Лекция 3

- седловая точка келдышевской σ-модели: квантовое кинетическое уравнение
- флуктуации; динамические купероны и диффузоны
- первая квантовая поправка к омической диссипации
  - адиабатика/формула Кубо
  - динамическая локализация
- унитарные матрицы с линейно растущим возмущением: четыре петли
- роль взаимодействия и возможность экспериментального наблюдения ДЛ

Keldysh σ-model

M. S. (2003)

Low-energy effective theory is formulated in terms of the matrix Q-field ($Q^2 = 1$)

$Q_{tt'}^{\alpha\beta} \in$

- **time** space: continuous index t
- 2×2 **Keldysh** space (σ_i)
- 2×2 **Particle-Hole** space (τ_i)

$$S[Q] = \frac{\pi i}{2\Delta} \operatorname{Tr} \tau_3 \hat{E} Q + \frac{\pi \Gamma}{8\Delta} \int dt\, dt' \, [\varphi(t) - \varphi(t')]^2 \operatorname{tr} Q_{tt'} Q_{t't}$$

E-term

responsible for the RMT energy **level statistics** encoded in the rich structure of $Q_{EE'}$

Altland & Kamenev (2000)

kinetic term

accounts for **interlevel transitions** due to time-dependent Hamiltonian $H[\varphi(t)]$

$$\left\langle \left(\frac{\partial E_i}{\partial \varphi} \right)^2 \right\rangle = \frac{2\Gamma\Delta}{\pi}$$

Saddle point

$$S[Q] = \frac{\pi i}{2\Delta} \,\mathrm{Tr}\, \tau_3 \hat{E} Q + \frac{\pi \Gamma}{8\Delta} \int dt\, dt' \, [\varphi(t) - \varphi(t')]^2 \,\mathrm{tr}\, Q_{tt'} Q_{t't}$$

The standard form of the Keldysh Green function ($F = 1 - 2f$):

$$Q_{tt'} = \begin{pmatrix} \delta_{tt'} & 2F_{tt'} \\ 0 & -\delta_{tt'} \end{pmatrix} \otimes \tau_3$$

Quantum kinetic equation

The saddle point equation gives the **quantum kinetic equation**:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial t'}\right) f^{(0)}_{tt'} = -\Gamma(\varphi(t) - \varphi(t'))^2 \, f^{(0)}_{tt'}.$$

The Wigner-transformed function $f(E,t) = \int d\tau e^{iE\tau} f_{t+\tau/2, t-\tau/2}$ after averaging over fast oscillations obeys the **diffusion equation in the energy space**:

$$\frac{\partial f^{(0)}(E,t)}{\partial t} = D \frac{\partial^2 f^{(0)}(E,t)}{\partial E^2}, \qquad D = \Gamma \overline{(d\varphi/dt)^2}$$

Energy absorption rate

$$W \equiv \frac{\partial \langle E \rangle}{\partial t} = \int E \, \frac{\partial f^{(0)}}{\partial t} \, \frac{dE}{\Delta} = \frac{D}{\Delta} \int E \, \frac{\partial^2 f^{(0)}}{\partial E^2} \, dE = -\frac{D}{\Delta} \int \frac{\partial f^{(0)}}{\partial E} \, dE = \frac{D}{\Delta}$$

Thus dissipation at the saddle point is Ohmic, coinciding with the result in the Kubo regime:

$$W_K = \frac{\Gamma}{\Delta} \overline{\left(\frac{d\varphi}{dt}\right)^2}$$

Q: where is the Adiabatic regime?

where is dynamic localization?

A: Fluctuations near the saddle point

Elimination of the distribution function

In the original formulation, F enters the definition of the Q-manifold:

$$Q = U_F^{-1}\tilde{Q}U_F, \qquad U_F = \begin{pmatrix} 1 & F_0 \\ 0 & 1 \end{pmatrix}, \qquad \tilde{Q} = U^{-1}\sigma_3\tau_3 U$$

Evolution of the distribution can be read off from the Keldysh block of $\langle Q \rangle$:

$$F_{tt'} = \frac{1}{2}\langle Q^K_{tt'} \rangle, \qquad W(t) = -\frac{\pi i}{\Delta}\frac{\partial}{\partial t}\frac{\partial}{\partial \eta}\bigg|_{\eta=0} F_{t+\eta/2,t-\eta/2}.$$

$$Q = \begin{pmatrix} R & K \\ \bar{K} & A \end{pmatrix}$$

For a noninteracting problem one can work in terms of the $\tilde{Q}$-matrix

D. Ivanov & M. S. (2006)

- Full diffuson: $\langle \tilde{Q}^K_{t+\eta/2,t-\eta/2} \tilde{Q}^{\bar{K}}_{t'-\eta'/2,t'+\eta'/2} \rangle = \dfrac{2\Delta}{\pi}\delta(\eta-\eta')\,\mathcal{D}_\eta(t,t')$
- Energy absorption rate:

$$W(t) = -\frac{1}{2\Delta}\frac{\partial}{\partial t}\frac{\partial^2}{\partial \eta^2}\bigg|_{\eta=0} \mathcal{D}_\eta(t,t_0)$$

switch-on time
of the perturbation

Structure of the Q-manifold

The matrix $\tilde{Q} = U^{-1}\sigma_3\tau_3 U$ obeys: 1) $\tilde{Q}^\dagger = \tilde{Q}$, 2) $\tilde{Q}^T = \sigma_1\tau_2\tilde{Q}\tau_2\sigma_1$.

