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ФИАН

Electron-Electron Interaction Effects and Related Phenomena in Low-Dimensional Electron Systems

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Династия

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Outline

- ❑ (1) appearance of the M-I transition in 2D
- ❑ (2) electron-electron interaction effects in 2D
- ❑ (3) electron-electron interaction effects in 1D

Intrigue:
a competition of
✓ interference & interaction for various dimensionalities,
✓ e-e interaction & disorder,
✓ magnetic field reduction in dimensionality & exchange driven spin and charge ordering

• Lecture (1) outline:

- ❑ Introduction. Semiclassical picture of the M-I transition (MIT)

- ❑ Quantum single-partical picture. Quantum transport at *B* =0
- ❑ Experimentally studied 2D electron systems
- ❑ Appearance of the MIT in 2D system at *B* =0

- ❑ Reentrant *QHE-I* transitions in $B_{\perp}$ -field:
- ✓ Extended states in quantizing fields

- ❑ M-I transition at *B* =0 as an electron “liquid-solid” transition
- ✓ Collective transport in the solid (insulator) phase

Lecture (2). Outline:

❑ Quantitative studies of the *e*-*e* interaction
✓Renormalization of χ^*, m^*, g^* in 2D electron liquid
✓Potential magnetic transition ?

❑ Quantitative description of the metallic-like transport in 2D in terms of the *e*-*e* - interaction:
✓ At high electron density, away of the MIT
✓ For partially and fully spin-polarized state

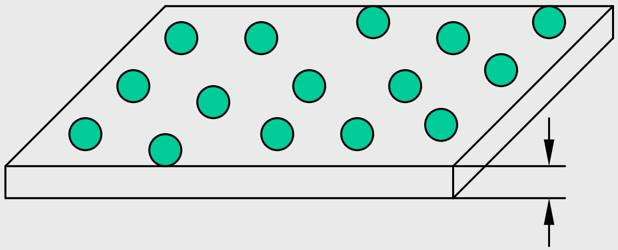
❑ Data analysis within framework of the RG
✓ Transport at low density, in the critical regime

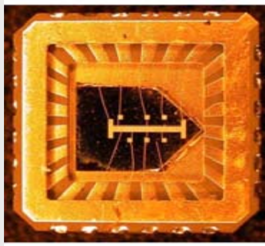
Lecture (3). Outline

- Quasi- 1D eletron system in a 3D host lattice
- Spin density wave
- Field induced spin-density wave
- Rapid oscillations in the spin-ordered insulating phase
- Inhomogeneous phase-mixed state in the vicinity of the phase transition

1. A few introductory cartoons

Studied object: 2D sheet of electrons at the semiconductor/insulator interface





When the bulk and interface are ideally clean, one can prepare the 2D-sheet of electrons with low density, ~10^{10}cm^{-2}

As a result, the e-e interaction energy $E_{ee} \propto e^2 n^{1/2}$ **becomes exctremely large ~ 70K,** much exceeding the kinetic (Fermi) energy $E_F \propto n$ ~ 5K

$$r_s = E_{ee}/E_F \propto 1/n^{1/2} >> 1$$

Motivation: strongly-correlated 2D system of charged fermions may manifest novel phases and novels physics

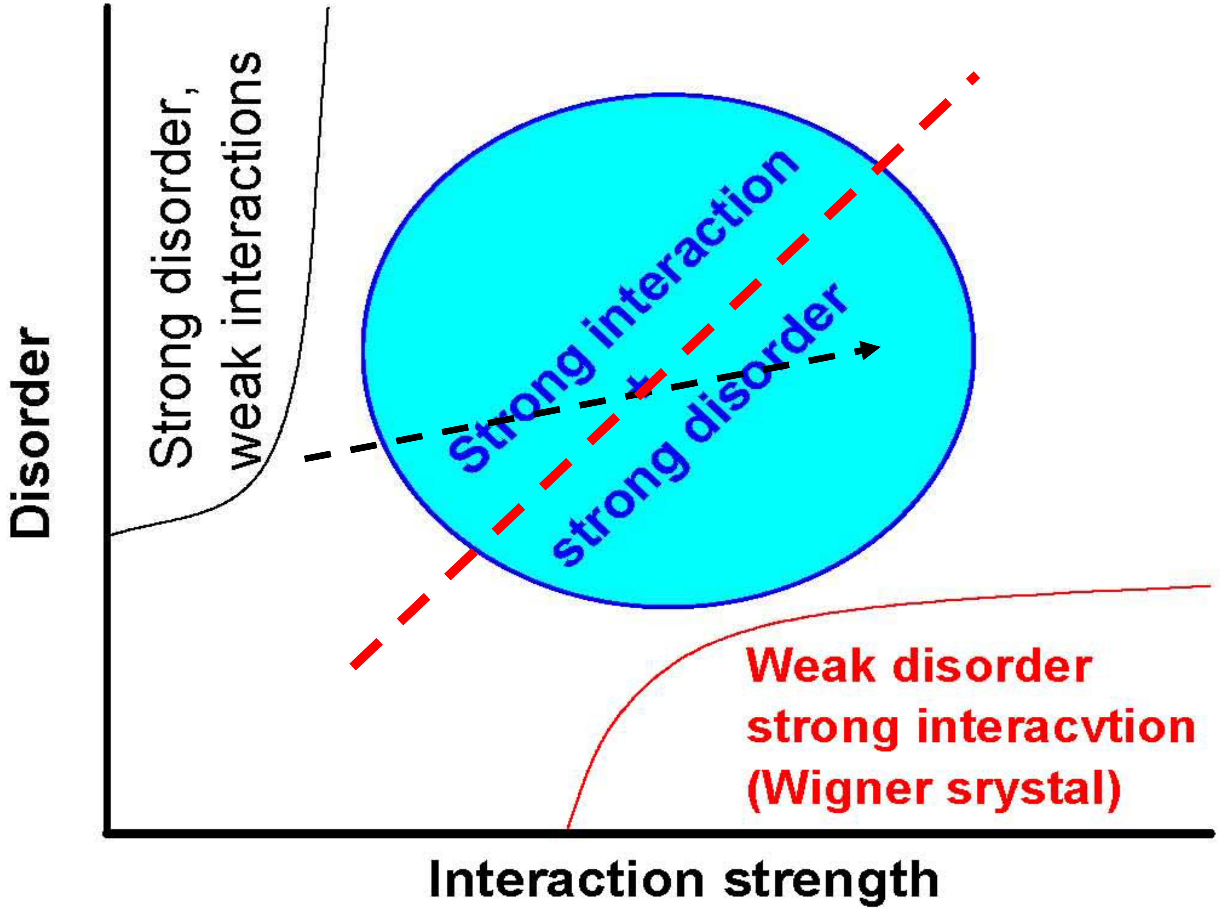


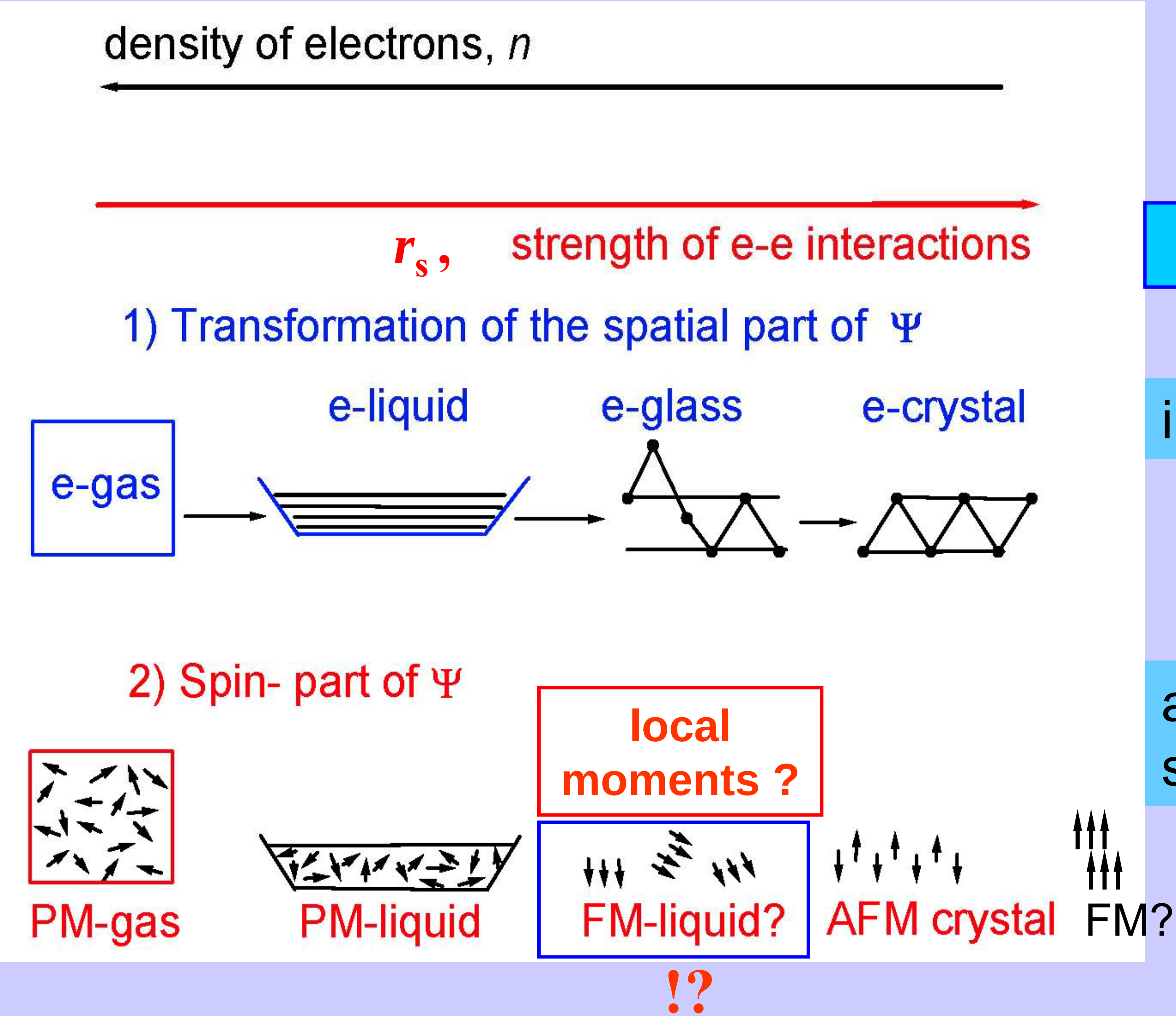
FIG. 2.

