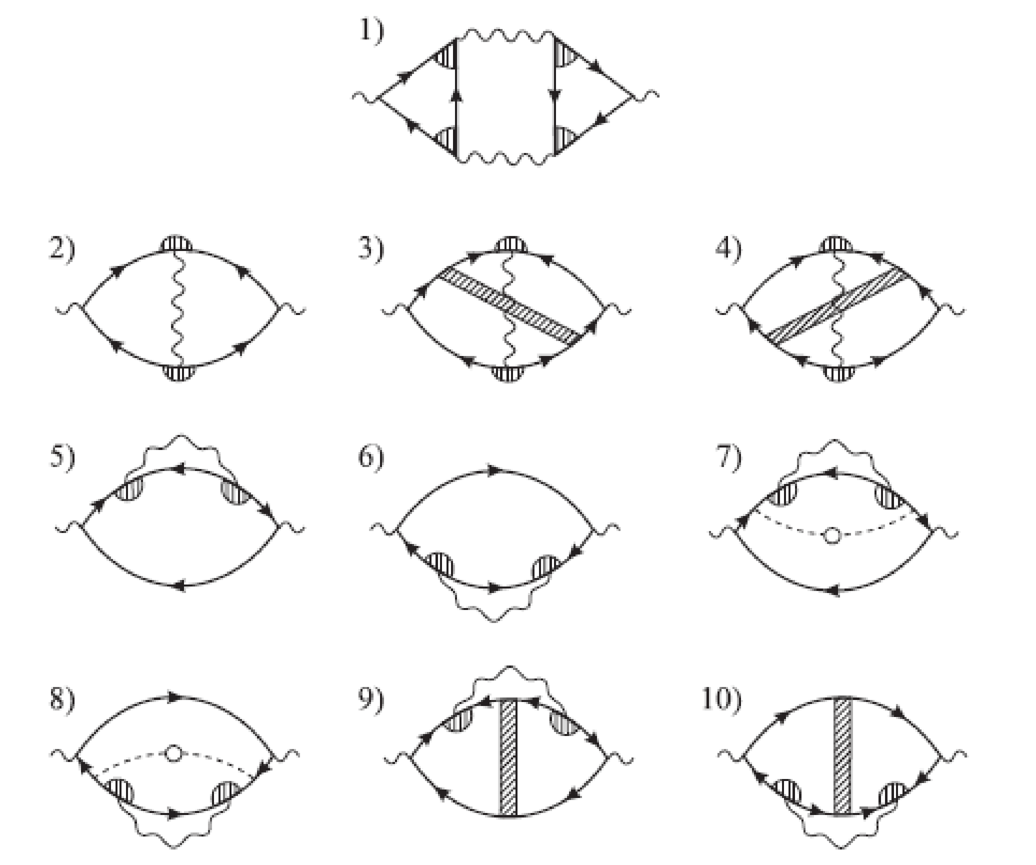
$$\frac{\partial G^{(0)}(\mathbf{p},\varepsilon_n)}{\partial \mathbf{p}} = \frac{\partial}{\partial \mathbf{p}} \frac{1}{i\varepsilon_n - \xi(\mathbf{p})} = G^{(0)}(\mathbf{p},\varepsilon_n)\mathbf{v}G^{(0)}(\mathbf{p},\varepsilon_n).$$


a)
4
b)
c)

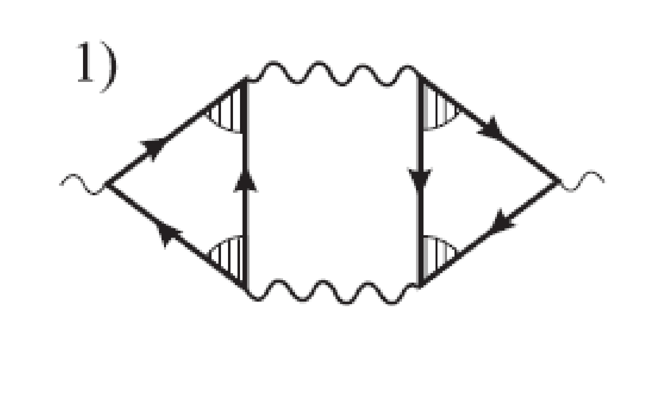
$$\mathbf{j}_\alpha(\mathbf{r}, t) = -\int Q_{\alpha\beta}(\mathbf{r}, \mathbf{r}', t, t')\mathbf{A}_\beta(\mathbf{r}', t')\, d\mathbf{r}'\, dt.$$

$$j_\alpha = \sigma_{\alpha\beta} E_\beta$$

$$\sigma_{\alpha\beta}(\omega) = -\frac{1}{i\omega}\left[Q_{\alpha\beta}\right]^R(\omega).$$



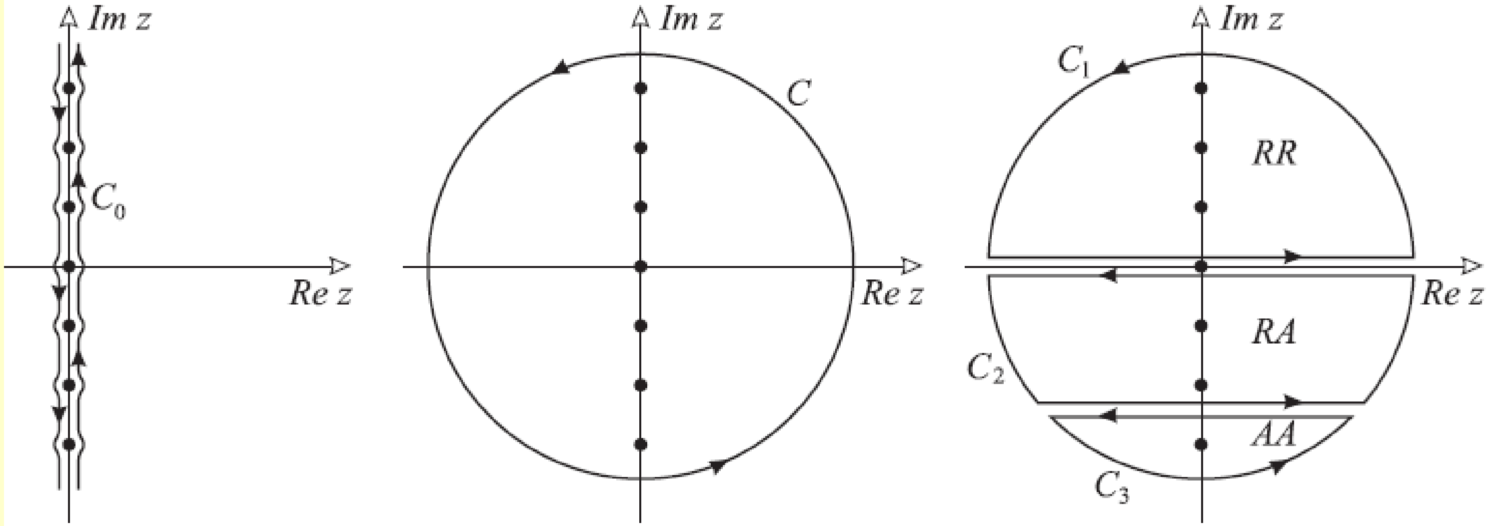
Парапроводимость



$$Q^{AL}_{\alpha\beta}(\omega_\nu) = -4e^2 T \sum_{\Omega_k} \int \frac{d^3\mathbf{q}}{(2\pi)^3} B_\alpha(\mathbf{q}, \Omega_k, \omega_\nu) L(\mathbf{q}, \Omega_k) \times$$
$$\times B_\beta(\mathbf{q}, \Omega_k, \omega_\nu) L(\mathbf{q}, \Omega_k + \omega_\nu)$$