Parametrization of the manifold:

$$\tilde{Q} = \sigma_3\tau_3 f(W) = \sigma_3\tau_3[1 + W + W^2/2 + \ldots], \qquad f(W)f(-W) = 1$$

$$W = \left(\begin{array}{cc|cc} 0 & a & b & 0 \\ -a^\dagger & 0 & 0 & -b^T \\ \hline -\,b^\dagger & 0 & 0 & a^T \\ 0 & b^* & -a^* & 0 \end{array}\right)_K$$

$\langle bb^\dagger\rangle$ = Diffuson

$\langle aa^\dagger\rangle$ = Cooperon

parametrizations

rational	$f(W) = \dfrac{1 + W/2}{1 - W/2}$	$J = 1$
sqrt	$f(W) = \sqrt{1 + W^2} + W$	cross diagram technique
exp	$f(W) = e^W$	$\rightarrow$ global parametrization

Q-matrix as a spin

Another type of sqrt-like parameterization:

$$f(W) = 1 + \frac{W^2}{2} + W\sqrt{1 + \frac{W^2}{4}}$$

$$W = \begin{pmatrix} 0 & B \\ -\bar{B} & 0 \end{pmatrix} \quad \Longrightarrow \quad Q = \begin{pmatrix} 1 - \frac{B\bar{B}}{2} & B\sqrt{1 - \frac{\bar{B}B}{4}} \\ \sqrt{1 - \frac{\bar{B}B}{4}}\,\bar{B} & -1 + \frac{\bar{B}B}{2} \end{pmatrix}$$

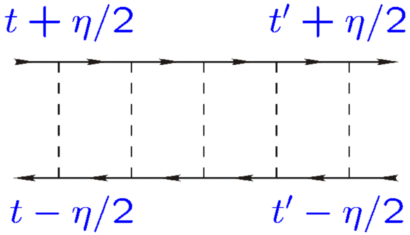
Holstein-Primakoff transformation
for quantum spin operators

$$\hat{S}^{+} = \hat{a}^{\dagger}\sqrt{2S - \hat{a}^{\dagger}\hat{a}}$$

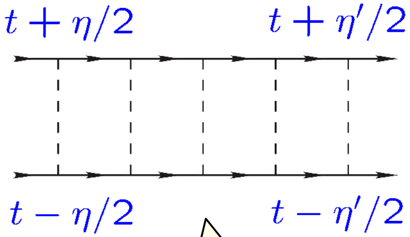
$$\hat{S}^{-} = \sqrt{2S - \hat{a}^{\dagger}\hat{a}}\;\hat{a}$$

$$\hat{S}^{z} = \hat{a}^{\dagger}\hat{a} - 2S$$

Soft modes: diffusons and cooperons



$$D_\eta(t,t') = \theta(t-t')\ \exp\left\{-\Gamma \int_{t'}^{t} [\varphi(\tau+\eta/2) - \varphi(\tau-\eta/2)]^2\, d\tau\right\}$$



$$C_t(\eta,\eta') = \theta(\eta-\eta')\ \exp\left\{-\frac{\Gamma}{2} \int_{\eta'}^{\eta} [\varphi(t+\tau/2) - \varphi(t-\tau/2)]^2\, d\tau\right\}$$

Each impurity line carries a nonzero frequency → both diffusons and cooperons decay with time.

Dephasing

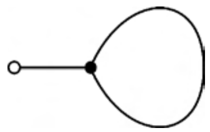
by the time-dependent perturbation

Vavilov, Aleiner (1999)

Yudson, Kanzieper, Kravtsov (2001)

One-loop quantum correction (GOE)

Fluctuations induce corrections to the distribution function $F_{tt'}$:

$\delta F =$ 

One-loop interference correction to the Kubo absorption rate W_K:

$$\delta W(t) = \frac{\Gamma}{\pi}\int_0^\infty \partial_t\varphi(t)\,\partial_t\varphi(t-\xi)\,C_{t-\xi/2}(\xi,-\xi)\,d\xi$$

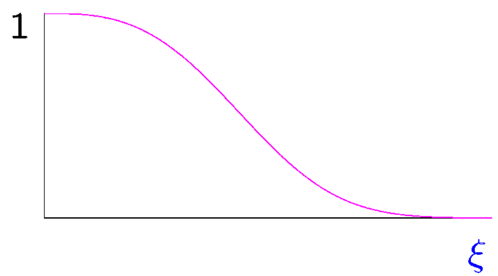
Holds for arbitrary $\varphi(t)$ and contains everything

Linear perturbation: GOE

Dynamic cooperon for $\varphi = vt$:

$$C_{t-\xi/2}(\xi, -\xi) = \exp\left\{-\frac{\Omega^3}{3}\xi^3\right\}$$

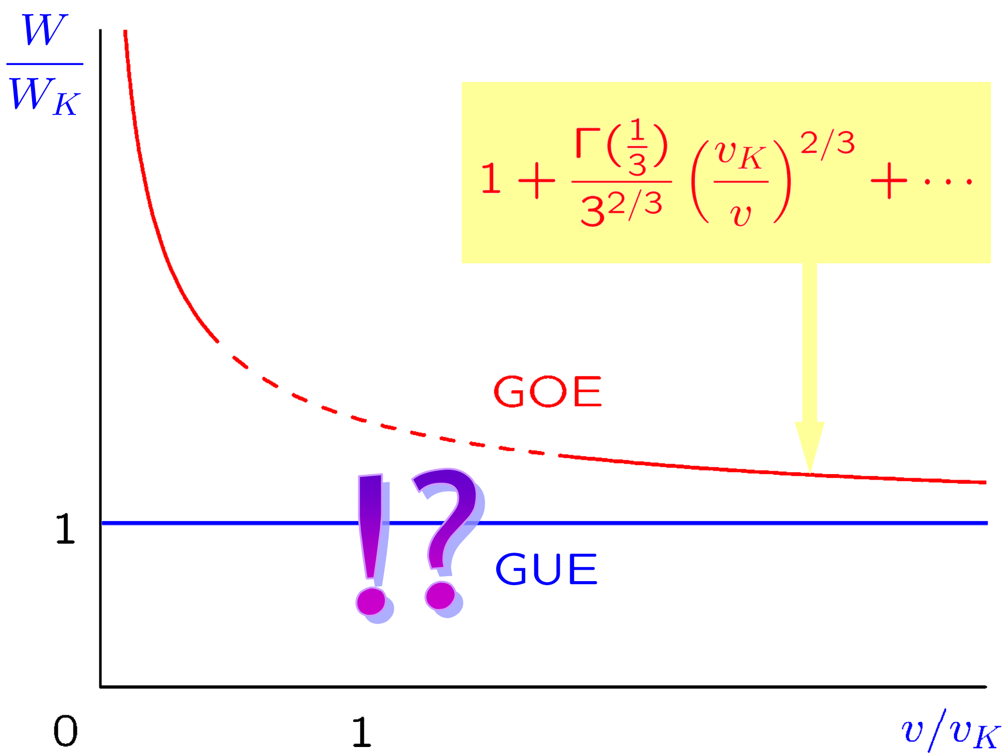
decays at the time scale Ω^{-1}, where $\Omega^3 = \Gamma v^2$.



Loop expansion parameter:

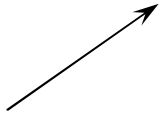
$$\frac{\Delta}{\Omega} = \pi\left(\frac{v_K}{v}\right)^{2/3}$$

Dissipation rate vs. velocity



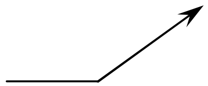
Linear perturbation: GUE

- **Linear perturbation** $\varphi = vt$



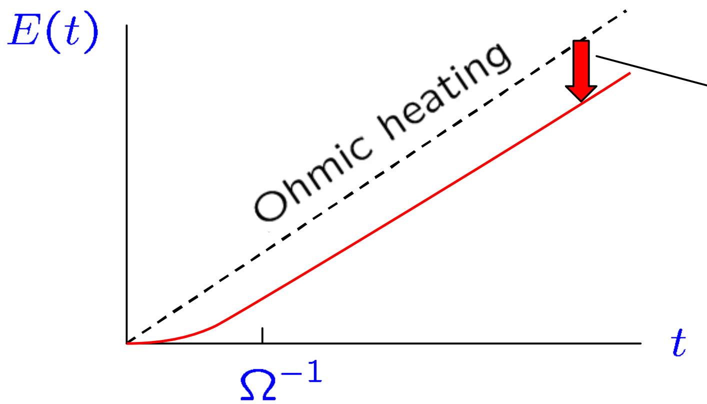
$$\delta W = \frac{\Omega^6 \Delta}{\pi^2} \int_0^\infty dx\, dy\, dz\, (-x^2 + 5xy) \exp\left\{-\Omega^3 (x+y)(y+z)(z+x)\right\} = 0$$

- **Suddenly switched** $\varphi = vt\,\theta(t)$



$$\delta W = \frac{\Omega^6 \Delta}{\pi^2} \int_0^{x+y+z<t} dx\, dy\, dz\, (-x^2 + 5xy) \exp\left\{-\Omega^3 (x+y)(y+z)(z+x)\right\}$$

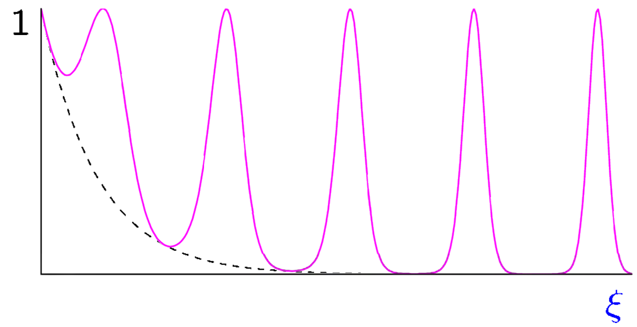
Long-time memory effects near the point of discontinuity:



$$\delta E(t) \simeq -\frac{\Delta}{\pi^2} \ln(\Omega t)$$

Result for a periodic perturbation (GOE)

For a periodic perturbation $\varphi(t) = \theta(t) \sin \omega t$ at $t, \xi \gg 1/\omega$:



$$C_{t-\xi/2}(\xi, -\xi) \approx \exp\left\{-2\Gamma\xi \cos^2[\omega(t-\xi/2)]\right\}$$

exponentially decays with ξ

no-dephasing points

Cooperon is equal to **1** at $\xi_k = 2t - \frac{2\pi}{\omega}(k + 1/2)$

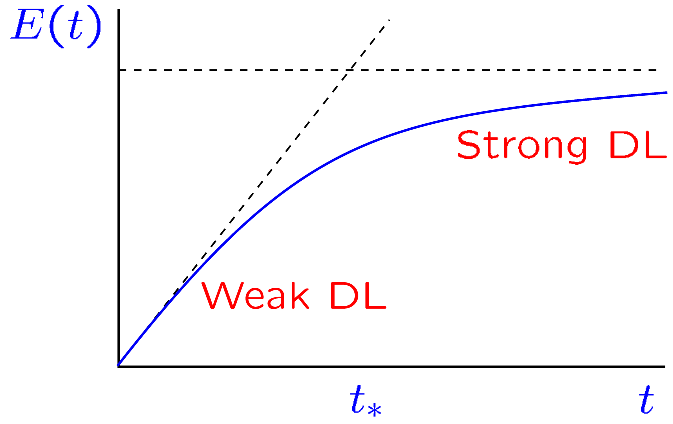
Integrating near ξ_k and summing over ξ_k we get

$$\frac{W(t)}{W_K} = 1 - \sqrt{\frac{t}{t_*}}, \qquad t_* = \frac{\pi^3 \Gamma}{2\Delta^2}$$