Strong interaction, zero disorder $\Rightarrow$ Wigner crystal

Weak interaction, weak disorder $\Rightarrow$ ordinary metals

Weak interaction, strong disorder $\Rightarrow$ Efros-Shklovskii case

e-e interaction is expected to cause ordering



$r_s = E_{ee}/E_F \propto 1/n^{1/2}$

in the charge

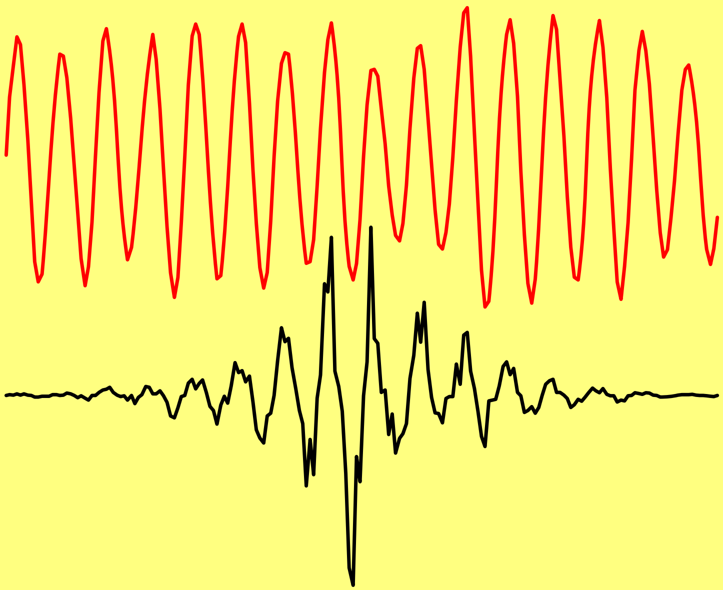
and spin systems

1.1. Historical milestones:

Difference between insulator and metal

✓Wave origin of the insulator and metal

Delocalized states
≡ Extended states



Localized states

$$l \sim \frac{1}{k_F}$$

Ioffe-Regel Criterion
A.F. Ioffe and A.R. Regel, *Progr. Semicond*. **4**, 237 (1960).

Abrahm Ioffe 1880-1960

Anatoly Regel 1915-1989

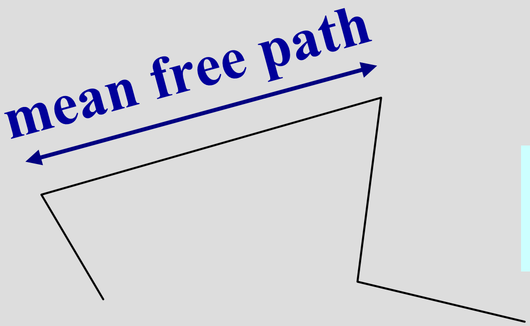
1.2. Emergence of the quantum transport concepts

Metal: classical physics $\sigma = \frac{ne^2\tau_{tr}}{m}$ $\sigma = ne\mu$

μ - **mobility**

Semiclassical physics

$l_{tr} >> \lambda$ (electron wave length)



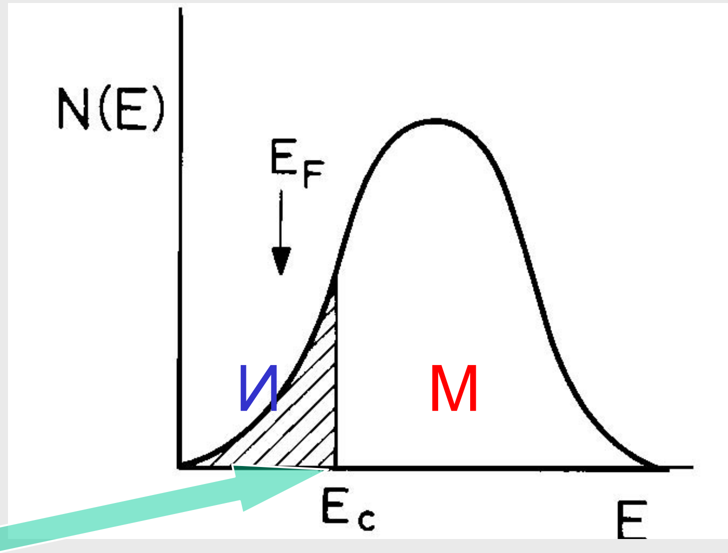
$\tau_{tr} = l_{tr} / v_F$

M-I Transition: $k_F l_{tr} \sim 1$ Ioffe – Regel; Mott

$$\sigma_{2d} = e^2 \frac{(p_F{}^2 / 2\pi\hbar^2)(l_{tr} / v_F)}{m} \sim e^2 /(2\pi\hbar)\ ;\ \ 2\pi\hbar / e^2 \approx 25.8\ \ k\Omega$$

Insulator: Anderson localization

1.3. Semiclassic concept of the metal-insulator transition



$$k_F l = 1$$

A critical disorder value causes "Anderson transition"

1977

Nevil Mott 1905-96

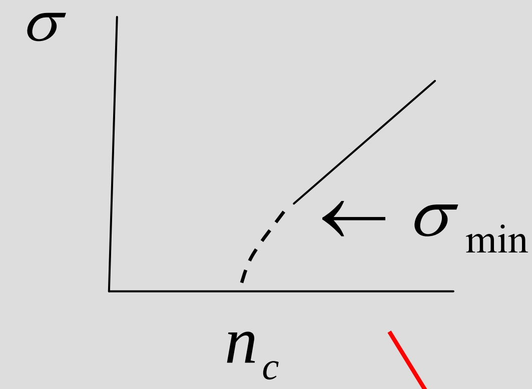
Phil Anderson

John van Flek

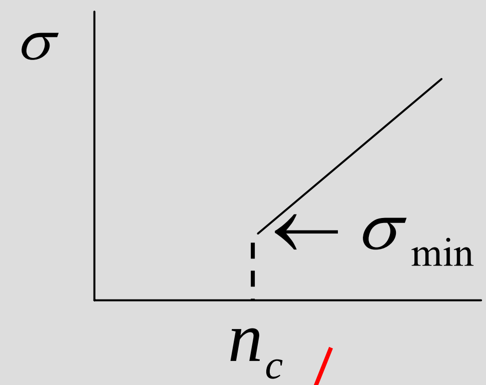
2000 Lars Onsager Prize to

David James Thouless

"For the introduction with J. Michael Kosterlitz of the theory of topological phase transitions, as well as fundamental contributions to our understanding of electron localization and the behavior of spin glasses."

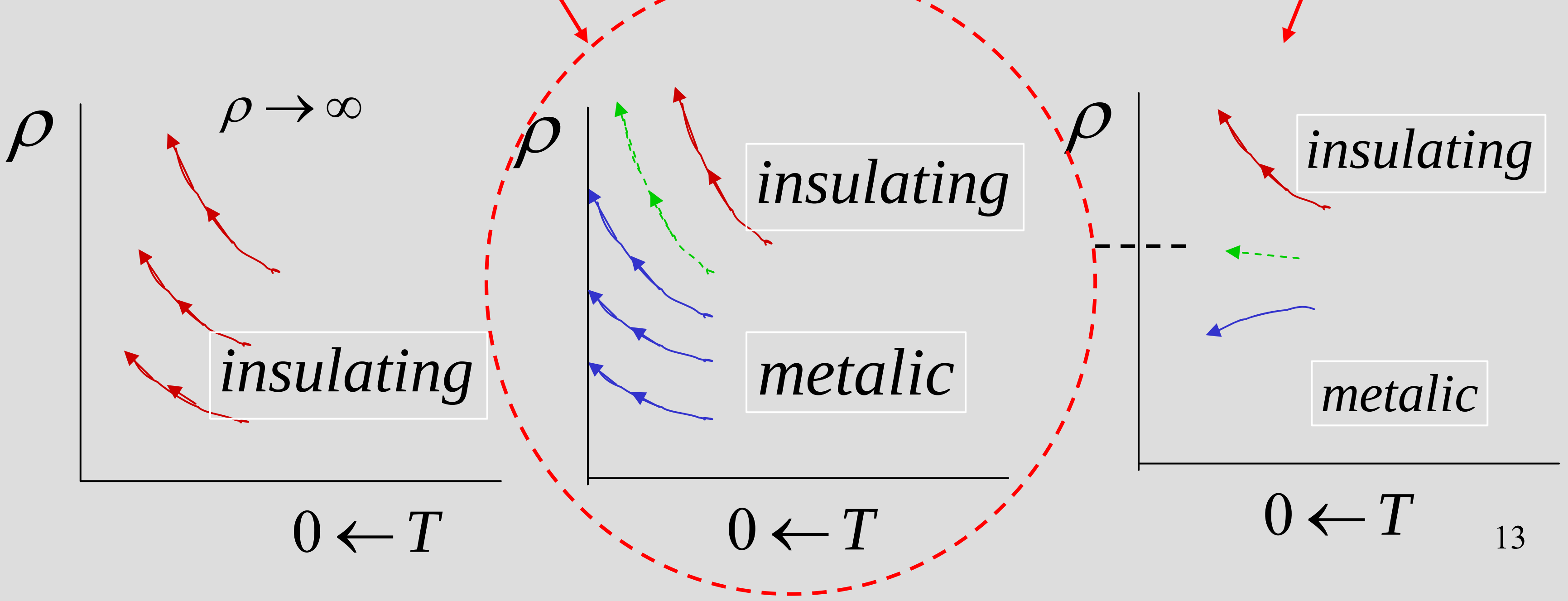


Metal - Insulator Transition at $T = 0$ in 3D

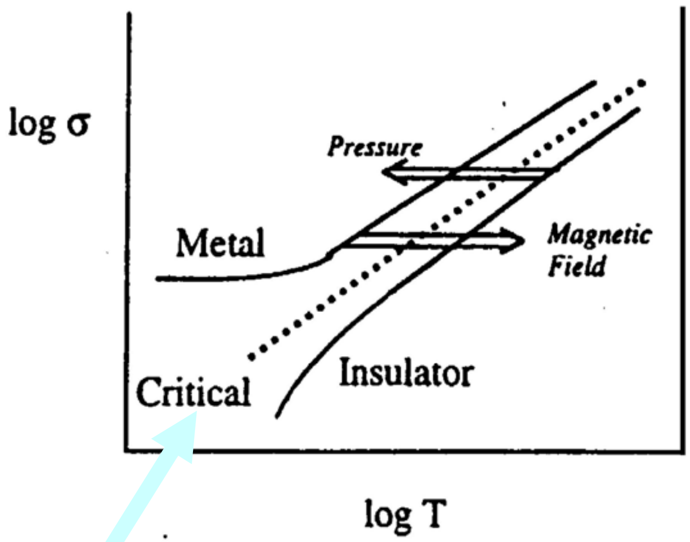


Possible behavior of the resistance :