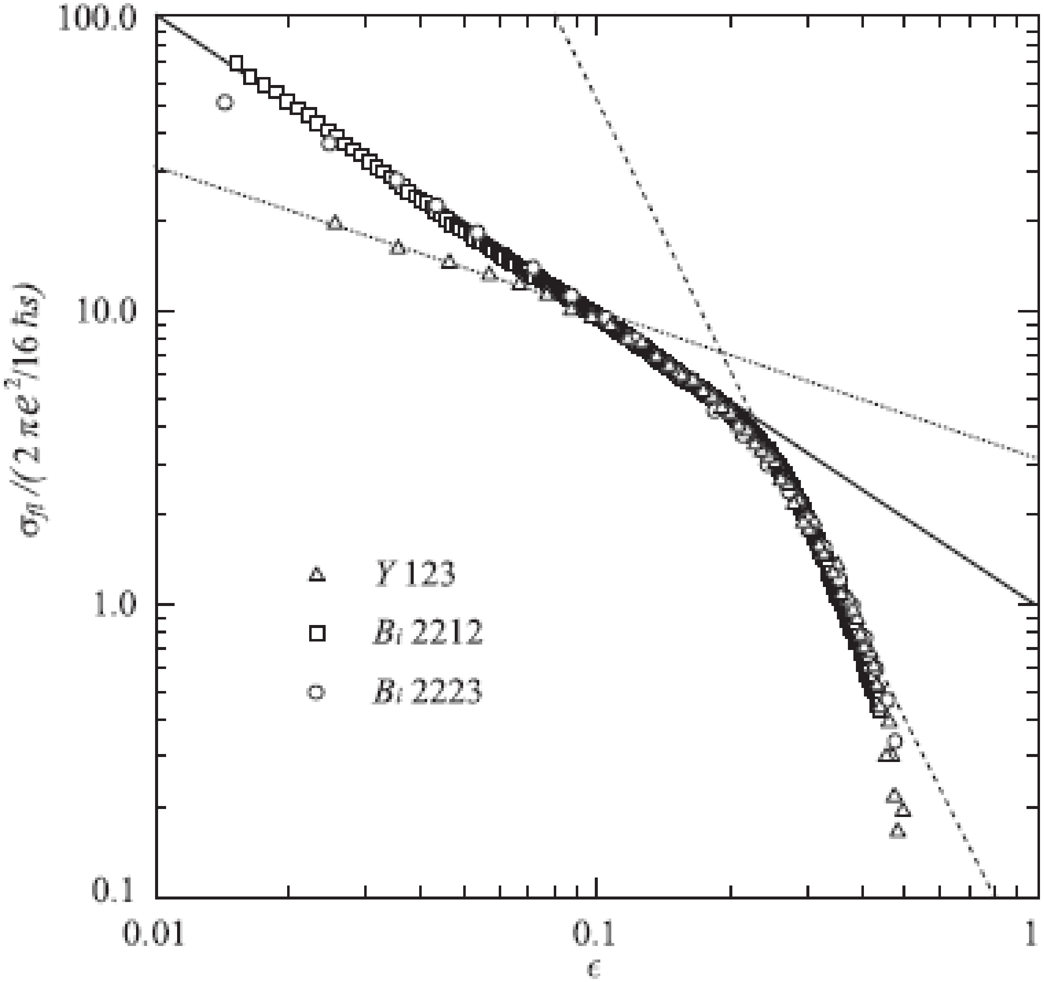
где

$$B_\alpha(\mathbf{q}, \Omega_k, \omega_\nu) = T \sum_{\varepsilon_n} \lambda(\mathbf{q}, \varepsilon_{n+\nu}, \Omega_k - \varepsilon_n) \lambda(\mathbf{q}, \varepsilon_n, \Omega_k - \varepsilon_n) \times$$
$$\times \int \frac{d^3\mathbf{p}}{(2\pi)^3} v_\alpha(\mathbf{p}) G(\mathbf{p}, \varepsilon_{n+\nu}) G(\mathbf{p}, \varepsilon_n) G(\mathbf{q} - \mathbf{p}, \Omega_k - \varepsilon_n$$



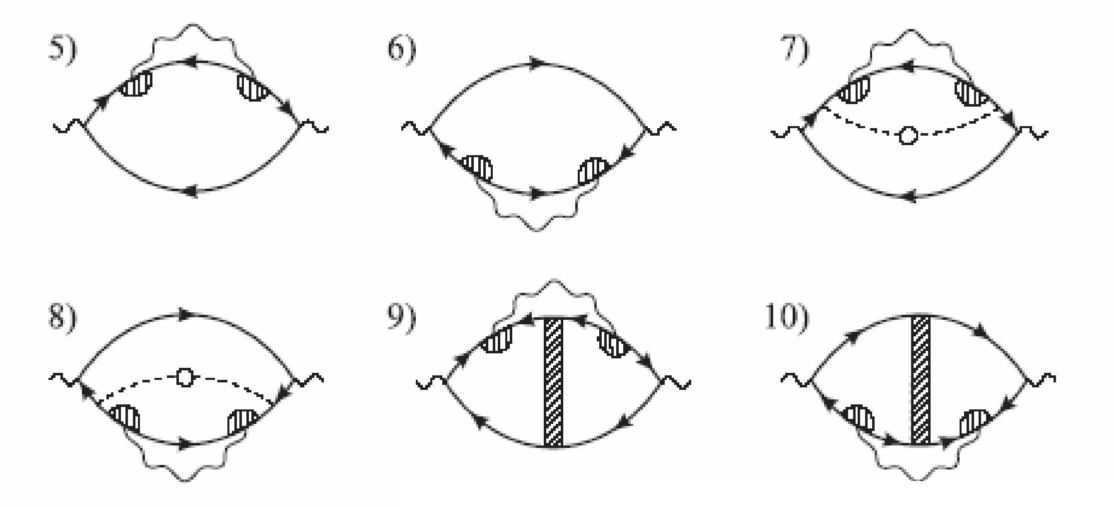
$$T\sum_{\Omega_k} f(\Omega_k) = \frac{1}{4\pi i}\oint_{C_0} dz\,\coth\frac{z}{2T}\,f(-iz),$$