Weak dynamic localization (GOE)

- General *periodic* perturbation with TRS,
  $\varphi(-t) = \varphi(t)$

$$\frac{\delta W(t)}{W_K} = -\sqrt{\frac{t}{t_*}}$$



- General *periodic* perturbation without TRS,
  $\varphi(-t) \neq \varphi(t)$

$$\frac{\delta W(t)}{W_K} = -\frac{t}{t_*} \qquad \text{(2 loops)}$$

- *Aperiodic* perturbation with d incommensurate frequencies

$d = 2$: $$\frac{\delta W(t)}{W_K} = -\frac{\Delta}{2\pi^2\Gamma} \ln \Gamma t$$

$d > 2$: $$\frac{\delta W(t)}{W_K} \propto -t^{1-d/2} \longrightarrow \text{const}$$

⟺ Weak Anderson localization in d dimensions

Analogously to the DL in the KQR with d incommensurate periods
Casati, Guarneri, Shepelyansky (1989)

Dynamic vs. Anderson localization

- Weak DL in periodically driven RMT ($\sqrt{t}$) = Weak DL in QKR
- Quantum kicked rotor can be mapped onto the 1D SUSY σ-model [Altland, Zirnbauer (1993)]
- Periodically driven RMT $\neq$ QKR!

  e.g., for a δ-kicked RMT localization time $t_* \to \infty$

Q: Is DL in periodically driven RMT analogous to the quasi-1D AL?

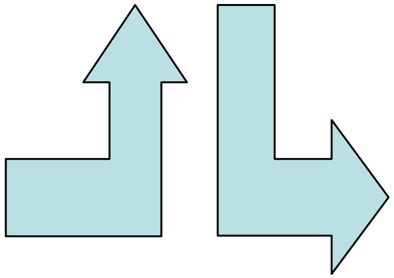
Dynamic vs. Anderson localization

If DL for the periodically driven RMT is equivalent to quasi-1D AL then [Altland (1993), Tian, Kamenev, Larkin (2004)]

$$\frac{W(t)}{W_0} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{-i\omega + 0} \frac{D(\omega)}{D_0}$$

Weak AL

$$\frac{\delta D(\omega)}{D_0} = \begin{cases} -\dfrac{1}{\sqrt{-i\omega t_{\text{loc}}}}, & \text{GOE,} \\ -\dfrac{1}{-6i\omega t_{\text{loc}}}, & \text{GUE} \end{cases}$$



Weak DL

$$\frac{\delta W(t)}{W_0} = -\begin{cases} \sqrt{\dfrac{t}{t_*}}, & \text{GOE,} \\ \dfrac{\pi t}{24 t_*}, & \text{GUE} \end{cases}$$

Q: Can periodically driven RMT be mapped onto the quasi-1D σ-model?

We believe, YES

GUE, linear perturbation, 4-loops

D. Ivanov & M. S. (2005)

Field theory for GUE: summary

- Dimensional action:

$$S[Q] = -\frac{\pi}{2}\,\mathrm{tr}\int dt\,(\partial_1 - \partial_2)Q_{tt} + \frac{\pi\alpha}{4}\,\mathrm{tr}\int\!\!\int dt\,dt'\,(t-t')^2\,Q_{tt'}Q_{t't}$$

$$\alpha = \frac{v^2}{v_K^2}$$

- Full diffuson: $\langle Q^{K}_{t+\eta/2,t-\eta/2}\,Q^{\bar{K}}_{t'-\eta'/2,t'+\eta'/2}\rangle = \frac{2}{\pi}\delta(\eta-\eta')\,\mathcal{D}_\eta(t-t')$

- Energy diffusion coefficient: $D(\alpha) = -\frac{1}{2}\frac{\partial}{\partial t}\frac{\partial^2}{\partial\eta^2}\Big|_{\eta=0}\mathcal{D}_\eta(t)$

- Parametrization: $Q = \sigma_3\,[1 + W + W^2/2 + \ldots],\qquad W_{tt'} = \begin{pmatrix} 0 & b_{tt'} \\ -b^\dagger_{tt'} & 0 \end{pmatrix}$

- Bare diffuson: $\mathcal{D}^{(0)}_\eta(t) = \theta(t)\exp(-\alpha\eta^2 t) \;\longrightarrow\; D^{(0)} = \alpha$

Perturbation theory

- **Rational parametrization**: $Q = \sigma_3 \frac{1 + W/2}{1 - W/2}, \qquad W_{tt'} = \begin{pmatrix} 0 & b_{tt'} \\ -\bar{b}_{tt'} & 0 \end{pmatrix}$

$$S[b, \bar{b}] = \frac{\pi}{2} \int\!\!\int dt_1\, dt_2\, \bar{b}_{12} \Big[(\partial_1 + \partial_2) + \alpha (t_1 - t_2)^2 \Big] b_{21}$$

$$+ \{ \text{higher vertices with } \partial_t \} + \{ \text{higher vertices with } \alpha(t_i - t_j)^2 \}$$

- Bare diffuson: $\mathcal{D}_\eta^{(0)}(t) = \theta(t) \exp(-\alpha\eta^2 t)$

- Diffuson equation of motion: $\partial_t \mathcal{D}_\eta^{(0)}(t) = \delta(t) - \alpha\eta^2 \mathcal{D}_\eta^{(0)}(t)$

