Квантово-когерентные явления в неравновесных мезоскопических системах (функциональные методы теории)

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ФОНД ДМИТРИЯ ЗИМИНА
ДИНАСТИЯ

Летняя школа

«Нанофизика низких температур»

Черноголовка, 2007

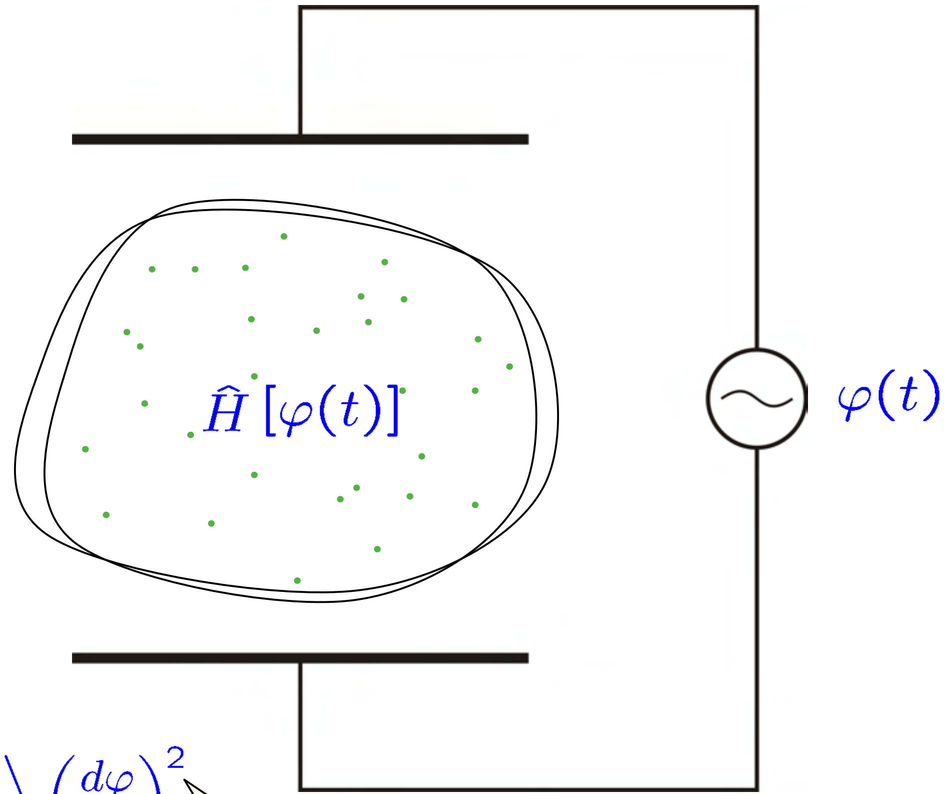
Physical motivation

disordered metal grain
or quantum dot

RMT spectral statistics
for $\omega \ll E_{\mathrm{Th}}$

$$W(t) \equiv \frac{d\langle E(t)\rangle}{dt} = ???$$

Standard approach:
Kubo formula



$$W = \frac{\pi}{\Delta^2}\left\langle \left(\frac{\partial E_i}{\partial t}\right)^2 \right\rangle = \frac{\pi}{\Delta^2}\left\langle \left(\frac{\partial E_i}{\partial \varphi}\right)^2 \right\rangle \left(\frac{d\varphi}{dt}\right)^2$$

Ohmic dissipation

Anything interesting beyond?

План курса

1. Случайные матрицы в физике твердого тела

2. σ-модели

3. Интерференционные эффекты в динамике сложных систем

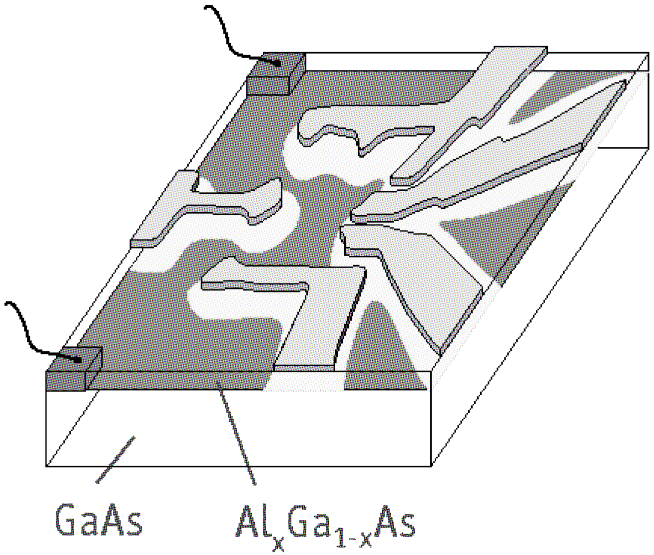
Лекция 1

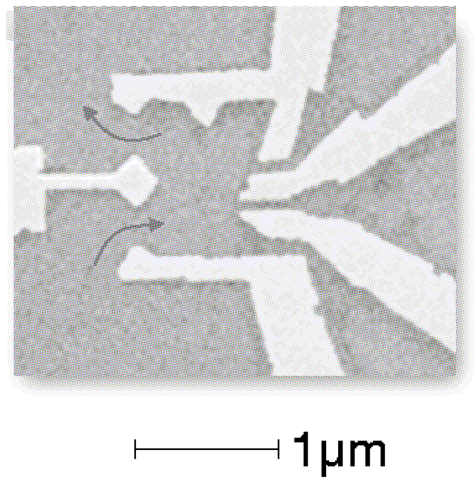
- Случайные матрицы в физике твердого тела
- Эволюция с зависящим от времени случайным гамильтонианом
- Квантовые интерференционные явления в динамике
  - адиабатика/формула Кубо
  - динамическая локализация

Квантовая точка

Пример КТ с хаотической динамикой

(C. Marcus group, Stanford)