no M - I Transition



Example of experimental data for the Anderson transition in 3D



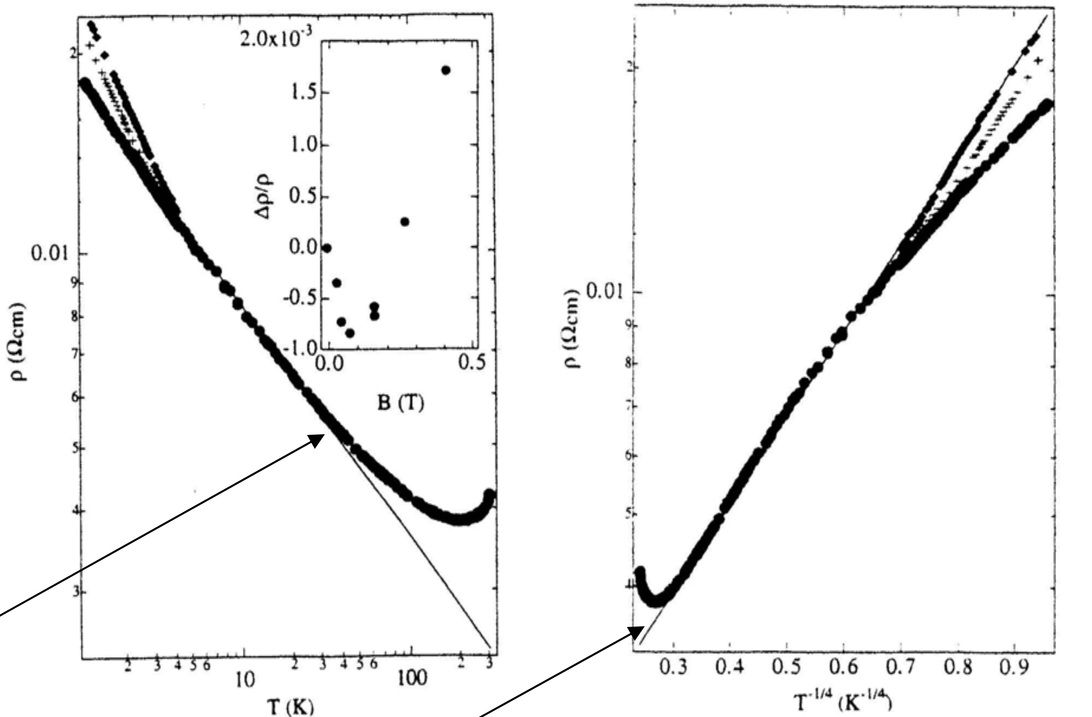


Figure 2: (Left) Log ρ vs log T for PANI-CSA; dots, H = 0, pluses, H = 4T, and diamonds, H = 8T. The solid line represents $\rho(T) \propto T^{-0.36}$. The inset shows the resistivity vs magnetic field at 1.2K.
(Right) Log ρ vs $T^{1/4}$ for PANI-CSA; dots, H = 0, pluses, H = 4T, and diamonds, H = 8T. The solid line represents VRH with $T_o = 56$K.

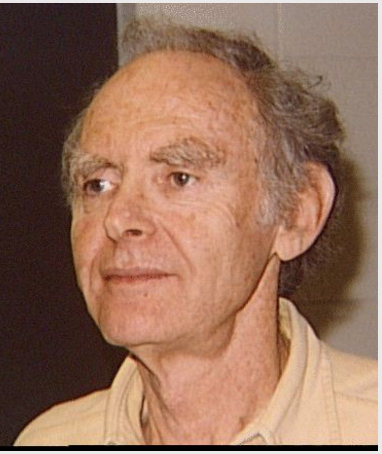
In the critical regime:

$$\sigma_{crit} = \left(\frac{T_1}{T}\right)^{\beta}$$

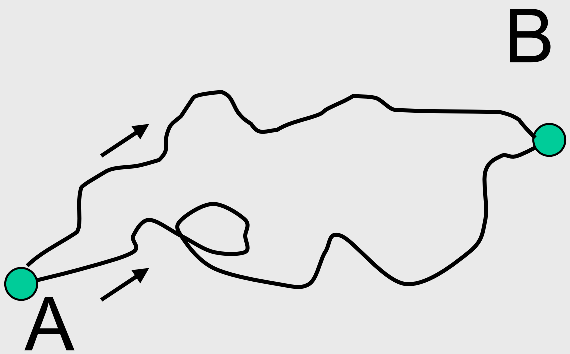
In the insulator:

$$\sigma_{ins} \propto \exp\left(\left(\frac{T_0}{T}\right)^{p}\right)$$

2. Quantum transport concept (1979):



Elihu Abrahams



Interference of electron waves is a driving force for localization

Dimensionality plays a crucial role

$$\sigma = \sigma_D + \frac{e^2}{\pi h} \ln(T\tau)$$

All electrons in 2D must be localized at $T \to 0$

$$\beta(\ln g) = \frac{d \ln g}{d\xi}$$

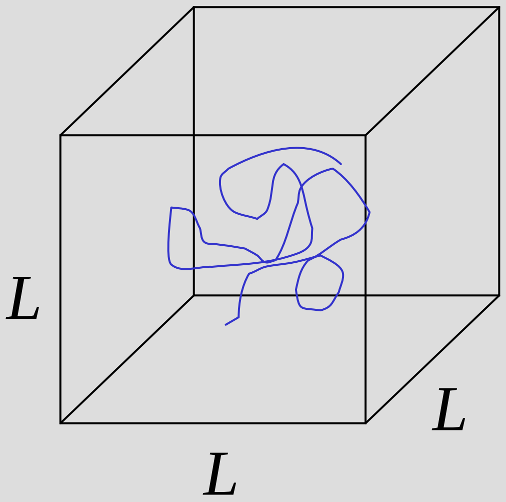
For $\ln(1/T\tau) \geq \sigma$

T.V. Ramakrishnan

Scaling ideas in transport :

Thouless (1974,77); Abrahams, Anderson, Licciardello, Ramakrishnan ('79); Wegner ('79).

Renorm. Group transformation idea:
increase the blocks size from l_{tr} to L and trace
$g(L)=\sigma(h/e^2)$ - the dimensionless conductance.



Dimension ***d=2*** is of a special importance;
the exponent in the Ohm's law, $g \sim (L/l_{tr})^{d-2}$,
vanishes at *d*=2.

$$\frac{d \ln g}{d \xi} = \beta(g) \; ; \; \xi = \ln\left(\frac{L}{l_{tr}}\right)$$

A. A. L. & R. (1979) applied the Gell-Mann - Low equation for the M-I transition:

$\beta(g; L/l_{tr})$ does not depend explicitly on *L*

M-I transition: $\beta(g = g_{crit}) = 0.$

One parameter scaling theory for noninteracting electrons

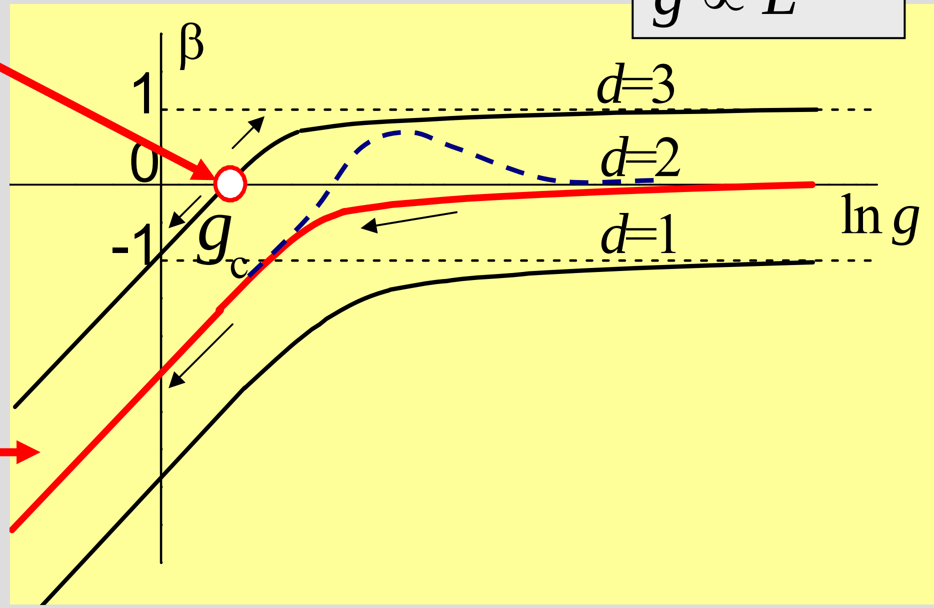
Abrahams, Anderson, Licciardello, and Ramakrishnan (1979)

$$\frac{d \ln g}{d\xi} = \beta(\ln g) \qquad \xi = \ln\left(\frac{L}{a}\right)$$

Conductance g is assumed to be the only scaling parameter

M-I transition in 3D

$g \propto L^{d-2}$



As a result of the interference, **all** electron states are fully localized when $\ln(1/T\tau) >> \sigma$