ALL vertices of the order higher than FOUR cancel in ALL orders of the perturbation theory

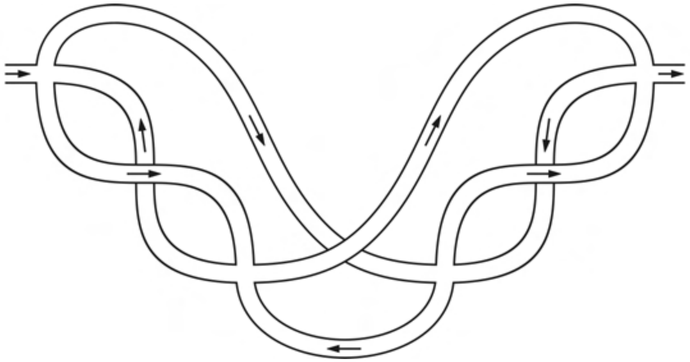
Effective matrix ϕ^4 theory

Initial Keldysh σ-model is perturbatively equivalent to the unconstrained matrix φ^4 theory:

$$S_{\text{eff}}[b,\bar{b}] = \frac{\pi}{2}\iint dt_1\, dt_2\, \bar{b}_{12}\Big[(\partial_1 + \partial_2) + \alpha(t_1 - t_2)^2\Big] b_{21} - \frac{\pi\alpha}{8}\int dt_1\, dt_2\, dt_3\, dt_4\, (t_1 - t_2)(t_3 - t_4)\, b_{12}\bar{b}_{23} b_{34}\bar{b}_{41}$$

Four loops

- *Just 20* 4-loop diagrams, e.g.



$$\int_0^\infty \ldots \int_0^\infty dt_1 \ldots dt_7 \, P_6(t_i) \, e^{-S_3(t_i)}$$

Result for the energy diffusion coefficient:

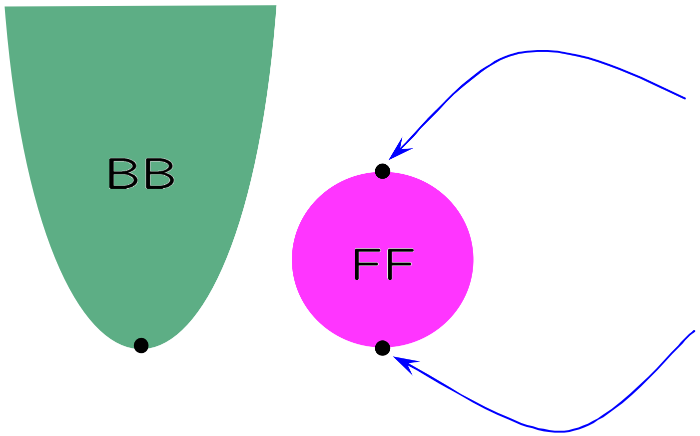
exact zero

$(7 \pm 7) \times 10^{-5}$

$$D(\alpha) = \alpha \left(1 + \frac{d_2}{\pi^2 \alpha^{2/3}} + \frac{d_4}{\pi^4 \alpha^{4/3}} + \ldots\right)$$

Features of the unitary symmetry

- Pair correlator via SUSY σ-model



$$\delta R(\omega) = 1 - \frac{1}{2(\pi\omega/\Delta)^2}$$

$$\delta R(\omega) = \frac{\cos(2\pi\omega/\Delta)}{(\pi\omega/\Delta)^2}$$

$$\Bigg\} \quad 1 - \frac{\sin^2(\pi\omega/\Delta)}{(\pi\omega/\Delta)^2}$$

Andreev, Altshuler (1995)

Saddle points + Gaussian fluctuations = Exact

Features of the unitary symmetry

- **Itzykson-Zuber integral:**

$$\int_{U(N)} DU\, e^{\mathrm{Tr}\, AU^{-1}BU} = \mathrm{const}\, \frac{\det ||e^{a_i b_j}||}{\Delta(a_k)\Delta(b_l)}$$

diagonal Hermitian

Saddle points: $U \to U e^{iT}$, $U^{-1} \to e^{-iT} U^{-1}$

$$\delta S = -i\, \mathrm{Tr}[A, U^{-1}BU]\delta T = 0 \quad \Longrightarrow \quad U_{\text{s.p.}} \in \mathrm{Sp}(N) \qquad \dim = N!$$

Fluctuations around $U = 1$: $\quad S = -\,\mathrm{Tr}\, Ae^{-iT}Be^{iT} \longrightarrow \frac{1}{2}\sum_{ij}(a_i - a_j)(b_i - b_j)|T_{ij}|^2$

Contribution of $U = 1$: $\quad \dfrac{e^{a_i b_i}}{\Delta(a_k)\Delta(b_l)}$

Contribution of $U = \sigma$: $\quad \dfrac{e^{a_i b_{\sigma_i}}}{\Delta(a_k)\Delta(b_{\sigma_l})} = (-1)^{\mathrm{sign}\,\sigma} \dfrac{e^{a_i b_{\sigma_i}}}{\Delta(a_k)\Delta(b_l)}$