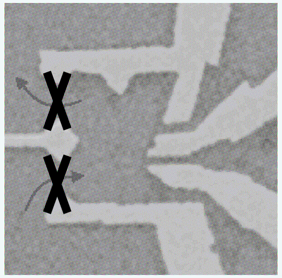




Сложная система со многими степенями свободы

Описание изолированной КТ

Изолированная КТ



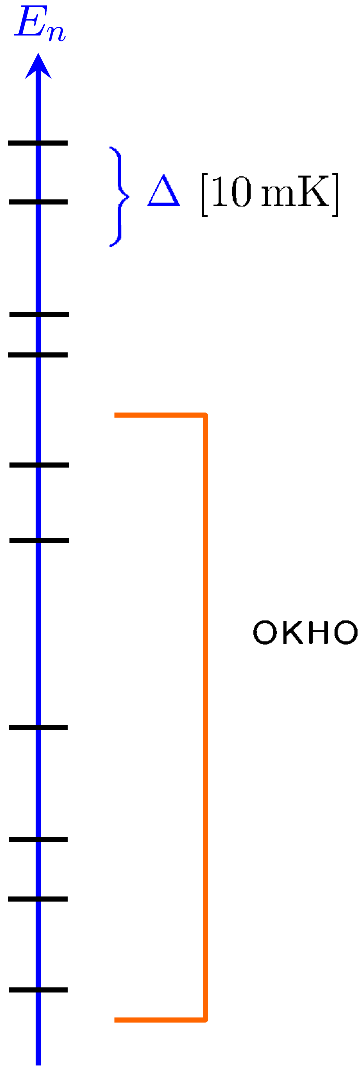
Случайные матрицы:

$$H\psi_n = E_n\psi_n$$

$$\mathcal{P}(H) \propto e^{-\mathrm{tr}\, H^2}$$

$E_i - E_j \ll E_{\mathrm{Th}}$

Efetov (1982)

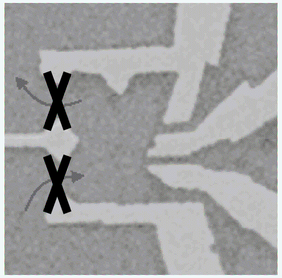


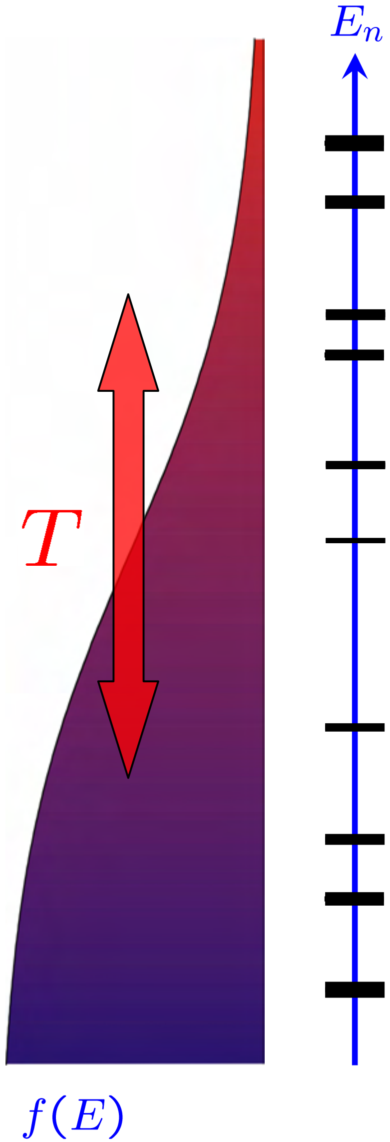
окно по энергии шириной

$E_{\mathrm{Th}} \sim \hbar/t$ [1–10 K]

Interaction in a closed QD

Изолированная КТ





$$\frac{1}{\tau_{ee}} \sim \Delta \frac{\pi^2 T^2 + E^2}{E_{\text{Th}}^2}$$

Sivan, Imry, Aronov (1994)

Spectrum is discrete
at $T < E_{\text{Th}}$

Random Matrix Theory

Spectral properties of an ensemble of Hermitian $N \times N$ matrices H

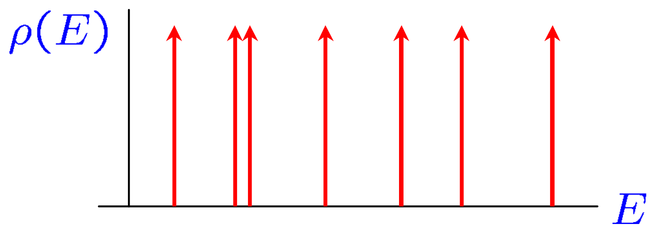
Probability density: $\mathcal{P}(H) = Z^{-1} e^{-\mathrm{Tr}\, H^2}$ (Gaussian ... ensemble)

Three standard Wigner-Dyson ensembles:

		symmetry	time-rev.	spin rot.	β
GOE	**Orthogonal**	$H^T = H$	$+$	$+$	1
GUE	**Unitary**		$-$	$+/-$	2
GSE	**Symplectic**	$H_{ij} = \begin{pmatrix} p_{ij} & q_{ij} \\ -q_{ij}^* & p_{ij}^* \end{pmatrix}$	$+$	$-$	4

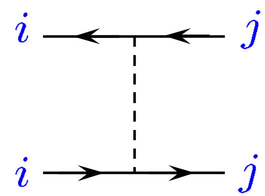
- **Eigenvalue problem**: $H\psi_n = E_n \psi_n$

  Density of states: $\rho(E) = \sum_n \delta(E - E_n)$



RMT: Average DOS (1)

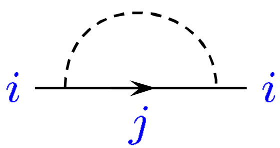
GUE, $\mathcal{P}(H) \propto \exp\left(-\frac{1}{2\alpha}\,\mathrm{Tr}\,H^2\right)$

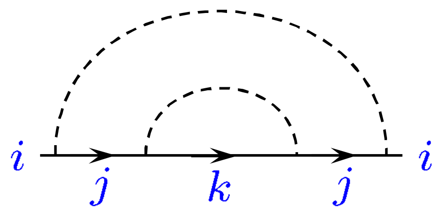


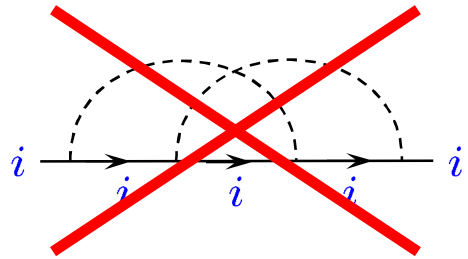
$$\mathrm{Tr}\,H^2 = \sum_i H_{ii}^2 + \sum_{i<j} |H_{ij}|^2 \quad\longrightarrow\quad \langle H_{ij}H_{kl}\rangle = \langle H_{ij}H_{lk}^*\rangle = \alpha\,\delta_{il}\delta_{jk}$$

$$\rho(E) = \mathrm{Tr}\,\delta(E-H) = -\frac{1}{\pi}\,\mathrm{Im}\,\mathrm{Tr}\,G^R(E) \qquad G_{ij}^R(E) = \left(\frac{1}{E-H+i0}\right)_{ij}$$

Bare Green function: $(G_0^R)_{ij} = \dfrac{\delta_{ij}}{E+i0}$



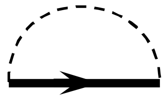




$$\Sigma = \alpha \sum_j G_0 = N\alpha G_0 \qquad \Sigma = \alpha^2 \sum_{jk} G_0^3 = N^2\alpha^2 G_0^3 \qquad \Sigma = \alpha^2 G_0^3$$