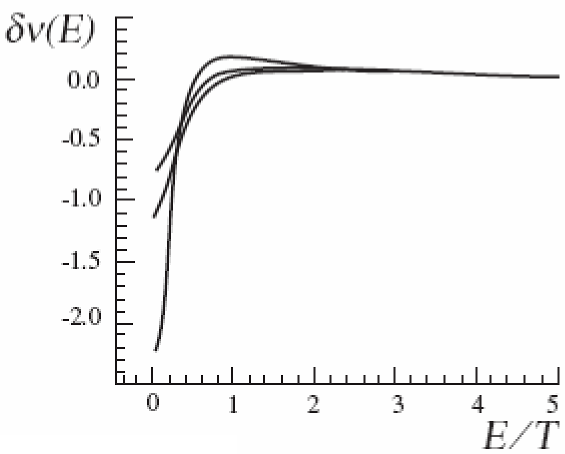
$$\sigma^{\alpha\beta}_{(D)} = \delta^{\alpha\beta}\begin{cases} \dfrac{e^2}{32\xi}\dfrac{1}{\sqrt{\epsilon}} & \text{3D case,} \\ \dfrac{e^2}{16d}\dfrac{1}{\epsilon} & \text{2D film, thickness}: d \ll \xi, \\ \dfrac{\pi e^2 \xi}{16S}\dfrac{1}{\epsilon^{3/2}} & \text{1D wire, cross-section: } S \ll \xi^2. \end{cases}$$



100.0
10.0
1.0
0.1
0.01
0.1
1
ϵ
σ_fl /(2 π e²/16 ħs)
Y 123
Bi 2212
Bi 2223

Вклад связанный с перенормировкой DOS



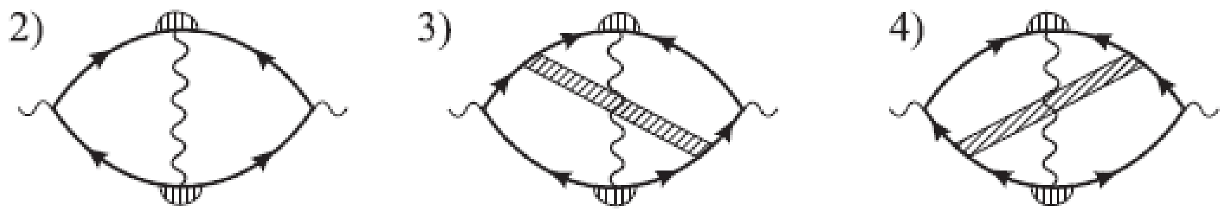


$$\delta\sigma_{xx}^{DOS} = -\frac{\Delta\mathcal{N}_e e^2\tau}{m} = -\frac{2n_S e^2\tau}{m}$$

$$n_s^{(2)} = \frac{1}{4\pi\alpha\xi^2}\frac{1}{s}\ \ln\frac{1}{\epsilon}$$

$$\Delta\sigma_{(2)}^{DOS} = -\ 0.1e^2/\hbar\ \ln(1/\varepsilon)$$

Вклад Маки-Томпсона



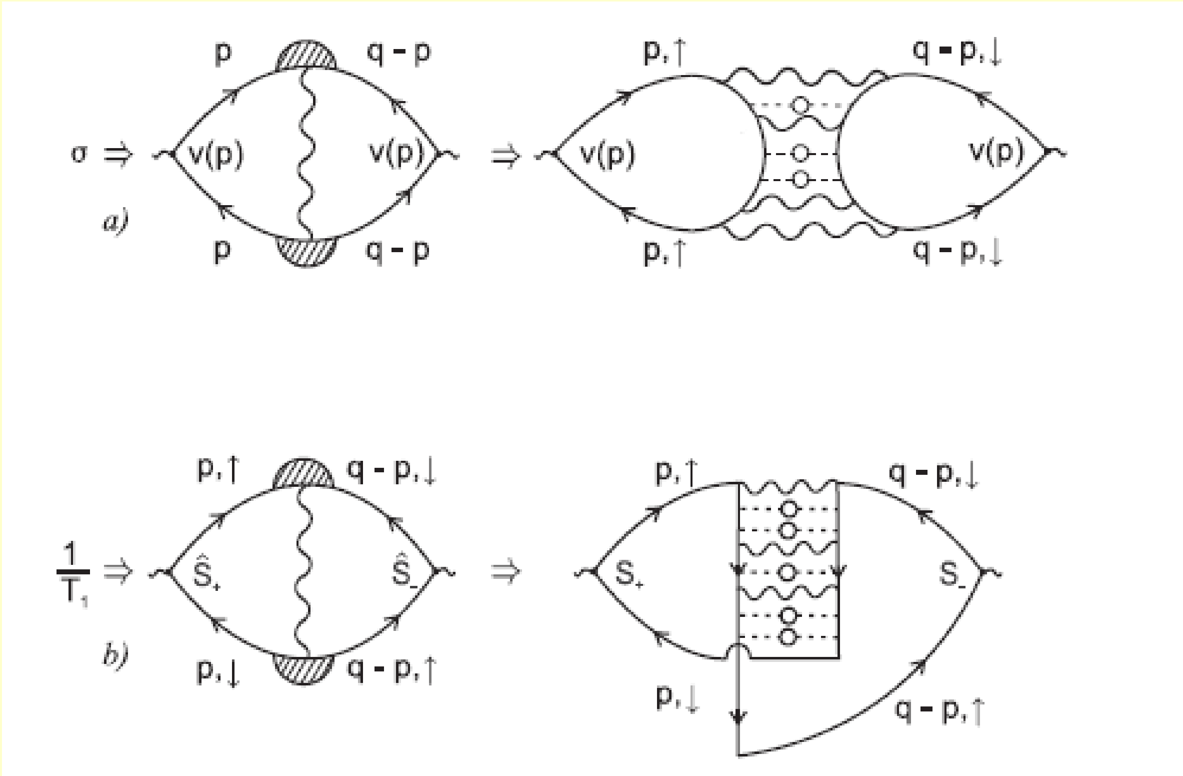
$$Q_{\alpha\beta}^{MT}(\omega_\nu) = 2e^2 T \sum_{\Omega_k} \int \frac{d^3\mathbf{q}}{(2\pi)^3} L(\mathbf{q}, \Omega_k) I_{\alpha\beta}(\mathbf{q}, \Omega_k, \omega_\nu),$$