$g \propto \exp(-L/L_c)$

Transport in low-mobility samples

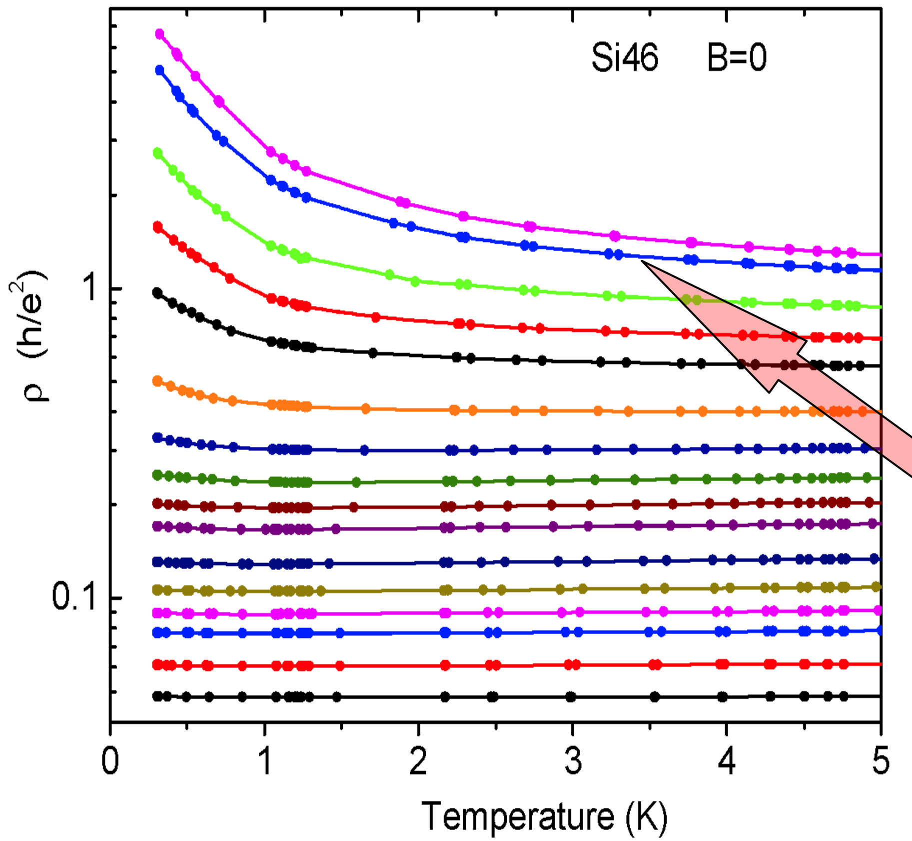


FIG. 20. Resistivity vs temperature for low mobility (disordered) Si-MOS sample

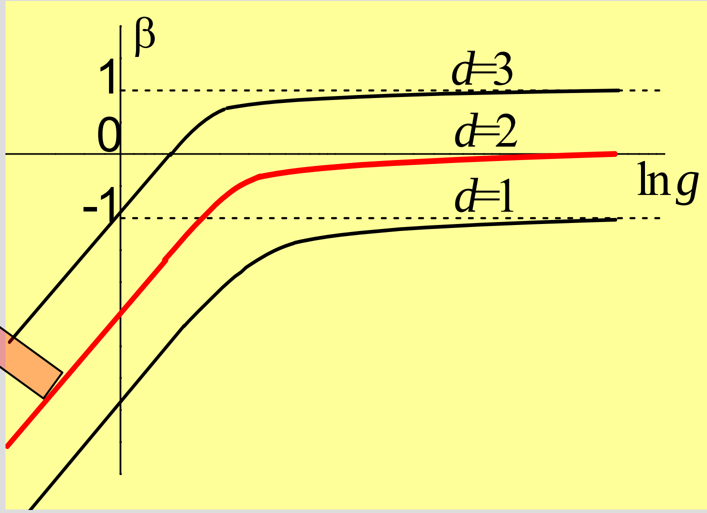
All electron states, indeed, become localized for

$$\ln(1/T\tau) \geq 1/\rho$$

One parameter scaling theory for free electrons (AALR-79) :

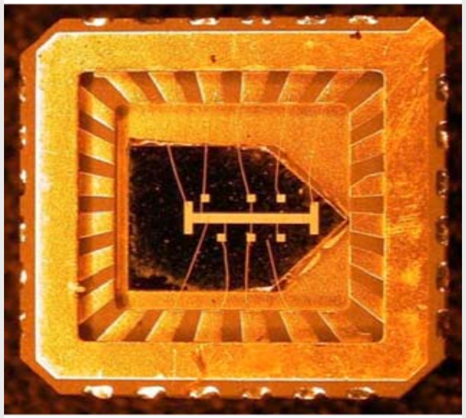
$$\frac{d \ln g}{d\xi} = \tilde{\beta}(\ln g) \; ; \; \xi = \ln \; (L/a).$$

$\tilde{\beta}(\ln g)$



$g \propto \exp(- L/L_c)$

3. Two-dimensional electron systems



Silicon Field Effect Transistor

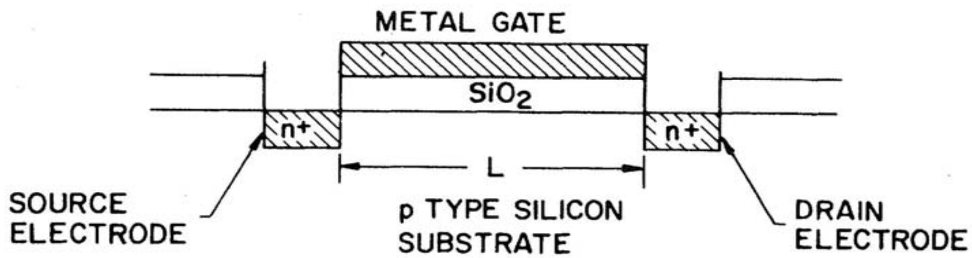


FIG. 12. Schematic structure of a 2DEG: Si-MOSFET

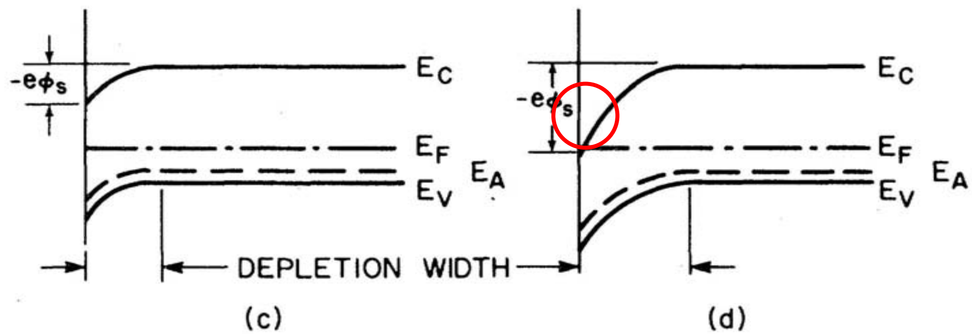
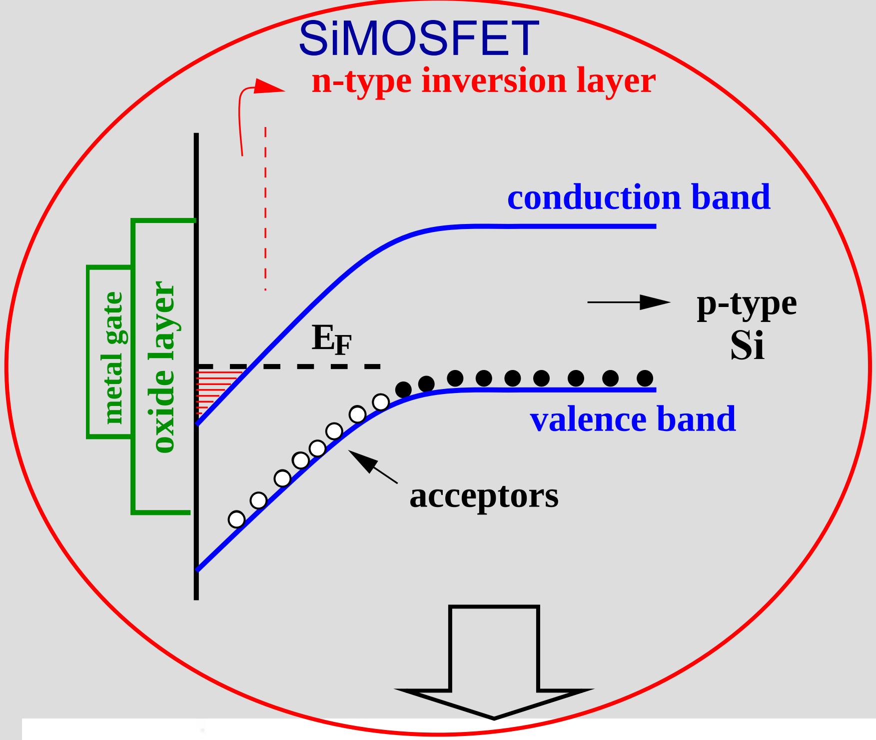