Random matrices: $N \gg 1$ ⟺ Disordered metal: $k_F l \gg 1$

RMT: Average DOS (2)

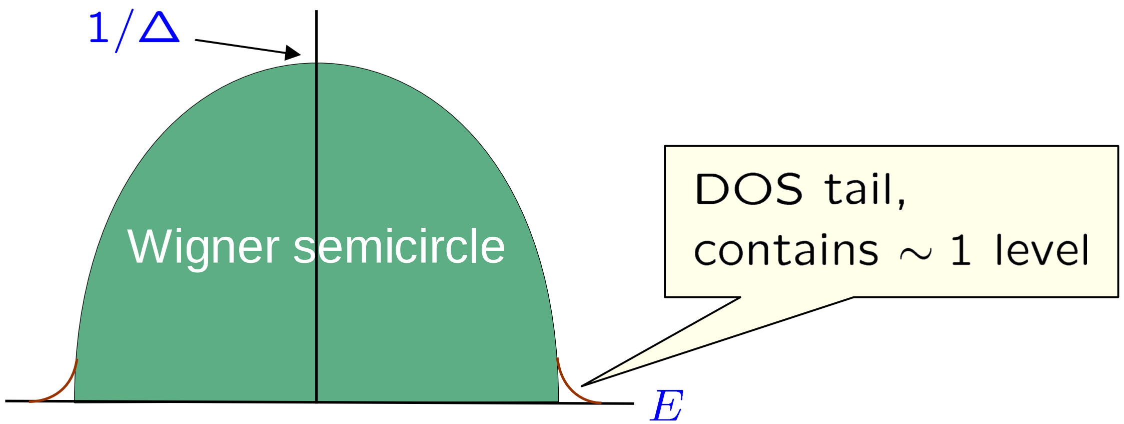
Self-Consistent Born Approximation: $\Sigma =$  $= \alpha \sum_j \langle G \rangle = N\alpha \langle G \rangle$

$$\langle G^R_{ij}(E) \rangle = \frac{\delta_{ij}}{E + i0 - \Sigma}$$

Solution of quadratic equation: $G^R(E) = \frac{E - i\sqrt{4N\alpha - E^2}}{2N\alpha}$

Average DOS: $\langle \rho(E) \rangle = \frac{\sqrt{4N\alpha - E^2}}{2\pi\alpha}$

$\Delta = \pi\sqrt{\alpha/N}$



RMT: Mathematical approach

$X = \mathrm{diag}(x_1, \ldots, x_n)$ — eigenvalues

Let us diagonalize: $H = U^{-1} X U$

unitary matrix

- **Step 1**: From N^2 to N variables:

$$\int \ldots e^{-\mathrm{tr}\, H^2} DH = \int \ldots e^{-\sum x_i^2} \left(\prod dx_k\right) DU\, J(X)$$

$$\beta = \begin{cases} 1, & \text{GOE} \\ 2, & \text{GUE} \\ 4, & \text{GSE} \end{cases}$$

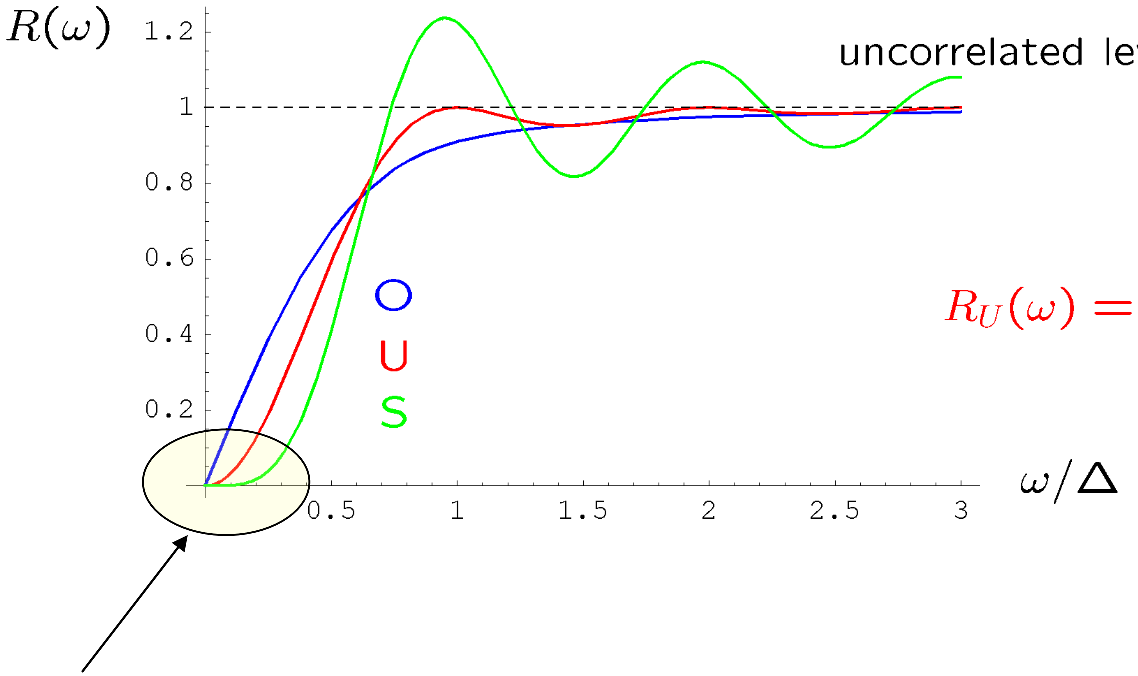
$$\prod_{i<j} |x_i - x_j|^\beta$$

Joint distribution function of eigenvalues: $\mathcal{P}(x_1, \ldots, x_N) = \# \prod_{i<j} |x_i - x_j|^\beta e^{-\sum_k x_k^2}$

- **Step 2**: From N to 2 variables — orthogonal polynoms

RMT: Pair correlation function

Correlation function of the DOS: $R(\omega) = \Delta^2 \langle \rho(E)\rho(E+\omega) \rangle$



$$R_U(\omega) = 1 - \frac{\sin^2(\pi\omega/\Delta)}{(\pi\omega/\Delta)^2}$$

Level repulsion at $\omega \ll \Delta$: $R(\omega) \propto \omega^{\beta}$

ensemble dependent: $\beta = \begin{cases} 1, & \text{GOE} \\ 2, & \text{GUE} \\ 4, & \text{GSE} \end{cases}$

RMT: Why do levels repel?