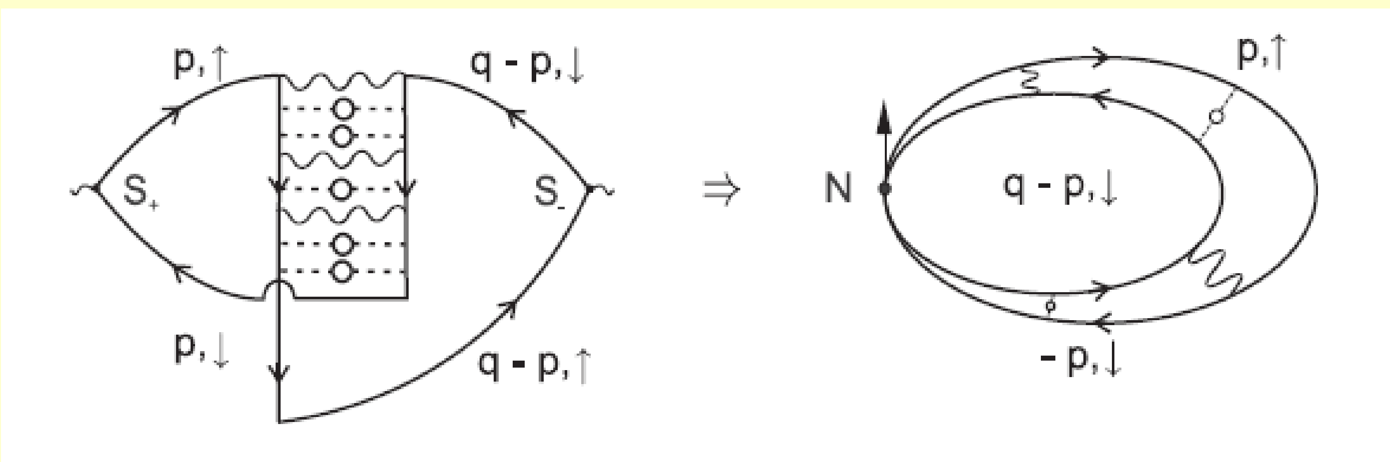
$$I_{\alpha\beta}^{MT}(q, 0, \omega_\nu) = \pi\nu \langle v_\alpha(p) v_\beta(q-p) \rangle_{FS} \times \qquad (7.$$

$$\times T \sum_{\varepsilon_n} \frac{1}{(|2\varepsilon_{n+\nu}| + \widehat{\mathcal{D}}q^2)} \frac{1}{(|2\varepsilon_n| + \widehat{\mathcal{D}}q^2)} \frac{1}{|\widetilde{\varepsilon}_{n+\nu}| + |\widetilde{\varepsilon}_n|}.$$

$$\sim \frac{1}{2(\omega_\nu + \widehat{\mathcal{D}}q^2)} \left[\sum_{n=0}^{\nu-1} \frac{1}{2\varepsilon_n + \widehat{\mathcal{D}}q^2} \right.$$

$$\delta\sigma^{MT} \sim \frac{e^2}{8\epsilon} \ln \frac{\mathcal{D}\tau_\varphi}{\xi^2(T)}.$$


p
q - p
p, ↑
q - p, ↓
σ ⇒
v(p)
v(p)
⇒
v(p)
v(p)
a)
p
q - p
p, ↑
q - p, ↓
p, ↑
q - p, ↓
p, ↑
q - p, ↓
1/T₁ ⇒
Ŝ₊
Ŝ₋
⇒
S₊
S₋
b)
p, ↓
q - p, ↑
p, ↓
q - p, ↑


p, ↑
q - p, ↓
S₊
S₋
p, ↓
q - p, ↑
⇒
N
p, ↑
q - p, ↓
- p, ↓

Diffusion Equation

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$

Diffusion constant

Einstein-Sutherland Relation for electric conductivity σ

$$\sigma = e^2 D \nu \qquad \nu \equiv \frac{dn}{d\mu}$$

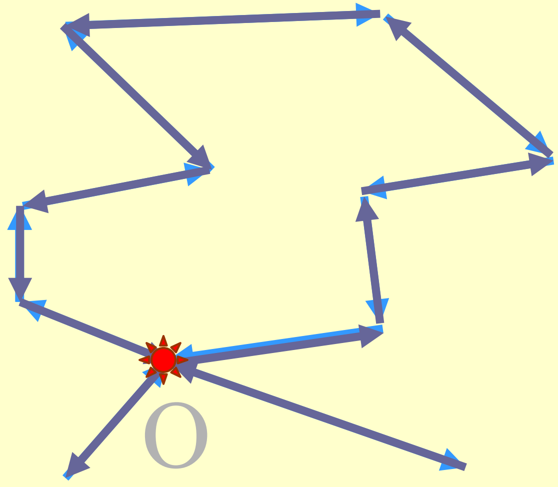
- Universality: the diffusion equation is valid as long as the process is marcovian

Einstein: there is no diffusion at too short scales –there is memory, i.e.,the process is not marcovian.

WEAK LOCALIZATION

$$\varphi = \oint \vec{p}\,d\vec{r}$$

Phase accumulated when traveling along the loop



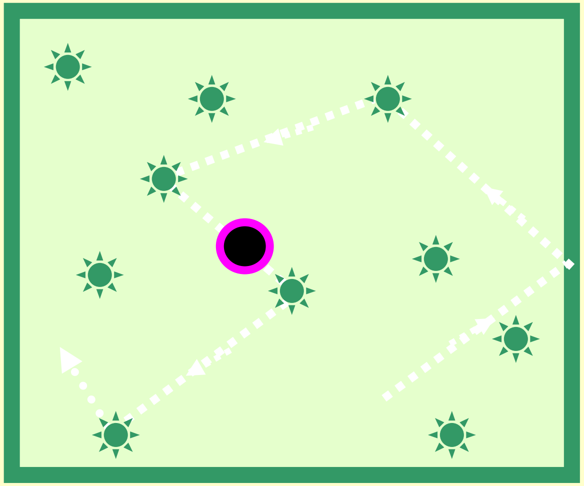
The particle can go around the loop in two directions

Memory!

$$\varphi_1 = \varphi_2$$

Constructive interference → probability to return to the origin gets enhanced → diffusion constant gets reduced. Tendency towards localization

Diffusion



Random walk

Density fluctuations $\rho(\mathbf{r},t)$ at a given point in space $\mathbf{r}$ and time t.

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$

Diffusion Equation

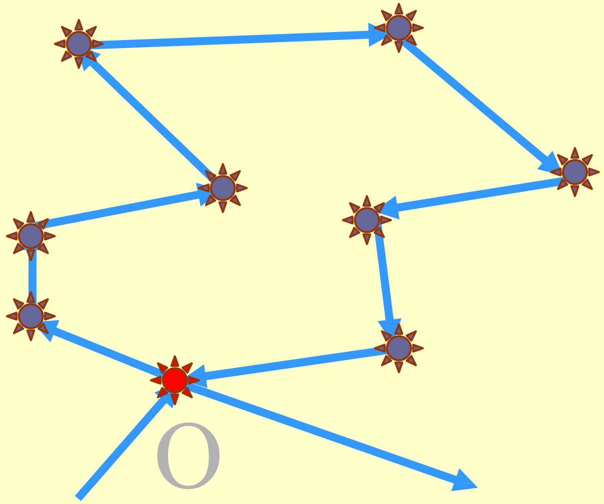
D - Diffusion constant

Mean squared distance from the original point at time t

$$\langle r(t)^2 \rangle = Dt$$

Probability to come back (to the element of the volume dV centered at the original point)