FIG. 13. Energy band diagram of the Si-MOS structure

Si-MOSFET is a perfect tool to study transport at various carrier densities



GaAs/AlGaAs heterostructure

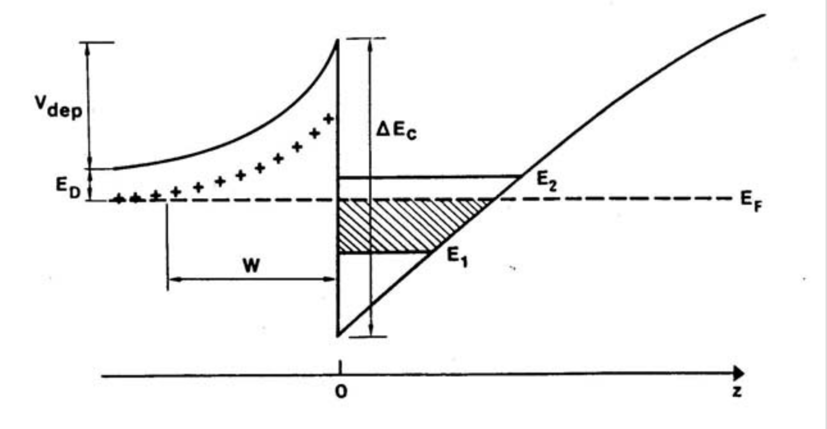
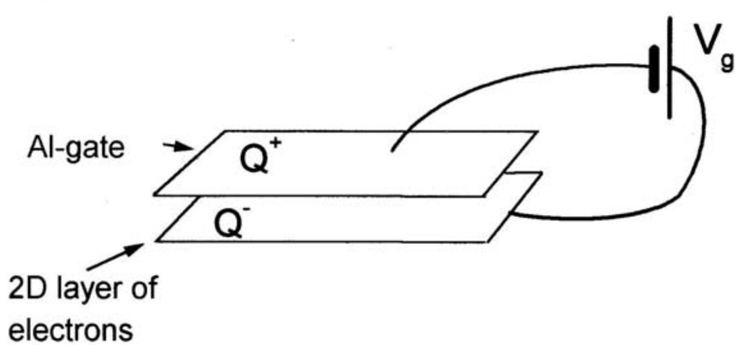


FIG. 11. Schematic energy diagram of a selectively doped GaAs/AlGaAs heterostructure. In order to increase carrier mobility, the doped layer is located away from the active layer



$$Q^+ = -Q^- = n\,e$$
$$n = \frac{1}{e}\, dQ/dV_g (V_g - V_t)$$

FIG. 14. Plane capacitor representation of the Si-MOS structure.

Electron density and, thus, the Fermi energy can be electrically tuned

4. Brief review of the transport in Si-MOSFETs

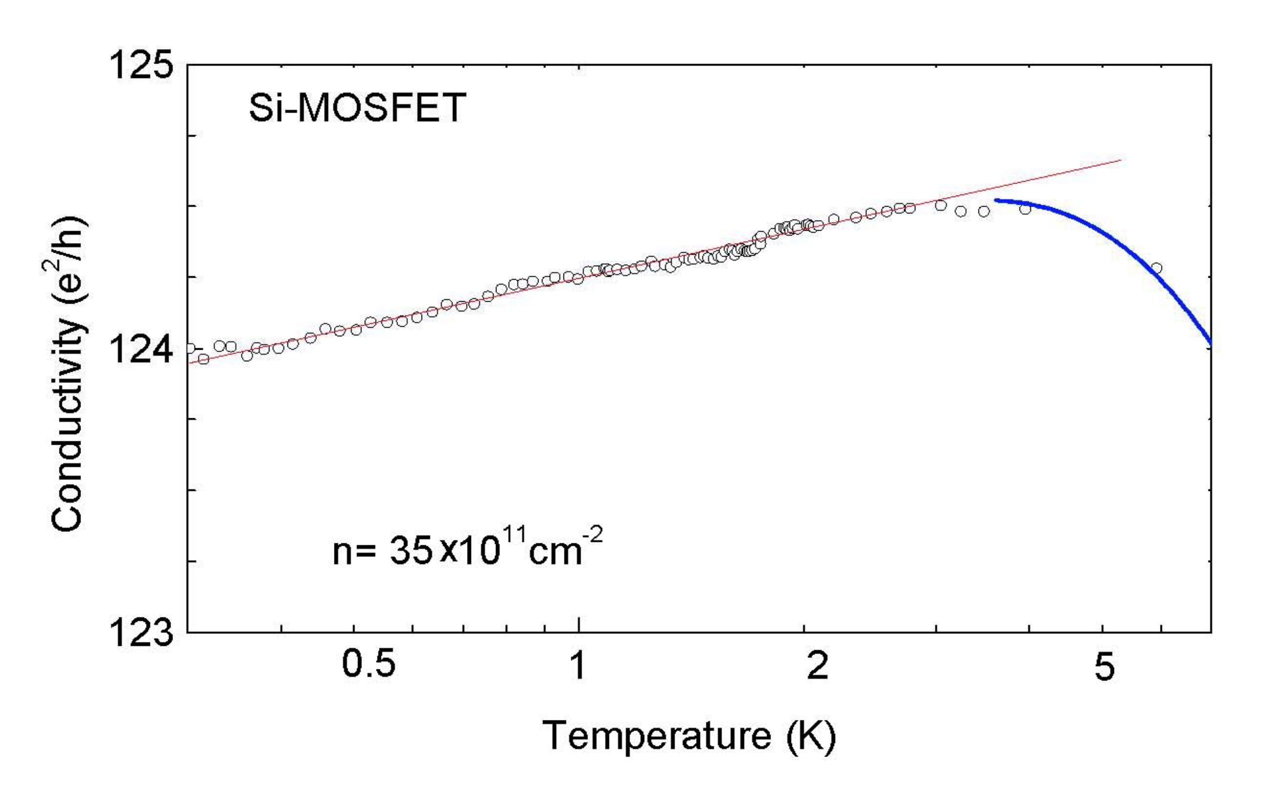
Most elaborated **theoretically** is the **diffusive regime** of transport and of (weak) *e-e* interaction in metals (low temperatures, high densities, low disorder)

$$T\tau \ll 1, \quad \tau_\varphi \gg \tau, \quad k_F l \gg 1$$

Is everything okay in this well-elaborated field of "metal physics"?

Quantum interference correction (backscattering)

$$\sigma = \sigma_D - \frac{e^2}{\pi h}\ln(\tau_\varphi/\tau) \sim \sigma_D + \frac{e^2}{\pi h}\ln(T\tau)$$



seems OK

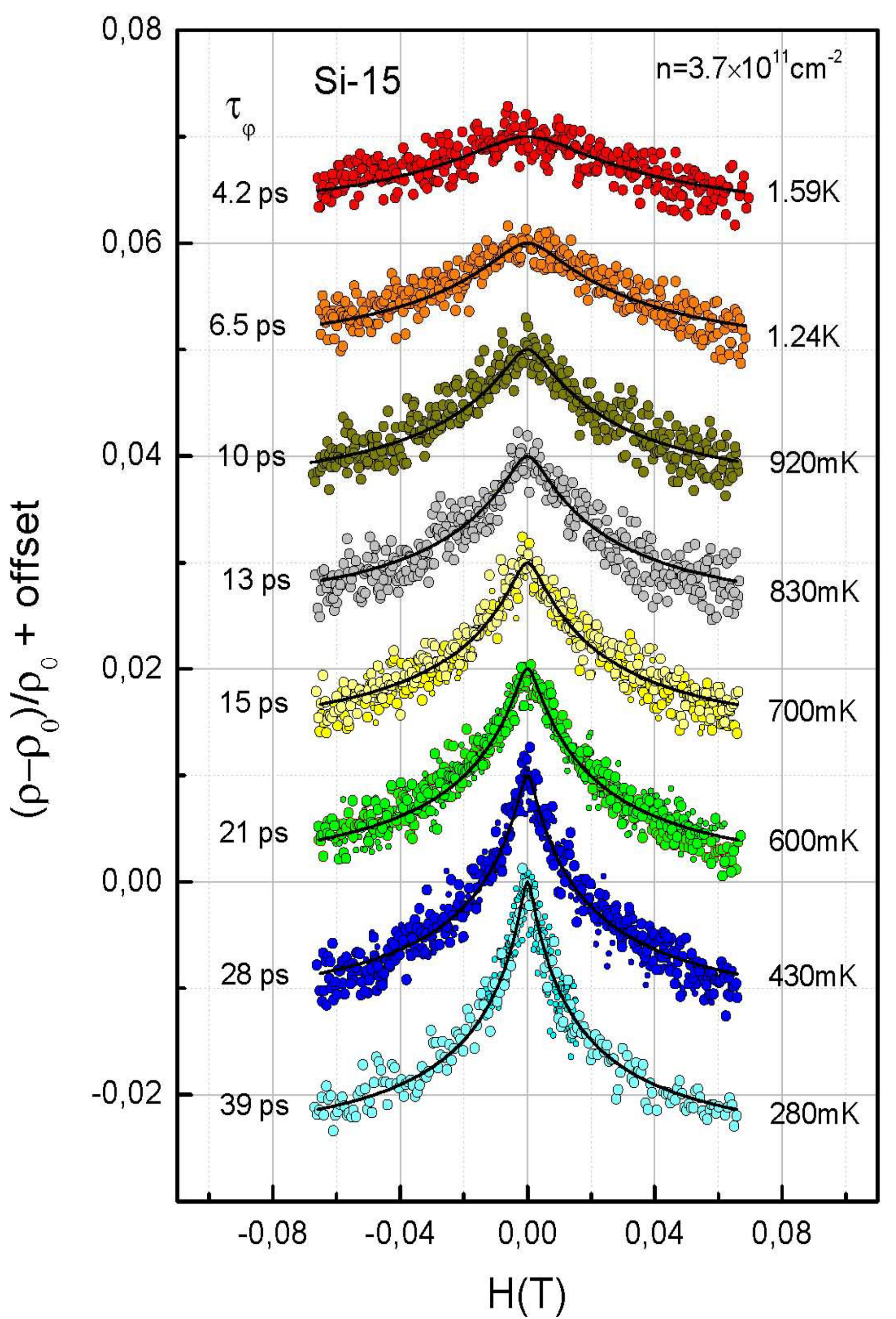
VP, G.Brunthaler, A.Prinz, G.Bauer, *Phys. Rev.* B **60**, 2154 (1999)