Toy example: 2×2 **matrix**

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{12}^* & h_{22} \end{pmatrix} \Longrightarrow E_{12} = \frac{h_{11}+h_{22}}{2} \pm \frac{1}{2}\sqrt{(h_{11}-h_{22})^2 + 4|h_{12}|^2}$$

Weight: $\mathcal{P}(H) \propto \exp\left(-\frac{1}{2\alpha}\operatorname{tr} H^2\right) = \exp\left(-\frac{1}{2\alpha}\left[h_{11}^2 + h_{22}^2 + 2|h_{12}|^2\right]\right)$

Pair correlator:

$$\langle \delta(E_1 - \varepsilon - \omega/2)\delta(E_2 - \varepsilon + \omega/2)\rangle \Longrightarrow \left\langle \delta\left(\sqrt{(h_{11}-h_{22})^2 + 4|h_{12}|^2} - \omega\right)\right\rangle$$

Degrees of freedom?

GOE: $d(h_{11}-h_{22})\, dh_{12}$

GUE: $d(h_{11}-h_{22})\, d(\mathrm{Re}\, h_{12})\, d(\mathrm{Im}\, h_{12})$

GOE: $R(\omega) \sim \omega$

GUE: $R(\omega) \sim \omega^2$

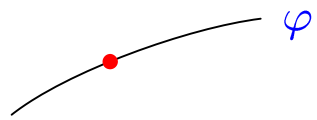
$\Big\}\ \omega^\beta$

Parametric level statistics

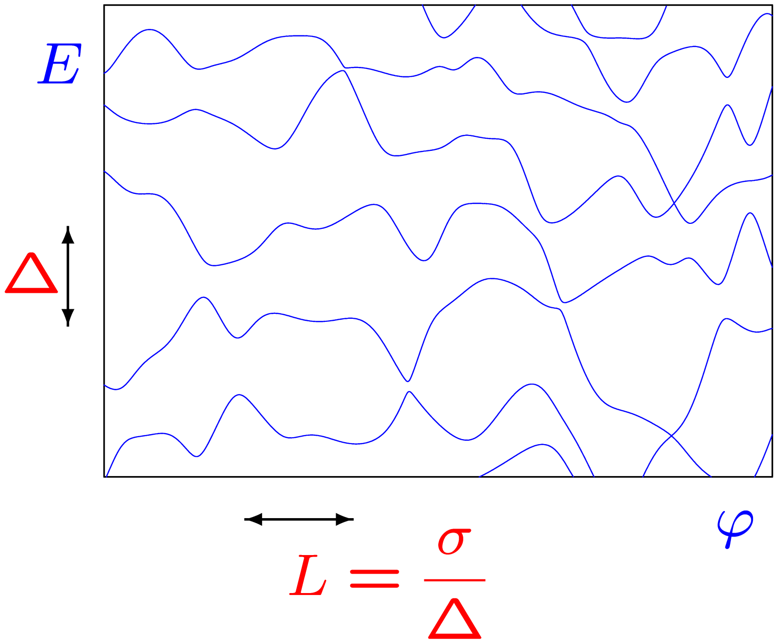
Altshuler, Simons (1993)

Consider a one-parameter trajectory in the space of random matrices:

$$H(\varphi) = H_1 + H_2\,\varphi$$



Still studying the spectrum: $H(\varphi)\psi_n(\varphi) = E_n(\varphi)\psi_n(\varphi)$



Parameters of the spectrum:

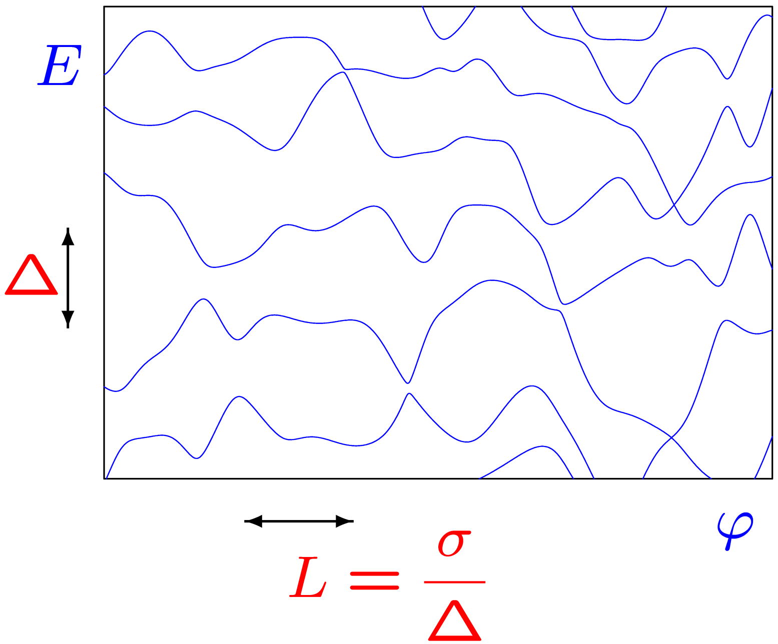
- Δ — mean level spacing
- σ — mean level velocity:

$$\sigma^2 = \langle (\partial E_i / \partial \varphi)^2 \rangle$$

Universality

Route to time-dependent random matrices

- **Eigenvalue statistics**: $H\psi = E\psi$ — everything is known
- **Parametric eigenvalue statistics**: $H[\varphi]\,\psi = E[\varphi]\,\psi$



- **Time-dependent problem**.

Let $\varphi(t)$ be a function of time:

$$i\frac{\partial \Psi(t)}{\partial t} = H[\varphi(t)]\,\Psi(t)$$

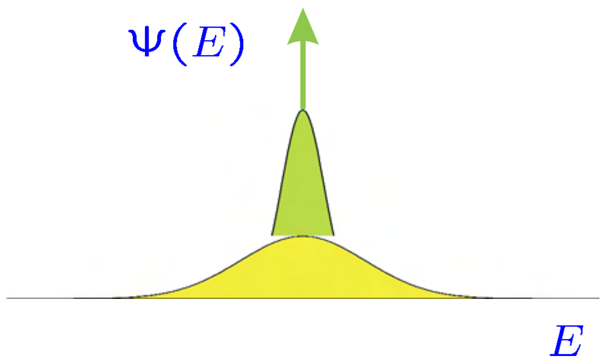
Energy is not a conserving quantity anymore

$$\langle E(t) \rangle = ???$$

What is to look for?