$$P\left(r(t)=0\right)dV = \frac{dV}{\left(Dt\right)^{d/2}}$$



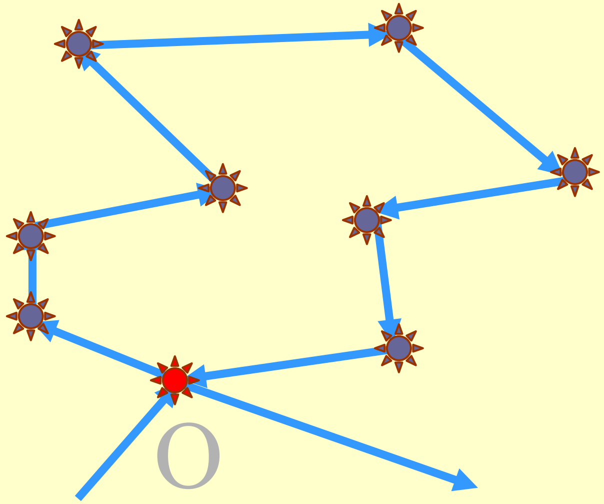
What is the probability ***P(t)*** that such a loop is formed within a time t ?

Probability to come back (to the element of the volume *dV* around the original point)

$$P\left(r\left(t\right)=0\right)dV=\frac{dV}{\left(Dt\right)^{d/2}}$$

Q: $dV=?$

A: $dV=\lambda^{d-1}v_F dt$



What is the probability *P(t)* that such a loop is formed within a time t ?

Probability to come back (to the element of the volume dV around the original point)

$$P(r(t)=0)\,dV = \frac{dV}{(Dt)^{d/2}}$$

$$\textbf{Q:}\quad dV = ?$$

$$\textbf{A:}\quad dV = \lambda\!\!\!^{-}{}^{d-1} v_F\, dt$$

$$P(t) = -\lambda\!\!\!^{-}{}^{d-1}\int_{\tau}^{t}\frac{v_F\,dt'}{(Dt')^{d/2}}$$

$$\frac{\delta g}{g} \approx P(t_{\max})$$

$$P(t) = -\lambda^{d-1}\int_{\tau}^{t}\frac{v_F\,dt'}{(Dt')^{d/2}}$$

$$\frac{\delta g}{g} \approx P(t_{\max})$$

Q: $t_{\max} = ?$

A: $t_{\max} \sim \min\left\{\frac{L^2}{D}, \frac{1}{\omega}, \tau_{\varphi,\ldots}\right\}$

Т. 18 Журнал экспериментальной и теоретической физики. Вып.

1948

ОБ ИЗМЕНЕНИИ ЭЛЕКТРИЧЕСКОГО СОПРОТИВЛЕНИЯ ТЕЛЛУРА В МАГНИТНОМ ПОЛЕ ПРИ НИЗКИХ ТЕМПЕРАТУРАХ

Р. А. Ченцов

R.A. Chentsov "***On the variation of electrical conductivity of tellurium in magnetic field at low temperatures***", *Zh. Exp. Theor. Fiz. v.18, 375-385, (1948).*

Таблица 2

Уменьшение сопротивления теллура в магнитном поле

Образец	Температура (°К)	Максимальное уменьшение сопротивления
Te-1	2,13	$0,7 \cdot 10^{-3}$
Te-2	2,15	$1,0 \cdot 10^{-3}$
Te-4	1,96	$1,1 \cdot 10^{-3}$
Te-5	1,96	$0,5 \cdot 10^{-2}$

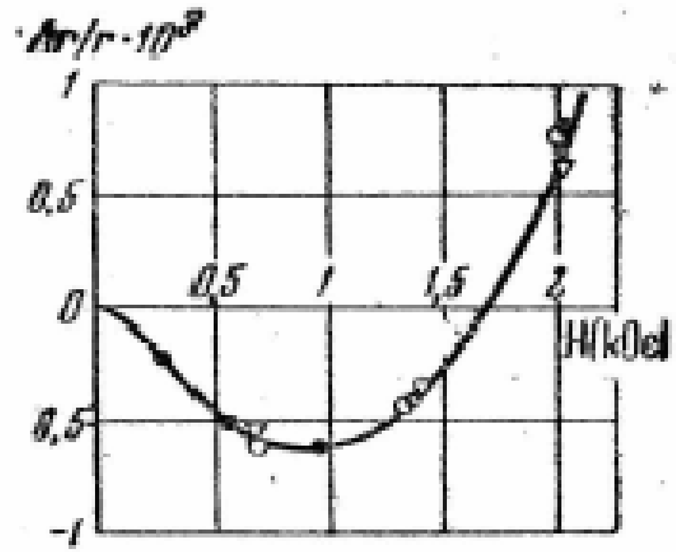
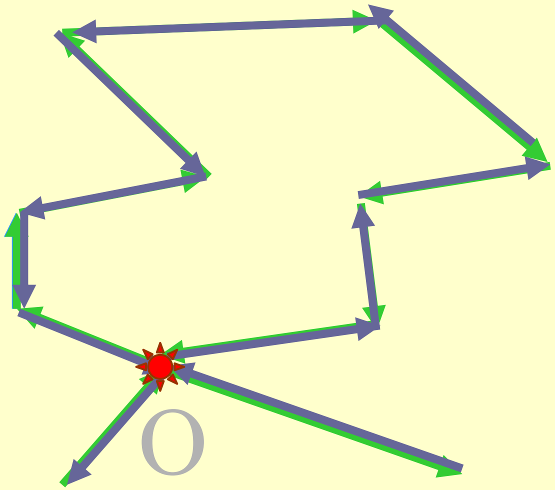
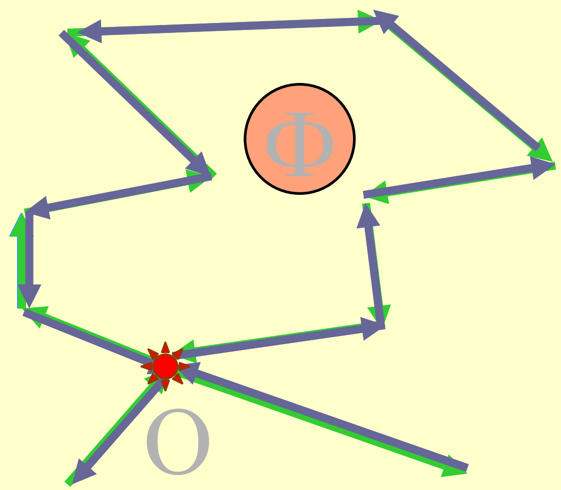