Saddle points + Gaussian fluctuations = Exact

Dyson-Maleev transformation

$$Q = \begin{pmatrix} 1 - bb^\dagger/2 & b - bb^\dagger b/4 \\ b^\dagger & -1 + b^\dagger b/2 \end{pmatrix}$$

violates hermiticity

Bilinear in Q action

$$S[Q] = \frac{\pi i}{\Delta} \mathrm{Tr}\, \hat{E} Q - \frac{\pi \Gamma}{4\Delta} \mathrm{Tr}[\varphi, Q]^2$$

becomes **quartic** in $b, b^\dagger$

Works for ANY unitary sigma-model

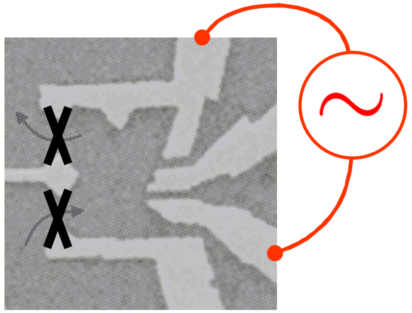
Dyson-Maleev transformation for quantum spin operators

$$\hat{S}^+ = (2S - \hat{a}^\dagger \hat{a})\hat{a}$$
$$\hat{S}^- = \hat{a}^\dagger$$
$$\hat{S}^z = S - \hat{a}^\dagger \hat{a}$$

Роль взаимодействия и возможность экспериментального наблюдения динамической локализации

Слабая динамическая локализация

Basko, Skvortsov, Kravtsov (2003)



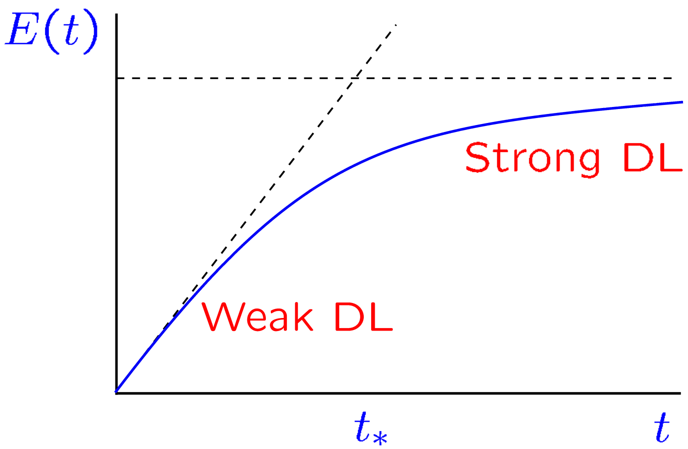
случайные матрицы + техника Келдыша

$$\varphi(t) = \theta(t)\sin\omega t$$

- **Квазиклассика:**

  омическая скорость поглощения W_0

- **Первая квантовая поправка (куперонная петля)**



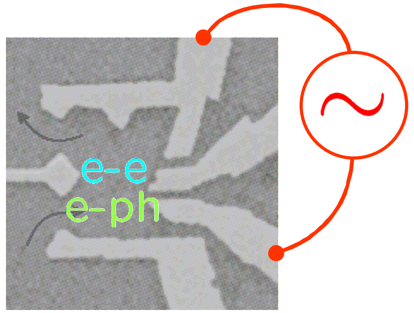
$$\frac{\delta W(t)}{W_0} = -\sqrt{t/t_*}$$

$$t_* = \frac{\pi^3\Gamma}{\Delta^2} \propto \frac{W_0}{\Delta}$$

растет со временем действия возмущения

(формальная аналогия с одномерной локализацией)

Влияние декогерентности



Источники сбоя фазы в КТ:

- e-e взаимодействие
- e-ph взаимодействие
- уход в контакты

$$\tau_\varphi^{-1} \equiv \gamma_\varphi = \gamma_{\text{e-e}} + \gamma_{\text{e-ph}} + \gamma_{\text{esc}}$$

охлаждение: $W_{\text{out}} = W_{\text{e-ph}} + W_{\text{esc}}$

$T^6/(1\,\text{K})^4$

Механизм поглощения при $t_* \ll \tau_\varphi$

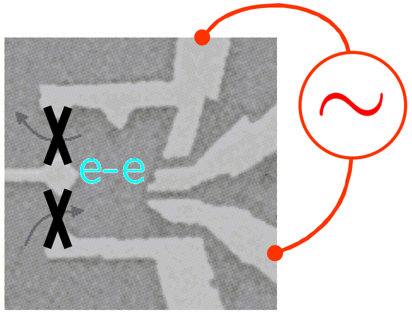
1. Сильная динамическая локализация устанавливается за время t_*.
2. Система живет в локализованном состоянии время τ_φ до следующего неупругого столкновения. Затем снова оказывается в состоянии поглощать энергию.
3. Go to 1.

Средняя скорость накачки энергии в КТ:

$$W_{\text{in}} \sim W_0 \gamma_\varphi t_*$$

меньше W_0,
но конечна

e-e взаимодействие в изолированной КТ



e-e scattering rate in a closed system:

$$\gamma_{\text{e-e}}(T) \sim \Delta \frac{T^2}{E_{\text{Th}}^2}$$

[Sivan, Imry, Aronov (1994)]

Динамика разогрева:

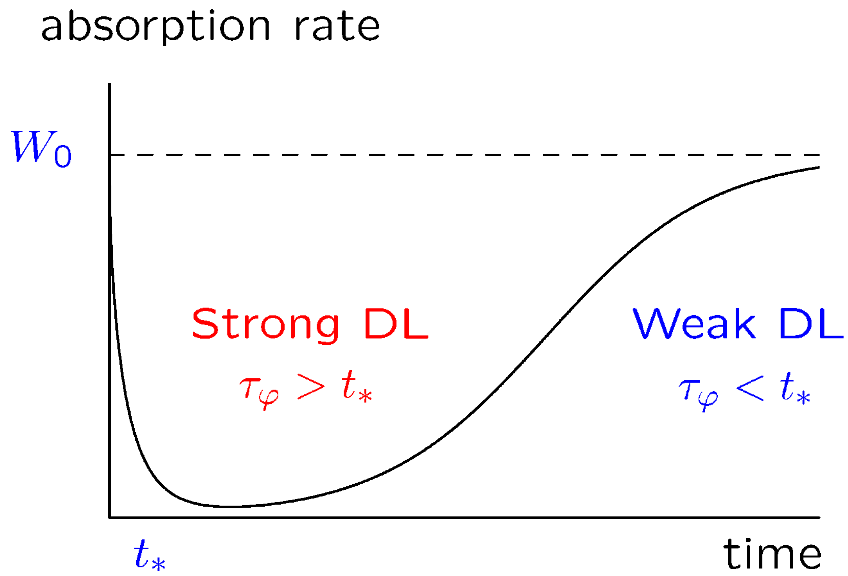
$$\frac{d\mathcal{E}}{dt} = W_{\text{in}}$$

$$\mathcal{E} \sim \frac{T^2}{\Delta}$$

$$\Downarrow$$

$$\frac{dT^2}{dt} \sim \frac{\Gamma^2\omega^2}{E_{\text{Th}}^2\Delta} T^2$$

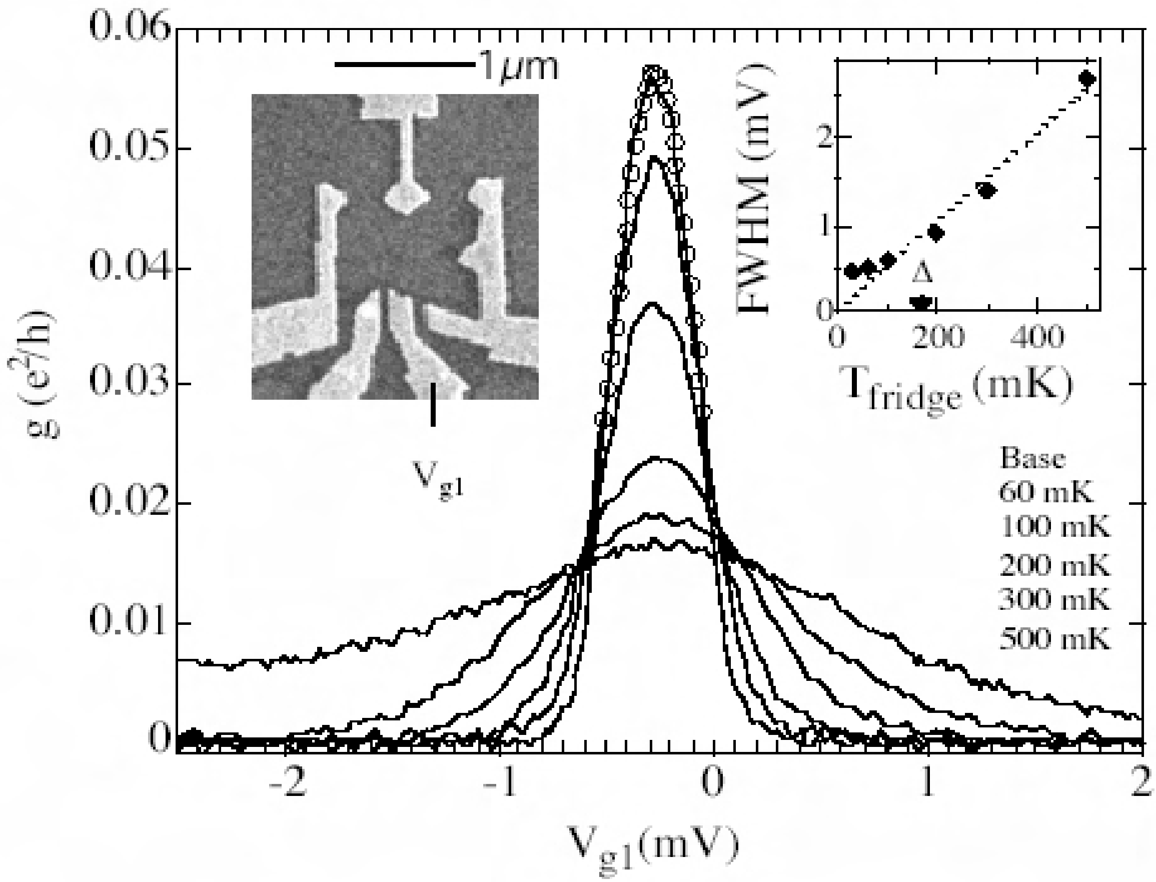
экспоненциальный разогрев!



e-e взаимодействие убивает ДЛ в замкнутой КТ [Basko (2003)]

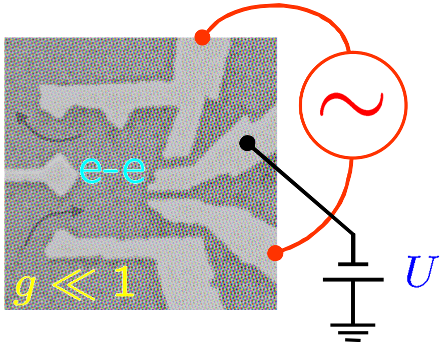
Открытая точка + кулоновская блокада

Как измерить эффективную температуру электронов в КТ?