Spreading of the wave function due to interlevel transitions

[math]
Evolution of the initial state
$\Psi_n(0) = \delta_{n,0}$



$$\langle \psi(t) | [H(t) - E_0]^2 | \psi(t) \rangle = Dt$$

diffusion coefficient
in the energy space

[phys]
Evolution of the distribution
of noninteracting fermions

$$\langle \mathcal{E}(t) \rangle = Wt$$

energy absorption rate

$$W = \frac{D}{\Delta}$$

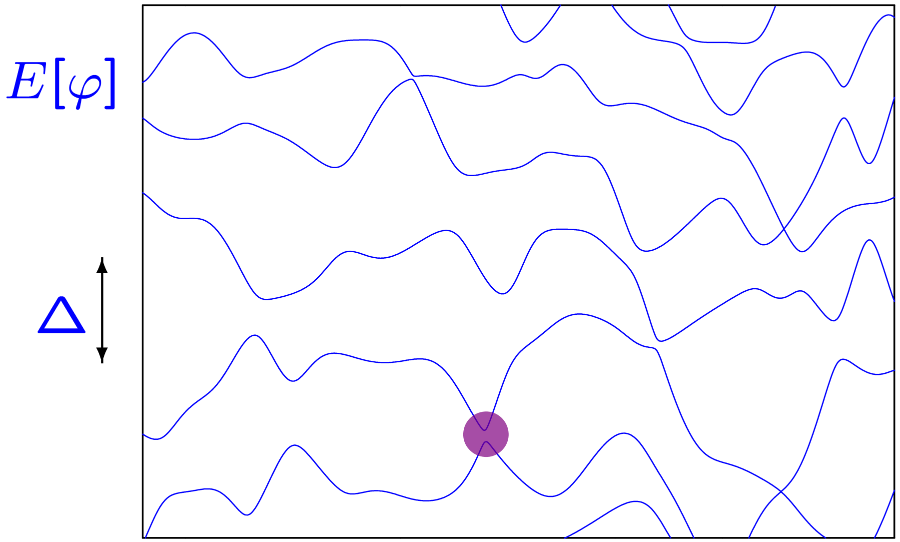
Two basic quantum phenomena

- **Adiabatic** & **Kubo** regimes of dissipation
  - distinguished by $v = d\varphi/dt$
  - local property
  - best example: $\varphi(t) = vt$

- **Dynamical localization** in the energy space
  - for re-entrant $\varphi(t)$
  - global property
  - best example: $\varphi(t) = \sin \omega t$

Two regimes of dissipation

Adiabatic spectrum

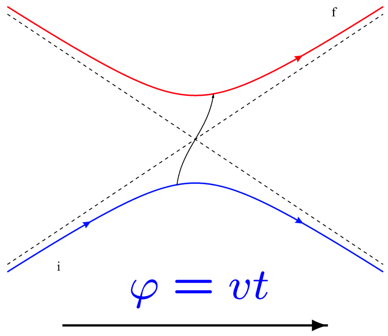


$$\sigma^2 = \langle (\partial E_i/\partial\varphi)^2 \rangle \qquad \varphi = vt$$

Critical velocity: $v_K \sim \dfrac{\Delta^2}{\sigma}$

Landau-Zener transition

$$H[\varphi] = \begin{pmatrix} A\varphi & \varepsilon \\ \varepsilon & -A\varphi \end{pmatrix}$$

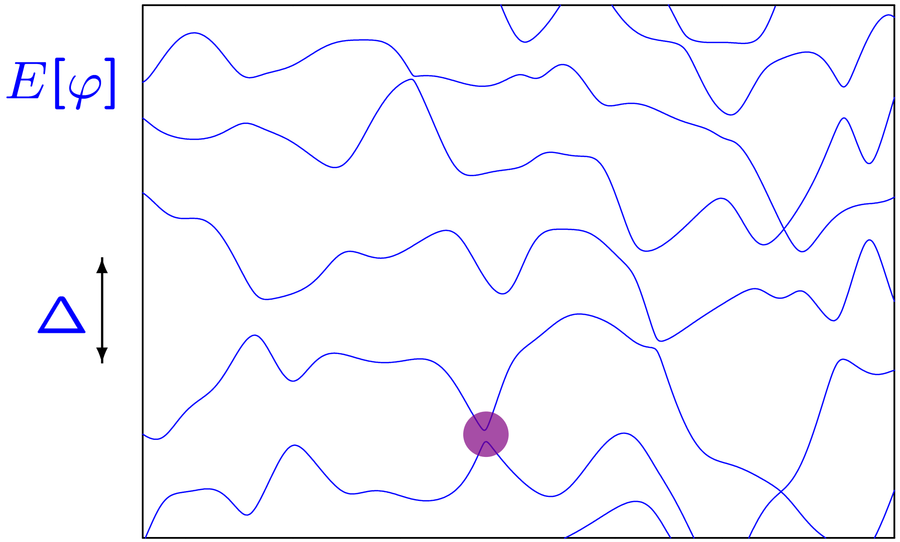


Probability to jump
to the other branch:

$$w_{\mathrm{LZ}} = \exp\left(-\frac{\pi\varepsilon^2}{Av}\right)$$

Two regimes of dissipation

Adiabatic spectrum



$\sigma^2 = \langle(\partial E_i/\partial\varphi)^2\rangle$ $\qquad$ $\varphi = vt$

Critical velocity: $v_K \sim \frac{\Delta^2}{\sigma}$

- **Adiabatic regime:** $v \ll v_K$

  discrete spectrum

  LZ transitions

  depends on statistics

- **Kubo regime:** $v \gg v_K$

  continuous spectrum

  Kubo formula

  statistics independent

Dissipation in the Kubo regime

Wilkinson (1988)

$$H(t) = H_0 + Vt \qquad i\frac{d}{dt}|t\rangle = V(t)t|t\rangle \qquad |t\rangle = T\exp\left(-i\int_{-\infty}^{t} V(t')t'dt'\right)|0\rangle$$

Linear response with respect to $v = d\varphi/dt$: $\quad |t\rangle \approx |0\rangle - i\int_{-\infty}^{t} V(t')t'dt'\,|0\rangle$

Absorption rate: $\quad W = \frac{d}{dt}\langle E\rangle = \frac{d}{dt}\langle t|H|t\rangle = \langle t|\frac{dH}{dt}|t\rangle = \langle t|V(t)|t\rangle$