Рис. 2

Magnetoresistance





No magnetic field

$$\varphi_1 = \varphi_2$$

With magnetic field H

$$\varphi_1 - \varphi_2 = 2*2\pi\,\Phi/\Phi_0$$

Length Scales

Magnetic length $L_H = (hc/eH)^{1/2}$

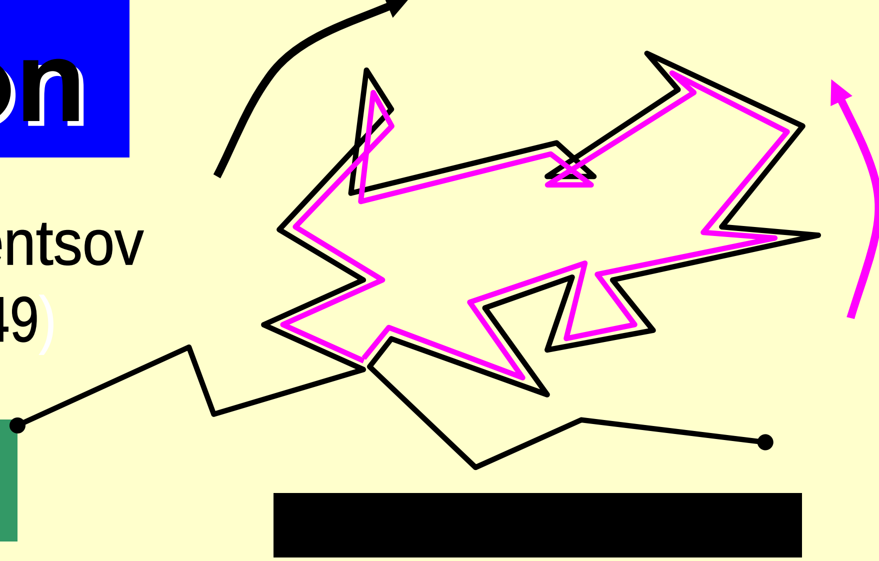
Dephasing length $L_\varphi = (D\,\tau_\varphi)^{1/2}$

$$\delta g(H) = f_d\left(\frac{L_H}{L_\varphi}\right)$$

Universal functions

Magnetoresistance measurements allow to study inelastic collisions of electrons with phonons and other electrons

Weak Localization



Negative Magnetoresistance

Chentsov (1949)

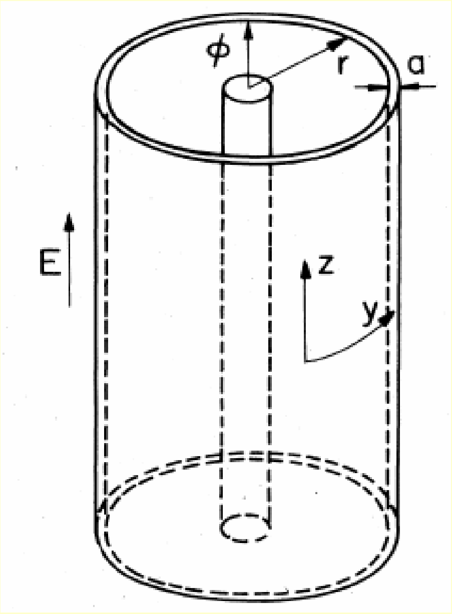
Aharonov-Bohm effect

Theory

B.A., Aronov & Spivak (1981)

Experiment

Sharvin & Sharvin (1981)



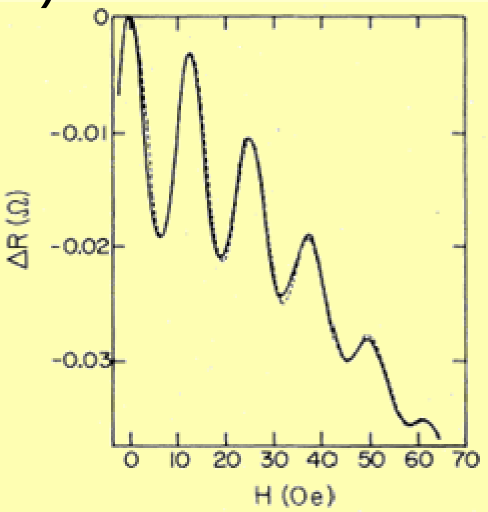
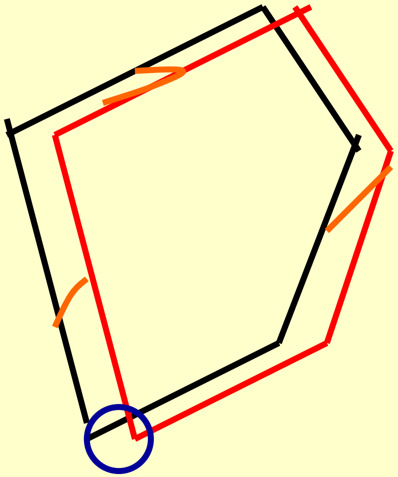


FIG. 8. Longitudinal magnetoresistance $\Delta R(H)$ at $T=1.1$ K for a cylindrical lithium film evaporated onto a 1-cm-long quartz filament. $R_{4.2}=2$ kΩ, $R_{300}/R_{4.2}=2.8$. Solid line: averaged from four experimental curves. Dashed line: calculated for $L_\varphi=2.2$ μm, $\tau_\varphi/\tau_{so}=0$, filament diameter $d=1.31$ μm, film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d=1.30\pm0.03$ μm (Altshuler *et al.*, 1982; Sharvin, 1984).

Физика вклада Маки-Томпсона

$$g_{\text{eff}} = \frac{g}{1 - \nu g \ln \dfrac{\omega_D}{2\pi T}} = \frac{1}{\ln \dfrac{T}{T_c}} \approx \frac{T}{T - T_c} = \frac{1}{\epsilon}.$$



$$n(p) \sim 1/\epsilon$$

$$R(t) \sim (\mathcal{D}t)^{1/2}$$

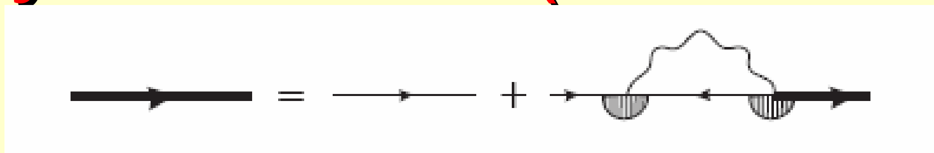
$$W \sim \int_{t_{\min}}^{t_{\max}} \frac{\lambda_F^{D-1}}{R^D(t)} v_F \, dt.$$