Suppression of the weak localization in $H_\perp$ field

The interference breaks down when $\Delta\varphi \sim 1$

$$\Delta\varphi \sim \frac{H D \tau_\phi}{\Phi_0} \sim 1 \qquad H \sim \frac{h}{e D \tau_\varphi}$$

seems OK

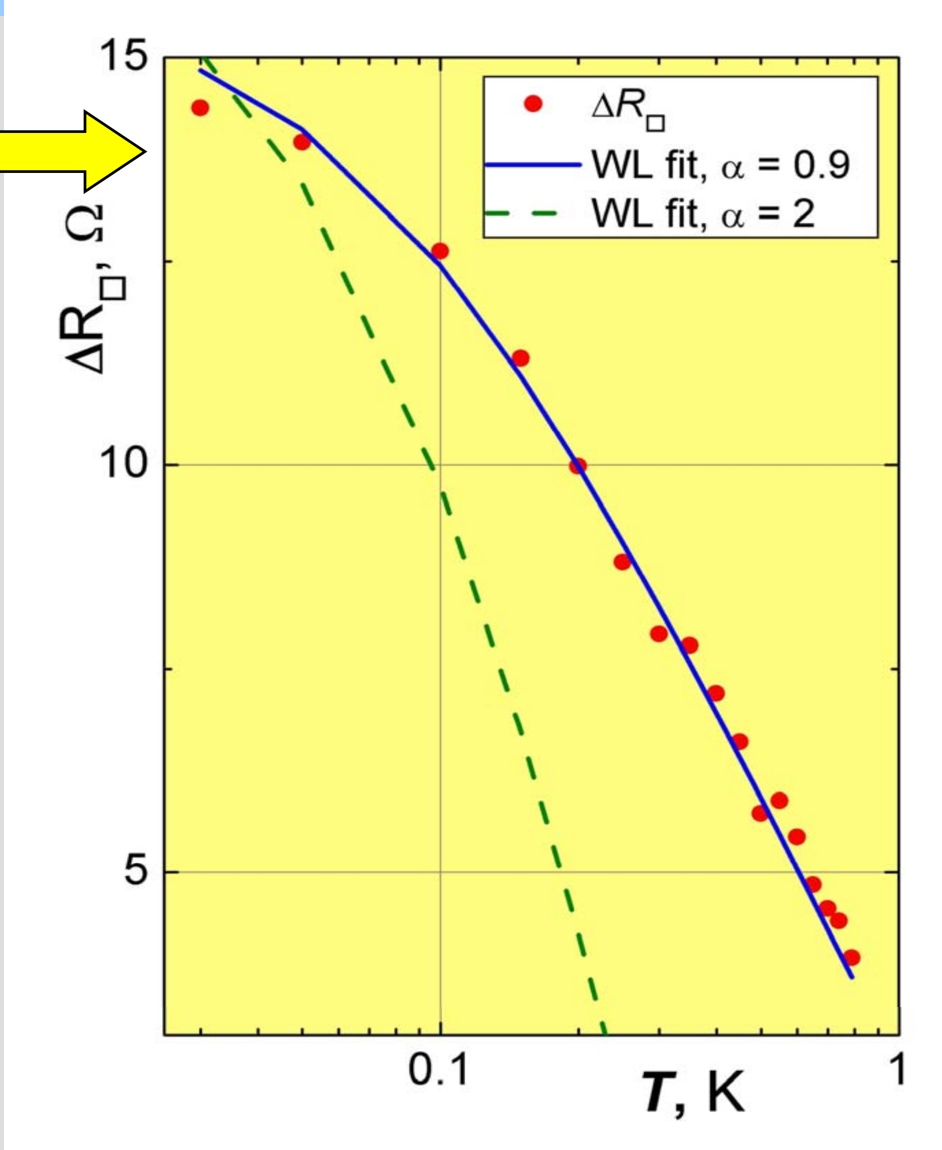


G.Brunthaler, A.Prinz, G.Bauer, V.Pudalov, PRL **87**, 096802 (2001)

Data on WL are quantitatively consistent with theory

Example:

$\Delta R = R(H_\perp = 0) - R(H_\perp = 125\text{Gs})$



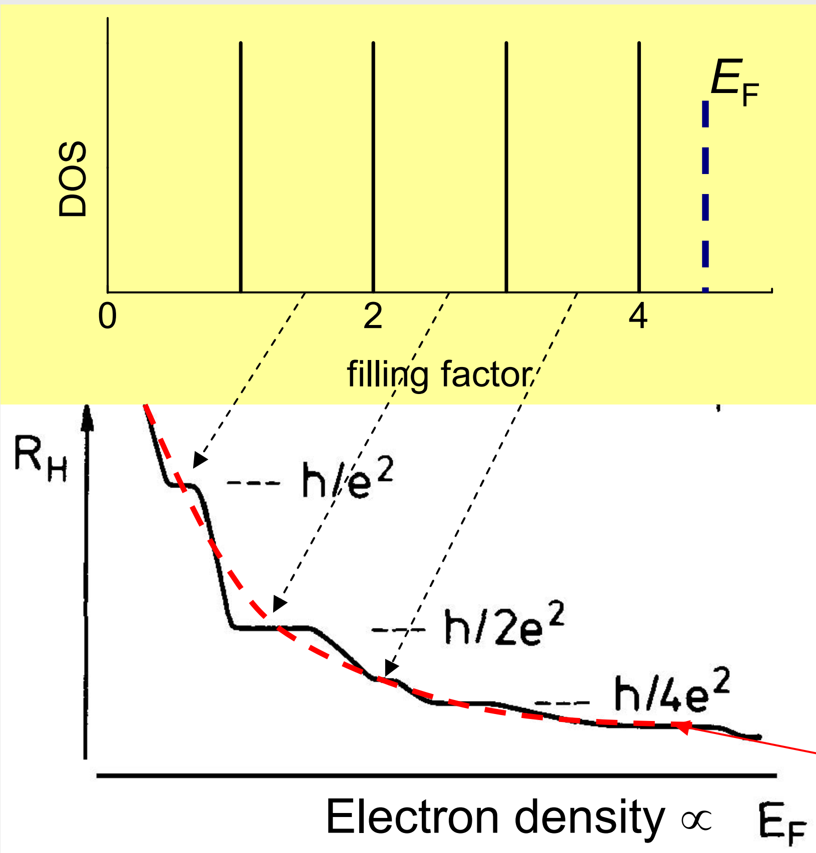
5. Extended states in quantizing magnetic fields.

Turn $B_\perp$ field on:

5.1. QHE: Quantum transport of a 2DE gas in strong magnetic fields

$$E_n = E_0 + \hbar\omega_c(n+1/2) \pm g\mu_B/2$$

$$\hbar\omega_c = \frac{e\hbar H}{m^*}$$



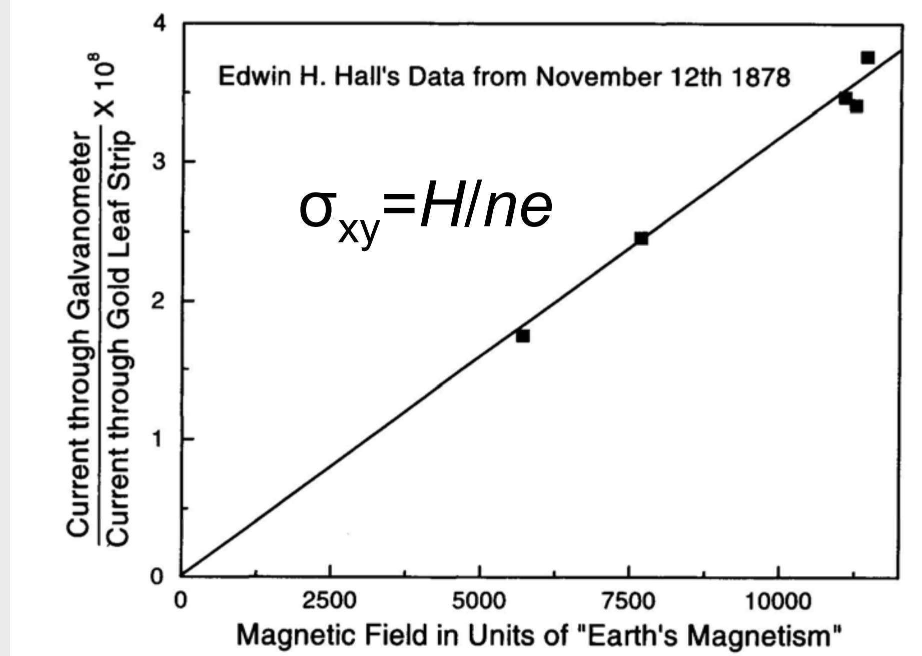


Figure 6. Edwin Hall's Hall data of 1878 as plotted from a table in his publication. The vertical axis is proportional to the Hall voltage, V_H of Fig. 5 and the horizontal axis is proportional to the magnetic field of Fig. 5. A linear relationship between V_H and B and hence between R_H and B is apparent. Since the days of Edwin Hall this strictly linear relationship has been confirmed by many, much more precise experiments.