Kubo formula: $\quad W = i\int_{-\infty}^{t} t'\,dt'\,\langle 0|[V(t'),V(t)]|0\rangle$

$$V(t) = \sum_{mn} V_{mn}\hat{a}^{\dagger}_{m}\hat{a}_{n}e^{i(E_n-E_m)t}$$

$$W = \sum_{mn}|V_{mn}|^2(f_m - f_n)\frac{i}{(E_m - E_n + i\gamma)^2}$$

$$E_{m,n} = E \pm \omega/2$$

$$W = |V_{m\neq n}|^2\int\frac{dE\,d\omega}{\Delta^2}\,\omega\frac{\partial f}{\partial E}\,\frac{i}{(\omega+i\gamma)^2} = \frac{\pi\sigma^2}{\Delta^2}v^2$$

$$W_K = \frac{\pi\sigma^2}{\Delta^2}v^2$$

makes sense for $\gamma \gg \Delta$

Dissipation in the adiabatic regime

Wilkinson (1988)

$$H = \begin{pmatrix} E_1 & \\ & E_2 \end{pmatrix} + \begin{pmatrix} h_{11} & h_{12} \\ h_{12}^* & h_{22} \end{pmatrix} \varphi \quad \Longrightarrow \quad E_+ - E_- = \sqrt{\varepsilon^2 + A^2(\varphi - \varphi_0)^2}$$

$$\varepsilon = E_1 - E_2 \qquad A = \sqrt{(h_{11} - h_{22})^2 + 4|h_{12}|^2} \qquad \varphi_0 = -\frac{\varepsilon}{A^2}(h_{11} - h_{22})$$

# of avoided crossings per unit time — LZ tunneling exponent

$n \to n+1$ **transition rate:** $R = \left\langle v\delta(\varphi_0) \; \exp\left(-\frac{\pi\varepsilon^2}{2Av}\right) \right\rangle_{\varepsilon,h}$

$$\mathcal{P}(\varepsilon)d\varepsilon \propto \varepsilon^{\beta} d\varepsilon$$

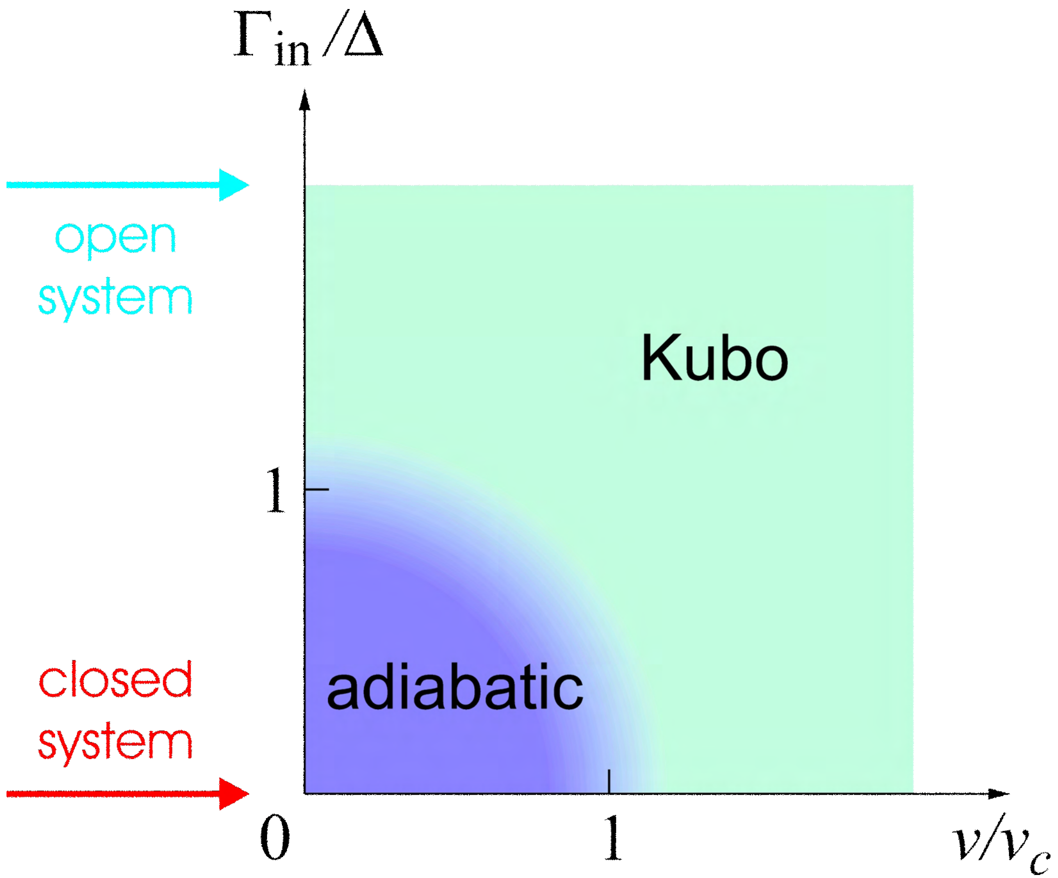
$$\mathcal{P}(h) \propto \exp\left(-\frac{\mathrm{tr}\, h^2}{2\sigma^2}\right)$$

$$\sigma^2 = \langle (\partial E_i/\partial\varphi)^2 \rangle$$

$$R \propto v \int_0^{\infty} \varepsilon^{\beta} \, d\varepsilon \; \frac{1}{\varepsilon} \; \exp\left(-\frac{\pi\varepsilon^2}{2Av}\right)$$