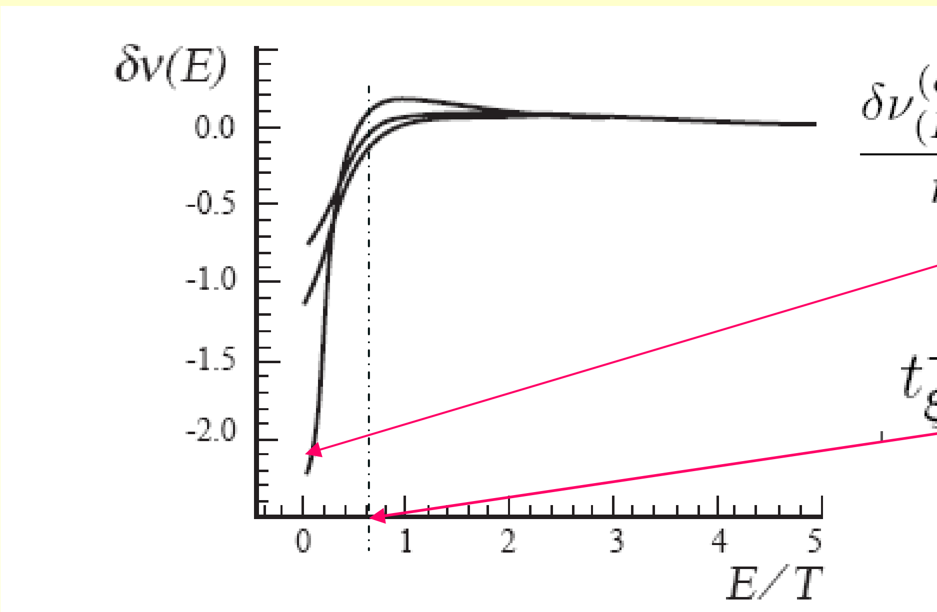
$$R(t_{\min}) \sim \xi(T)$$

$$\delta\sigma^{MT}/\sigma = W \; g_{\text{eff}}.$$

$$\delta\sigma^{MT} \sim \frac{e^2}{8\epsilon} \ln \frac{\mathcal{D}\tau_\varphi}{\xi^2(T)}$$

Fluctuation Renormalization of the density of states (Redi et al. 1970)





$$\frac{\delta\nu^{(d)}_{(D)}(0,\epsilon)}{\nu_{(D)}} \sim -\begin{cases} \sqrt{Gi_{(3,d)}}\,\epsilon^{-3/2}, & D=3, \\ Gi_{(2,d)}\epsilon^{-2}, & D=2. \end{cases}$$

$$t_\xi^{-1} = \mathcal{D}\xi^{-2}(T) \sim \tau_{\mathrm{GL}}^{-1} \sim T - T_{\mathrm{c}}$$

$$\int_0^\infty \delta\nu(E)\,dE = 0$$

Tunneling Hamiltonian: Ambegaokar-Baratoff formula

$$I_T = e\left\langle \frac{d\widehat{N}_L(t)}{dt} \right\rangle$$

$$\widehat{\mathcal{H}}_T = \sum_{\mathbf{p},\mathbf{k}} \left(T_{\mathbf{p},\mathbf{k}} \widehat{a}^+_{\mathbf{p}} \widehat{b}_{\mathbf{k}} + T^*_{\mathbf{k},\mathbf{p}} \widehat{a}_{\mathbf{p}} \widehat{b}^+_{\mathbf{k}} \right)$$

$$I_{qp}(V) = \frac{1}{eR_n \nu_L \nu_R} \int_{-\infty}^{\infty} \left(\tanh\frac{E+eV}{2T} - \tanh\frac{E}{2T} \right) \nu_R(E) \nu_L(E+eV) dE,$$

Zero bias anomaly in disordered metal (Altshuler, Aronov, 1979)

$$G_{qp}(V) = \frac{I_{qp}\ (V)}{dV} = \frac{1}{2TR_n\nu_R} \int_{-\infty}^{\infty} \cosh^{-2}\left(\frac{E+eV}{2T}\right)\ \nu_R(E)dE.$$

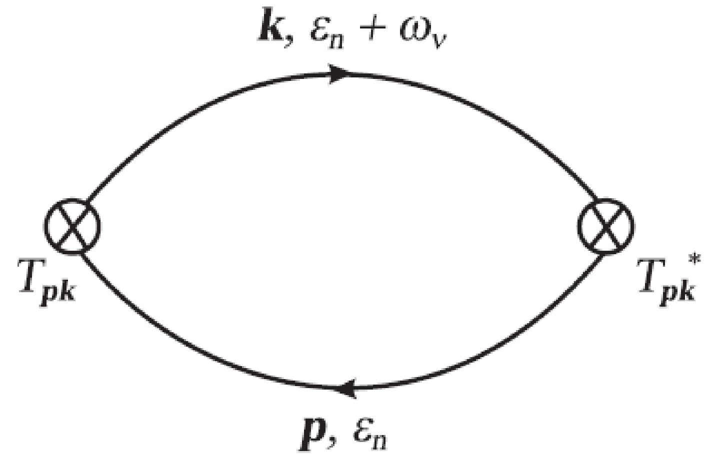
$$\tau^{-1} \gg T$$

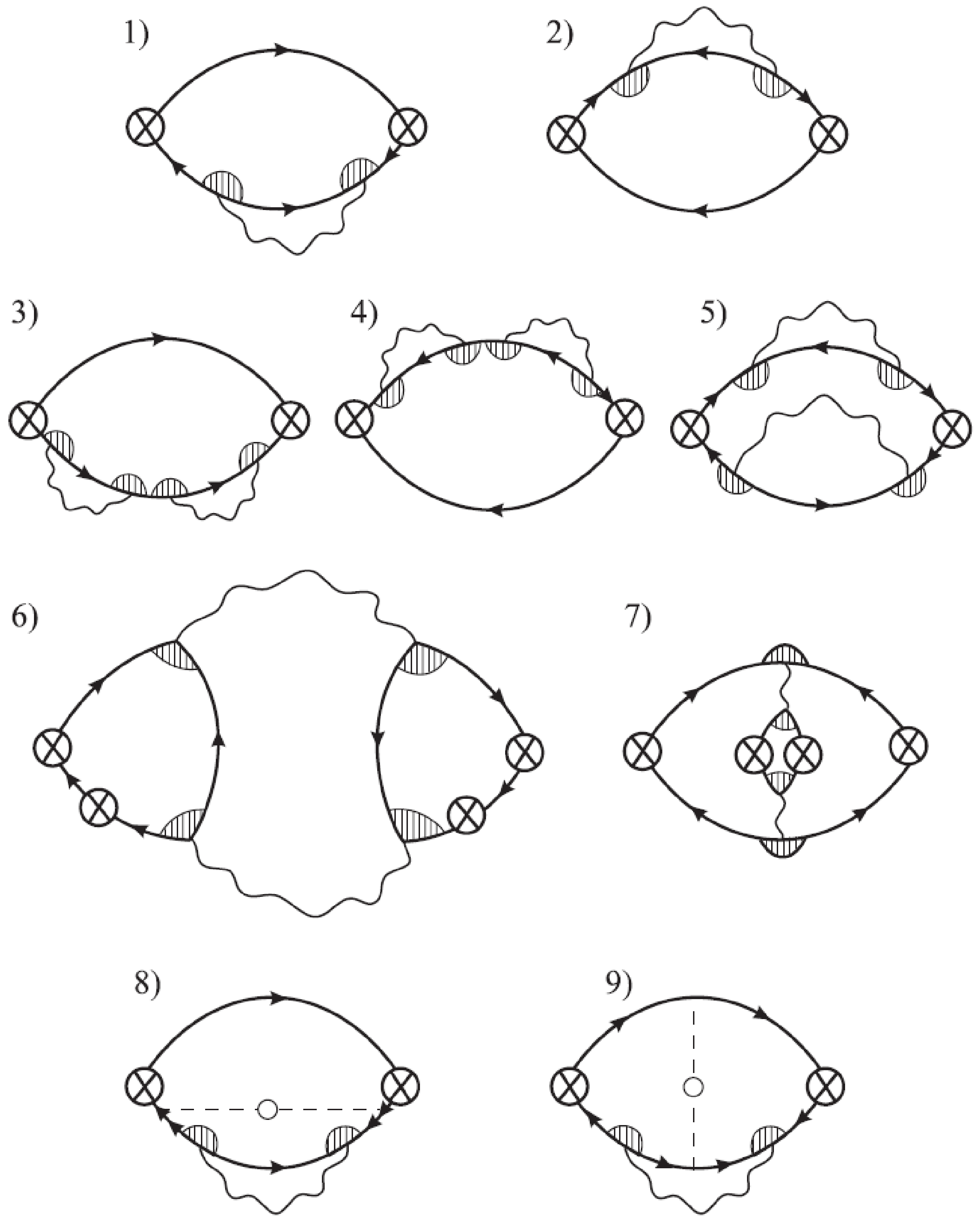
$$\frac{\delta G(V)}{G_n(0)} = \frac{\delta\nu(eV)}{\nu(0)}.$$