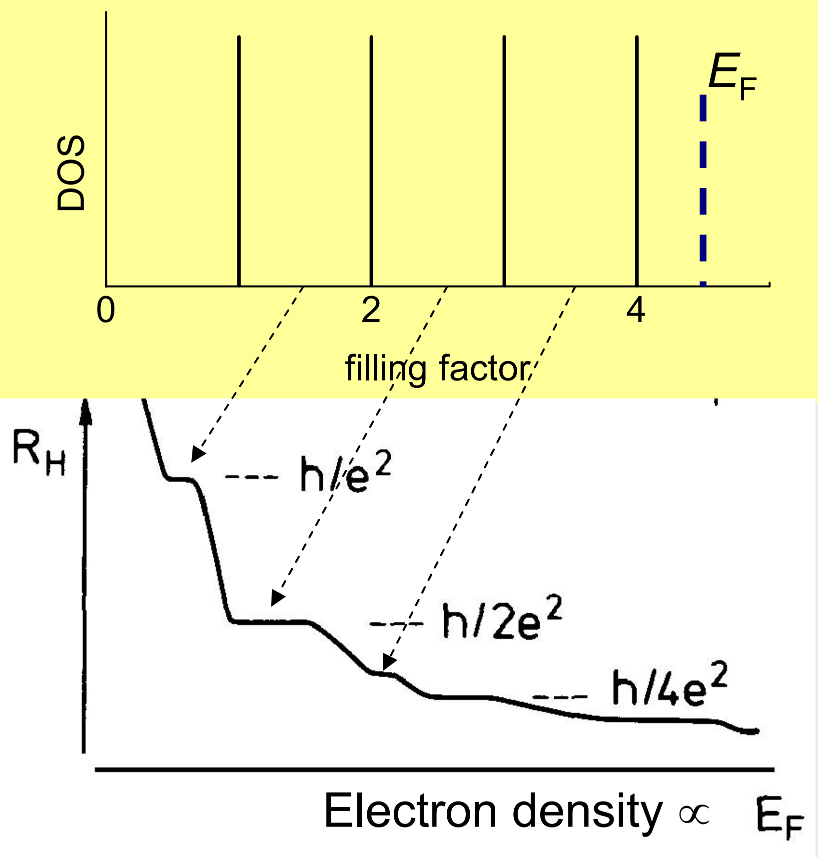
$\rho_{xy}=ne/\underline{H}$

5.1. QHE: Quantum transport of a 2DE gas in strong magnetic fields

$$E_n = E_0 + \hbar\omega_c(n+1/2) \pm g\mu_B/2$$

$$\hbar\omega_c = \frac{e\hbar H}{m^*}$$

Disorder causes broadening of the LL's



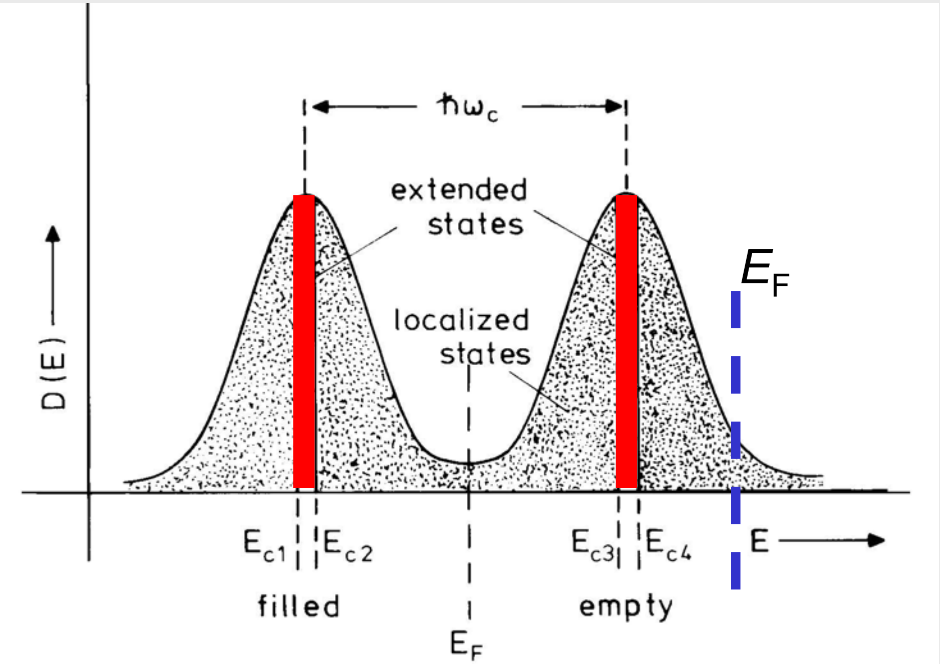
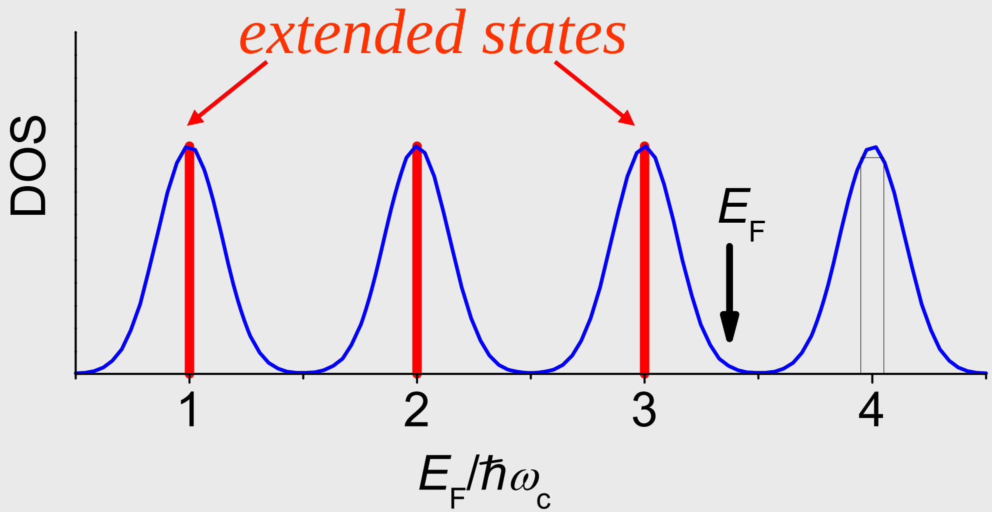


Fig. 6. Model for the broadened density of states of a 2 DEG in a strong magnetic field. Mobility edges close to the center of the Landau levels separate extended states from localized states.

5.2. Reentrant QHE-Insulator transitions in $\boldsymbol{B}_{\perp}$ fields:



$\omega_c\tau \to 0$ limit

Khmelnitskii (1984)

Laughlin (1984)

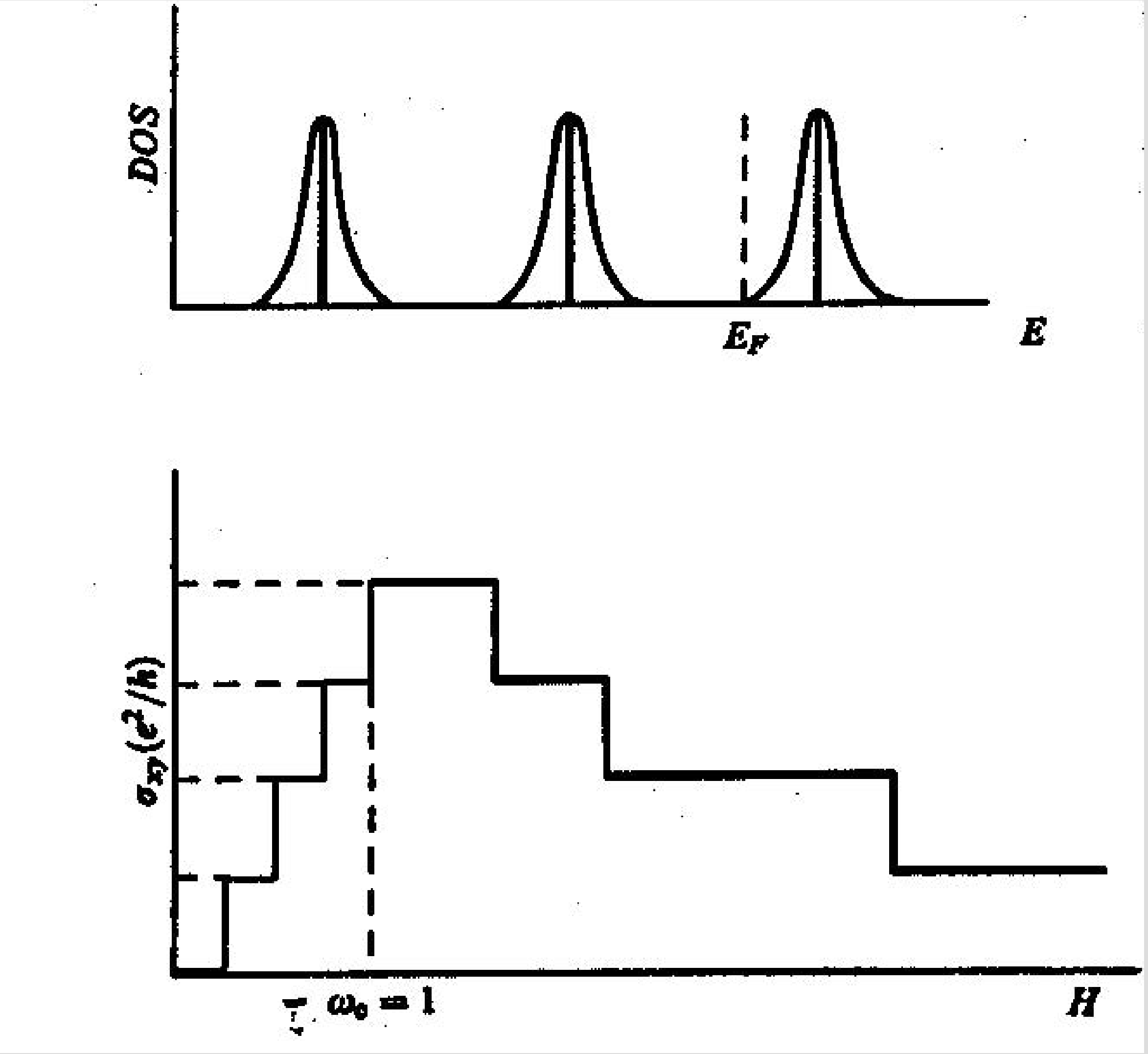
$\Longrightarrow$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c \left[\frac{1 + (\omega_c\tau)^2}{(\omega_c\tau)^2}\right]$$