Ширина пика кулоновской блокады знает про температуру электронов

Открытая точка + кулоновская блокада



Уравнение теплового баланса:

$$W_{\rm out} = W_{\rm in}$$

уход в контакты

е-е взаимодействие

Закон Видемана-Франца:

$$W_{\rm out} \sim G\,T(T - T_{\rm fridge}) \to G\,T^2$$

$G(U,T)$

$$W_{\rm in} \sim \frac{\Gamma^2\omega^2}{\Delta^2 E_{\rm Th}^2} T^2$$

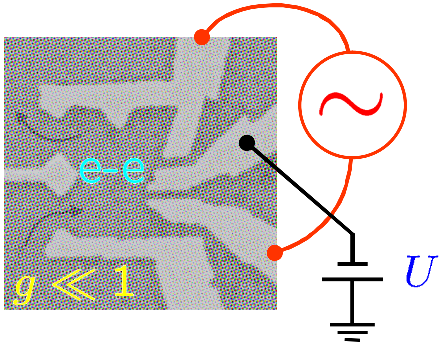
G не зависит от T и U:

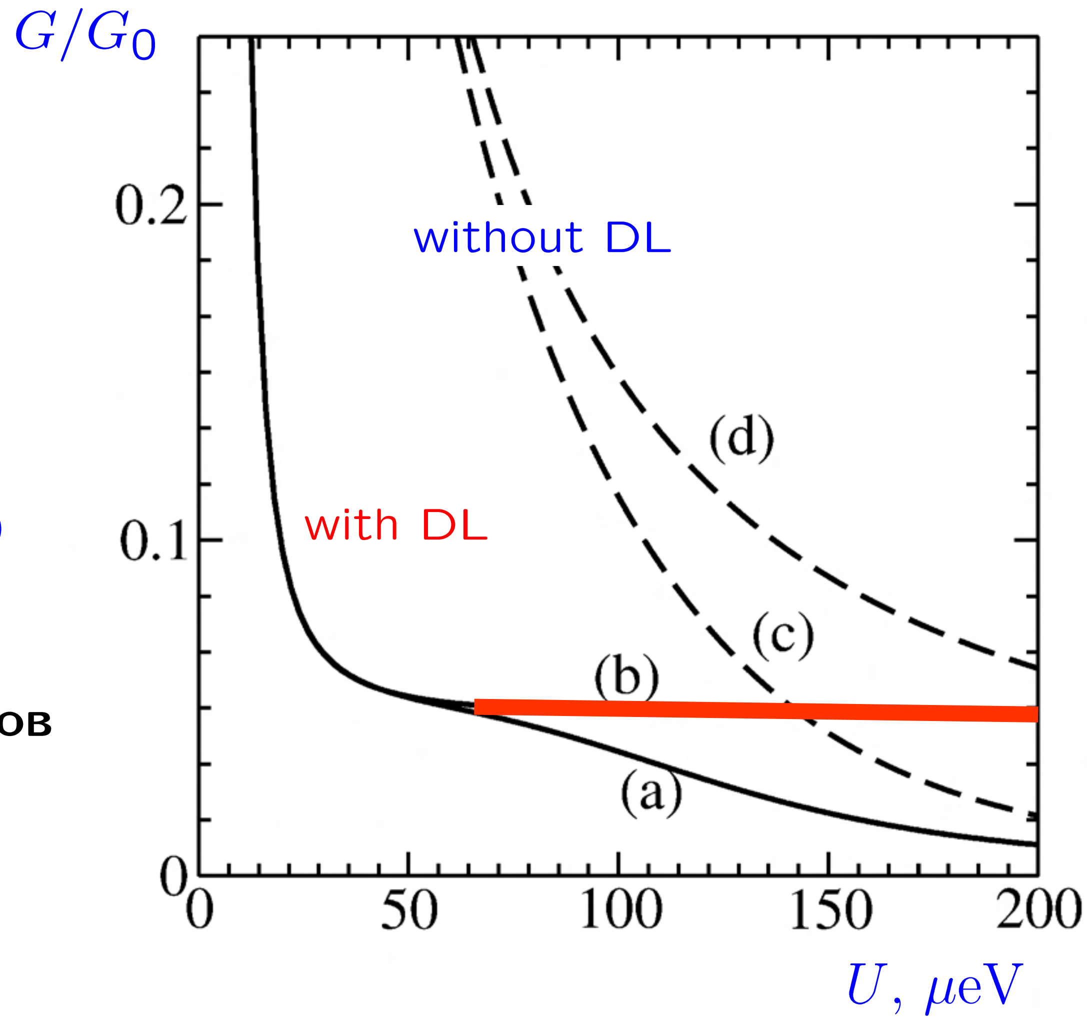
$$G \sim \frac{\Gamma^2\omega^2}{\Delta^2 E_{\rm Th}^2}$$

Плато в зависимости $G(U)$!

Плато в зависимости кондактанса от U

Basko, Kravtsov (2004)





Охлаждение за счет ухода

d) омический режим ($W = W_0$)

b) режим ДЛ

Охлаждение за счет фононов

c) омический режим

a) режим ДЛ

Dot: size $\sim 1\mu$m, $\Delta = 0.3\ \mu$eV, $E_{\text{Th}} = 100\ \mu$eV

Field: $\Gamma = 1\ \mu$eV, $\omega = 3\ \mu$eV ($\omega/2\pi \approx 0.7$ GHz)

Выводы

- Динамическая локализация возможна в сложной твердотельной системе со многими степенями свободы
- e-e взаимодействие убивает ДЛ в изолированной КТ
- Чтобы увидеть ДЛ, нужно устроить хорошее охлаждение, сохранив когерентность. В этом режиме нужно ожидать плато в зависимости кондактанса от затворного напряжения.

Что осталось сделать?

- Померить...
- При $T \ll (E_{\mathrm{Th}}\Delta)^{1/2}$ ожидается локализация в фоковском пространстве: $\gamma_{ee} = 0$.
  Спасает ли это динамическую локализацию?

The end