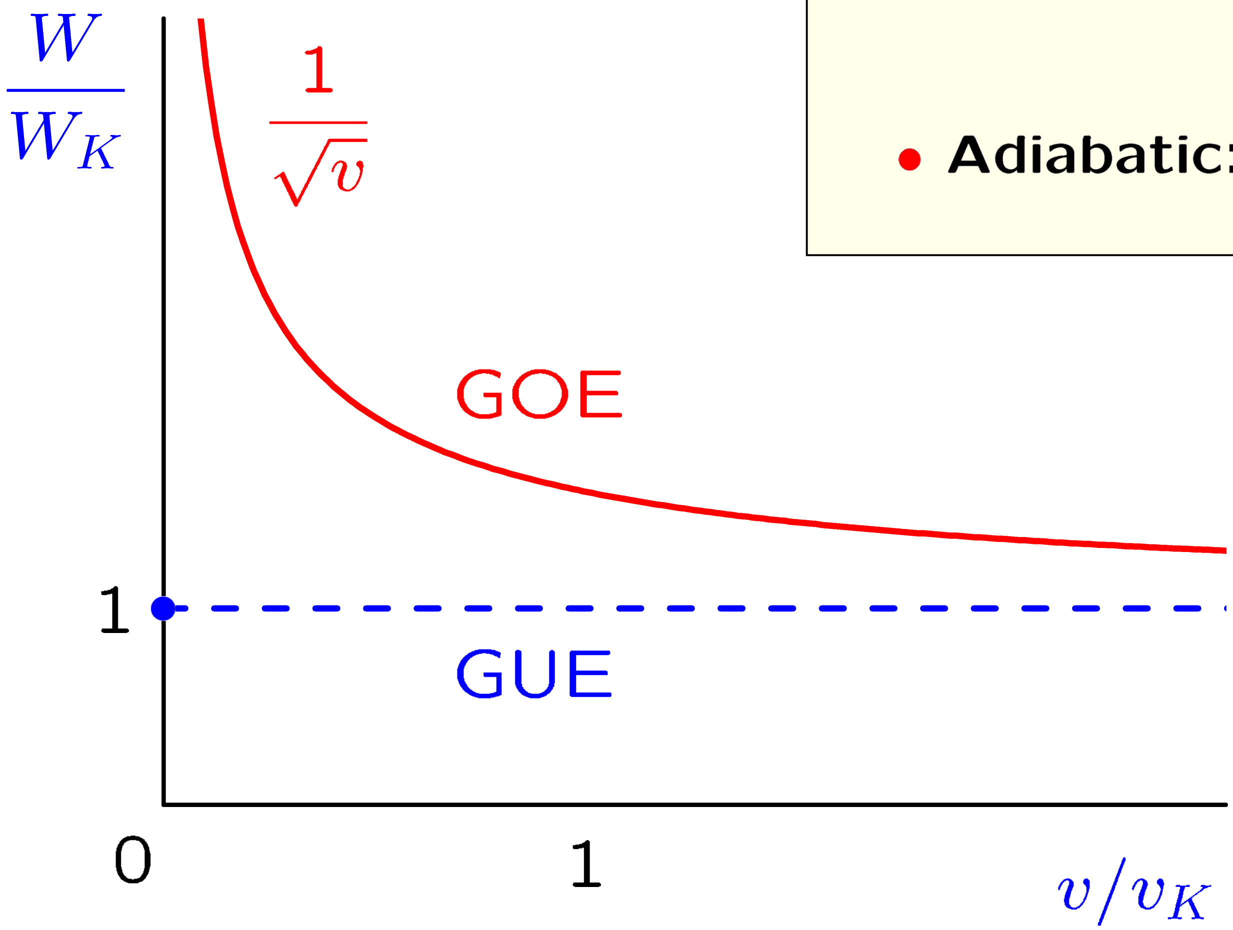
$$R \sim v^{1+\beta/2}$$

Linear response?



NO linear response in a closed system!

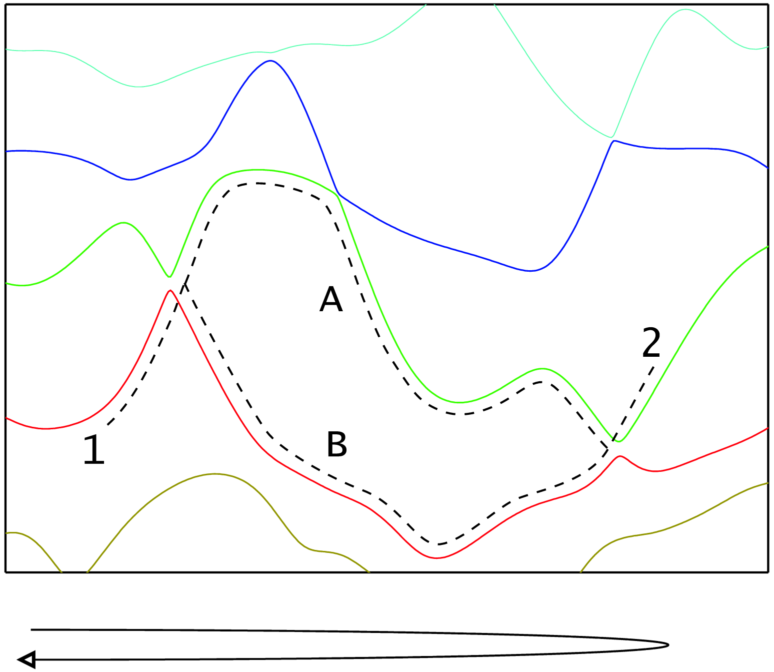
Linear perturbation: summary



- **Kubo:** $W_K = \frac{\pi\sigma^2}{\Delta^2} v^2$
- **Adiabatic:** $W \propto v^{1+\frac{\beta}{2}}$

Localization in the energy space

(Handwaving arguments in the adiabatic regime)



- **Monotonous** $\varphi(t)$
  interference is ineffective

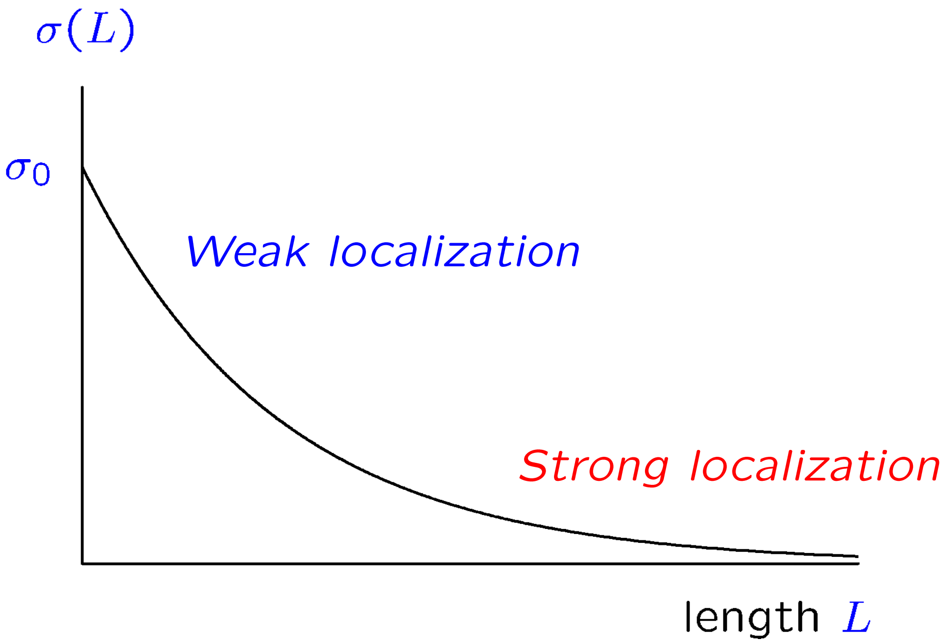
$$|A+B|^2 = |A|^2 + |B|^2 + 2\,\mathrm{Re}(AB^*)$$

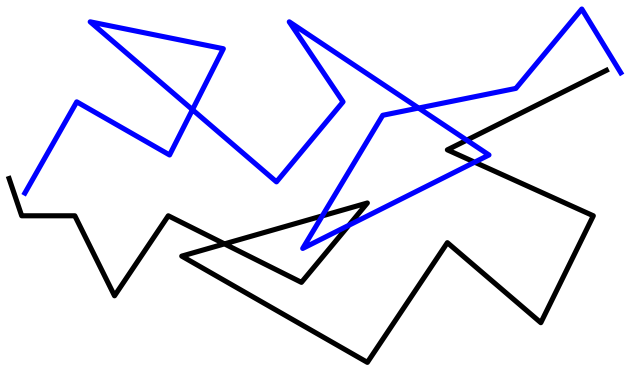
- **Re-entrant** $\varphi(t)$
  interference *may be* important
  for well-tuned perturbations