Fluctuation Pseudogap in N(S)-I-N Junction (Varlamov, Dorin, 1983)

$$I_T(V) = -e \ \mathrm{Im}\ K^R \ (\omega_\nu \to -ieV) \ ,$$

$$K \ (\omega_\nu) = 4T \sum_{\varepsilon_n} \sum_{\mathbf{p},\mathbf{k}} |T_{\mathbf{p},\mathbf{k}}|^2 G_R \ (\mathbf{p}, \ \varepsilon_n + \omega_\nu) \ G_L \ (\mathbf{k}, \ \varepsilon_n)$$

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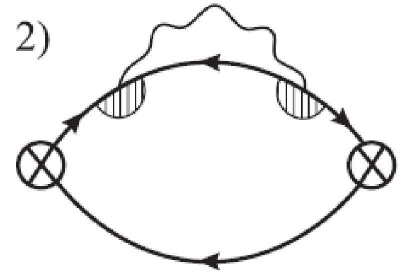
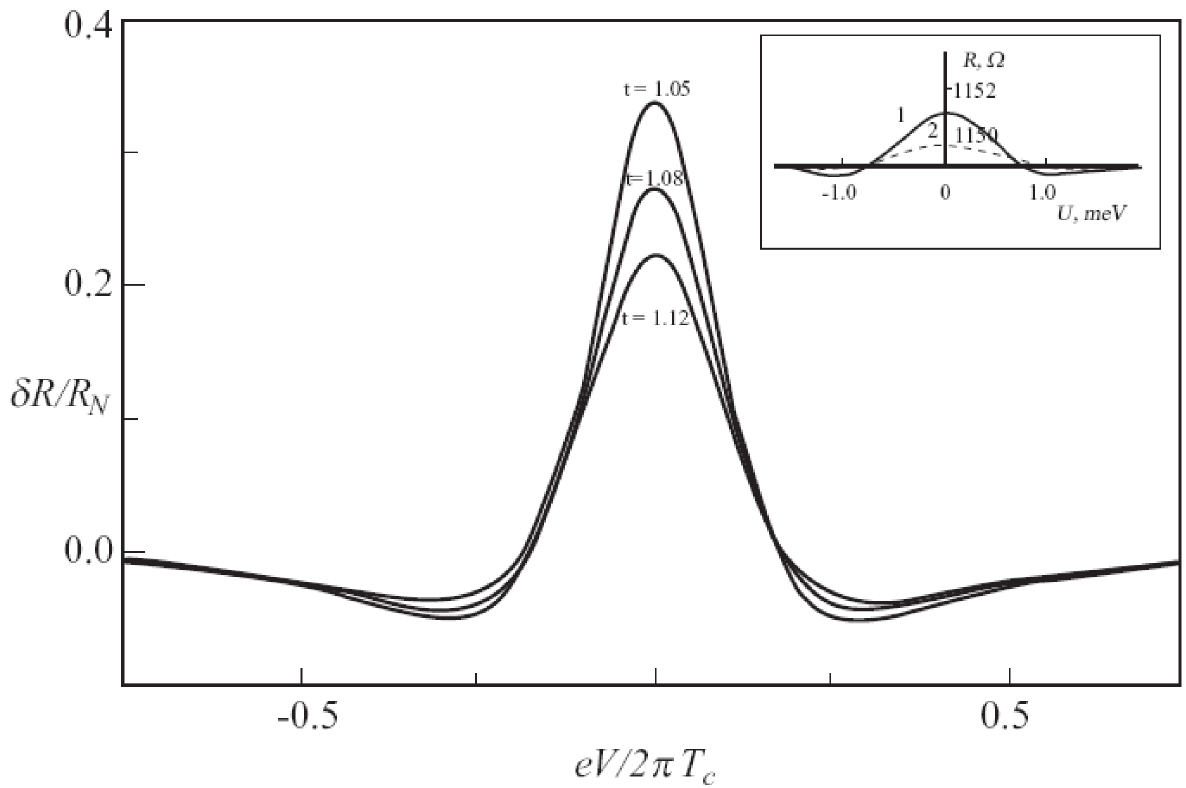
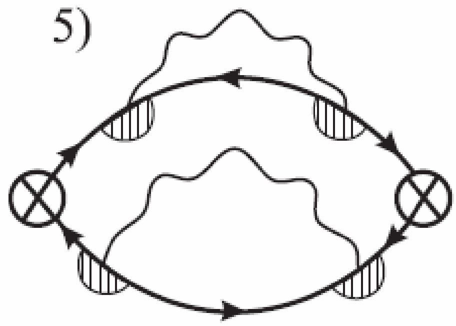


$$G_{qp}^{(1)}(V) = \frac{Gi_{(2d)}}{7\zeta\ (3)\ R_n} \left(\ln \frac{1}{\epsilon}\right) \text{Re}\ \psi'' \left(\frac{1}{2} - \frac{ieV}{2\pi T}\right)$$





$$\delta G_{\mathrm{fl}}^{(2d)}(0,\epsilon) \sim \int_{-\infty}^{\infty} \frac{dE}{\cosh^2\left(\frac{E}{2T}\right)} \left[\delta\nu_{(2)}^{(d)}(E,\epsilon)\right]^2 \sim \frac{Gi_{(2d)}^2}{\epsilon^3}.$$