Квантовые интерференционные эффекты

1) Андерсоновская локализация в реальном пространстве

Подавление диффузии на больших расстояниях
за счет деструктивной интерференции
между различными траекториями





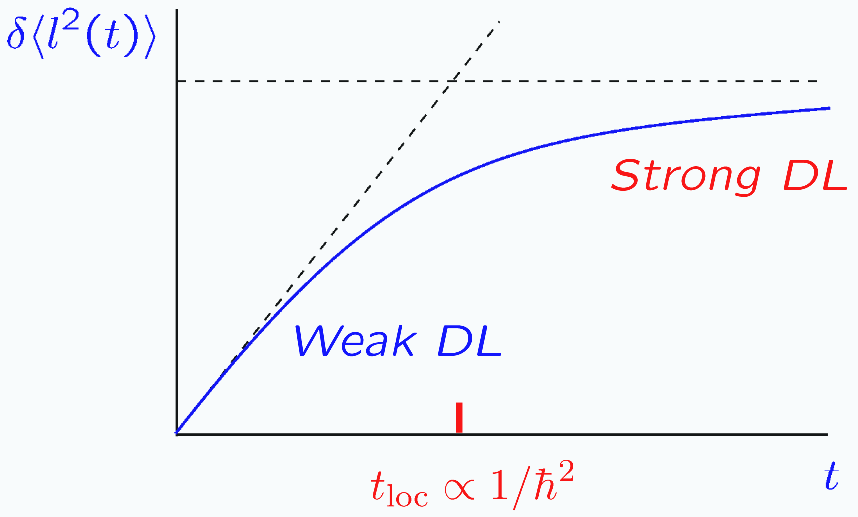
Квантовые интерференционные эффекты

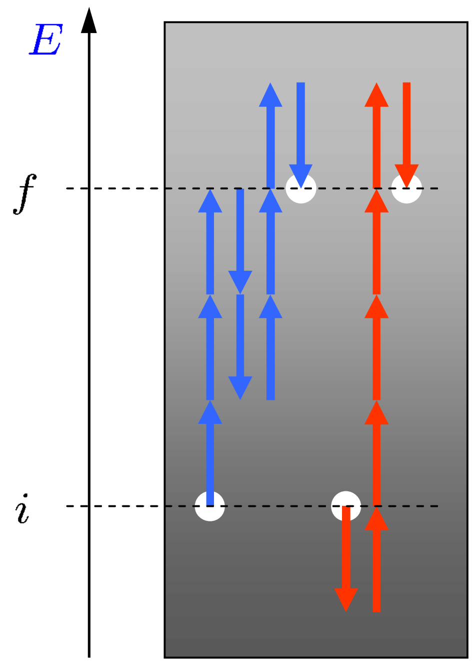
2) Динамическая локализация в пространстве энергий

Quantum kicked rotor

Casati, Chirikov, Ford, Izrailev (1979)

$$\hat{H} = -\frac{\hbar^2}{2}\frac{\partial^2}{\partial\theta^2} + K\cos\theta\sum_n \delta(t-n)$$



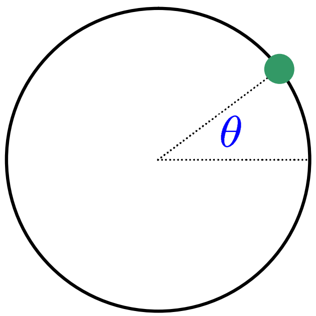


Экспериментальная реализация:

ультрахолодные атомы в поле модулированной стоячей лазерной волны [Moore et al (1994)]

Localization in Quantum Kicked Rotor

Casati, Chirikov, Ford, Izrailev (1979)



$$\hat{H} = \frac{\hat{l}^2}{2} + K\cos\theta \sum_n \delta(t-n)$$

momentum operator: $\hat{l} = -i\kappa \frac{\partial}{\partial\theta}$

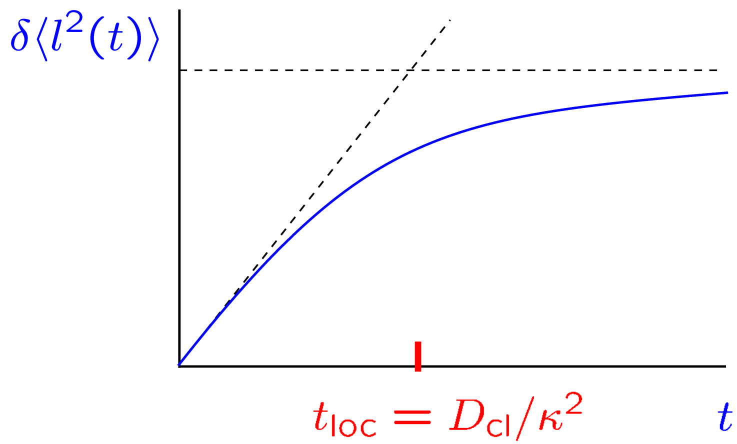
- **Diffusion in momentum space:**

$$\delta\langle l^2(t)\rangle \equiv \langle [l(t) - l(0)]^2 \rangle = 2D_{\rm cl}\, t$$

$$D_{\rm cl} \approx \frac{K^2}{4}, \qquad K \gg 1$$

A problem with one degree of freedom!

- **Dynamic localization**



quantum time

The model

We consider a time-dependent matrix Hamiltonian: $H(t) = H_0 + V\varphi(t)$,

where H_0 and V are independent random $N \times N$ matrices from the GOE (GUE) chosen to give the mean level spacing Δ and the sensitivity of the spectrum to variation of φ:

$$\left\langle \left(\frac{\partial E_n}{\partial \varphi} \right)^2 \right\rangle = \frac{2\Gamma\Delta}{\pi}$$

Schrödinger equation:

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H(t)|\psi(t)\rangle \qquad \Longrightarrow \qquad |\psi(t)\rangle = T\,\mathrm{exp}\left(-i\int_0^t H(t')dt'\right)|0\rangle$$

Physical observable:

$$\langle\psi(t)|A(t)|\psi(t)\rangle = \langle 0|\underbrace{\hat{T}^{-1}\,\mathrm{exp}\left(i\int_0^t H(t')dt'\right)}_{\text{backward evolution}} A(t)\,\underbrace{\hat{T}\,\mathrm{exp}\left(-i\int_0^t H(t')dt'\right)}_{\text{forward evolution}}|0\rangle$$

DISORDER + KINETICS = ???