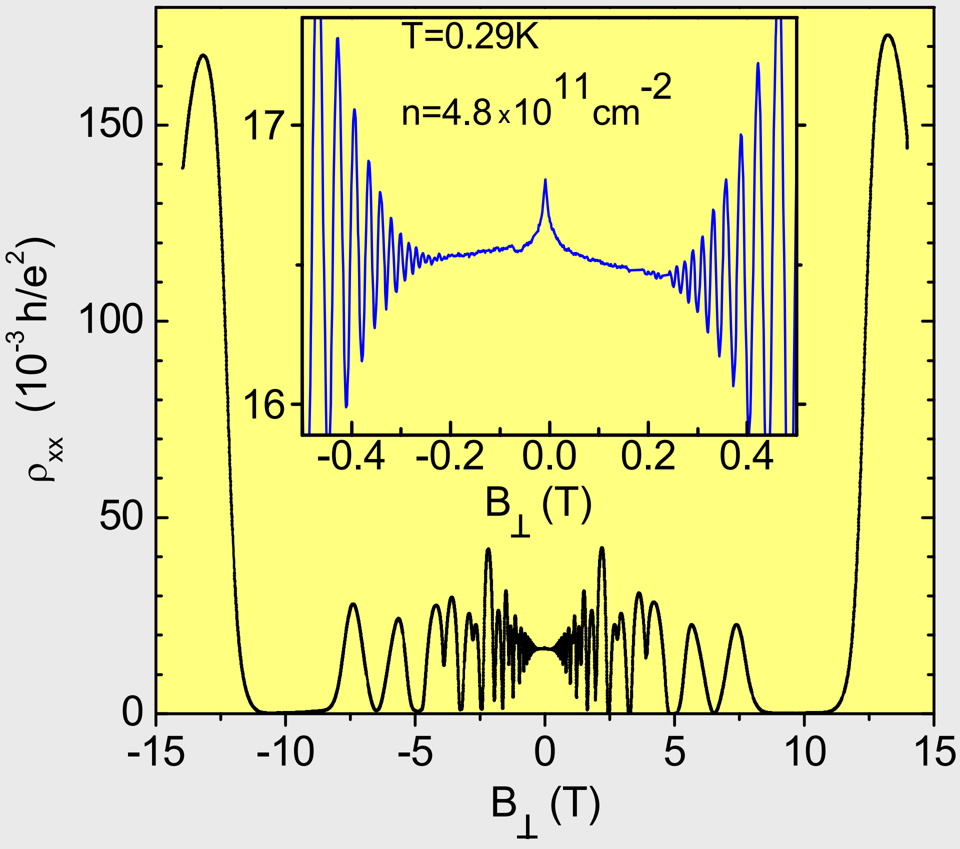
Theoretical single-particle picture:

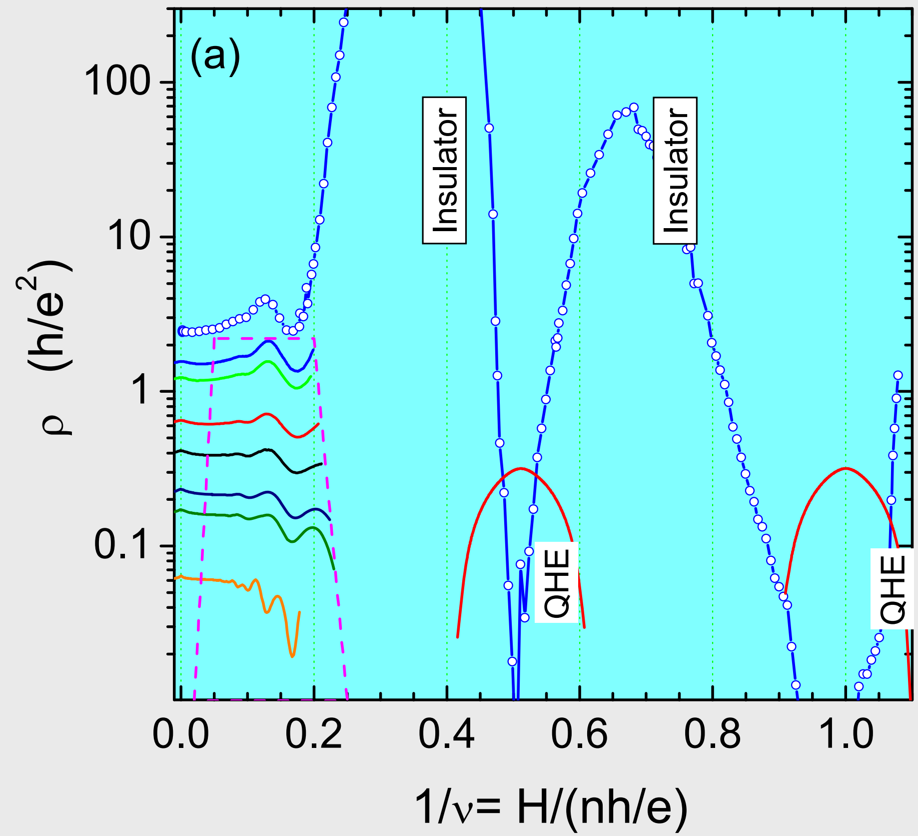
"Floating" of the extended states

Anticipated ρ_{xy} behavior for floating the extended states up



SdH oscillations ► reentrant QHE-Insulator transitions





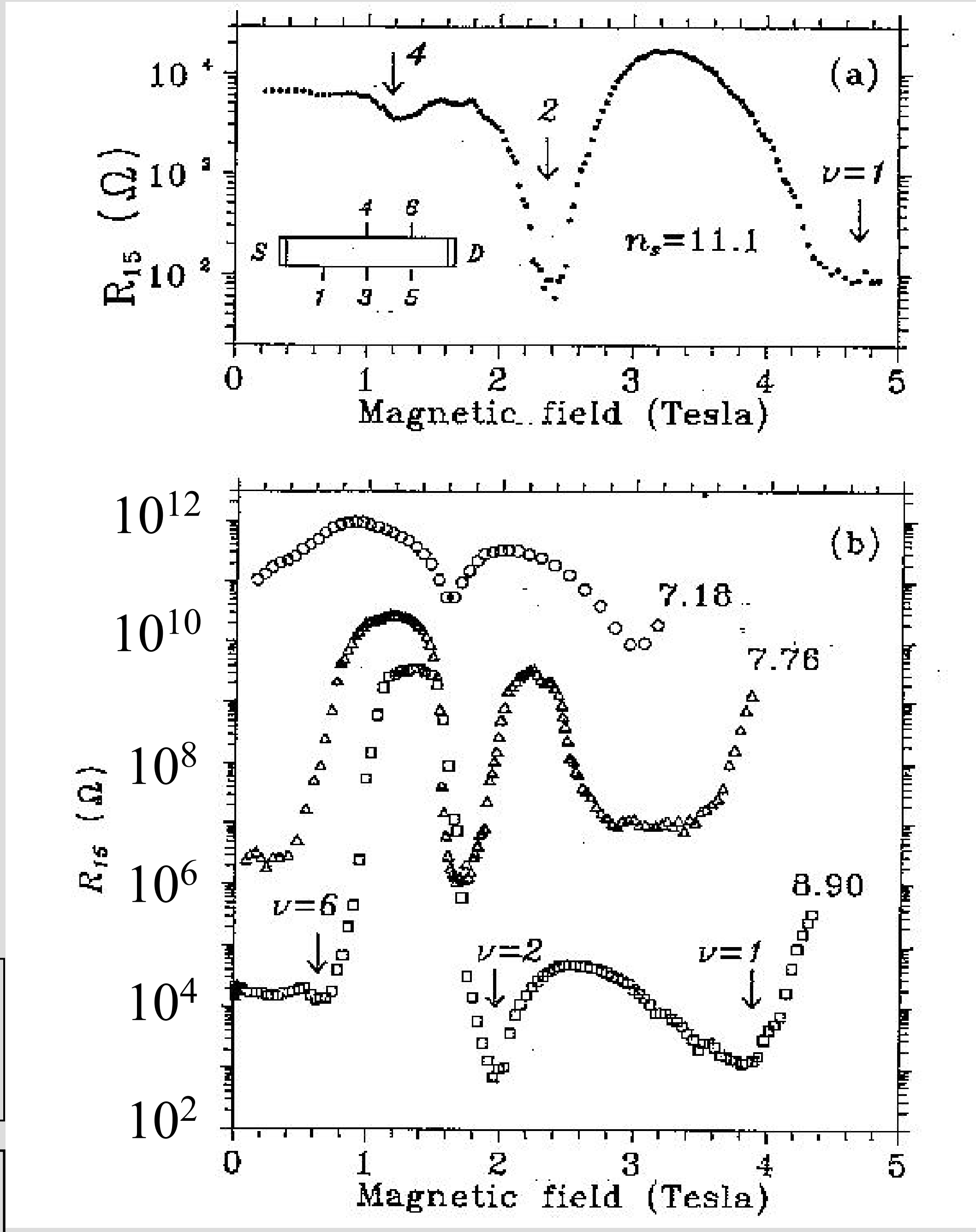
M.D'Iorio, V.Pudalov, S.Semenchinsky. *Phys. Lett.* A **150**, 422 (1990); *PRB* **46**, 15992 (1992)

Experiment

regular SdH effect

Density decreases by 20% only !

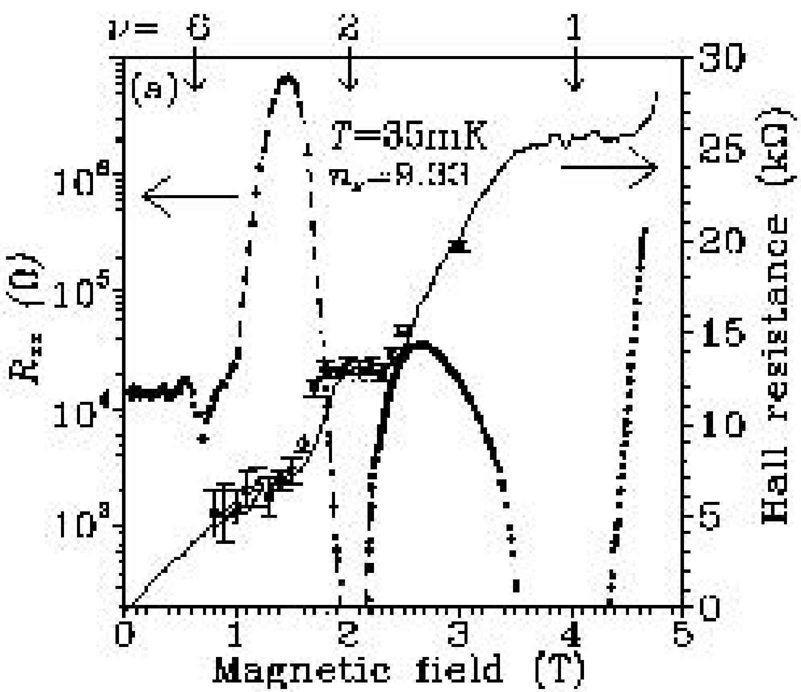
QHE-I transitions

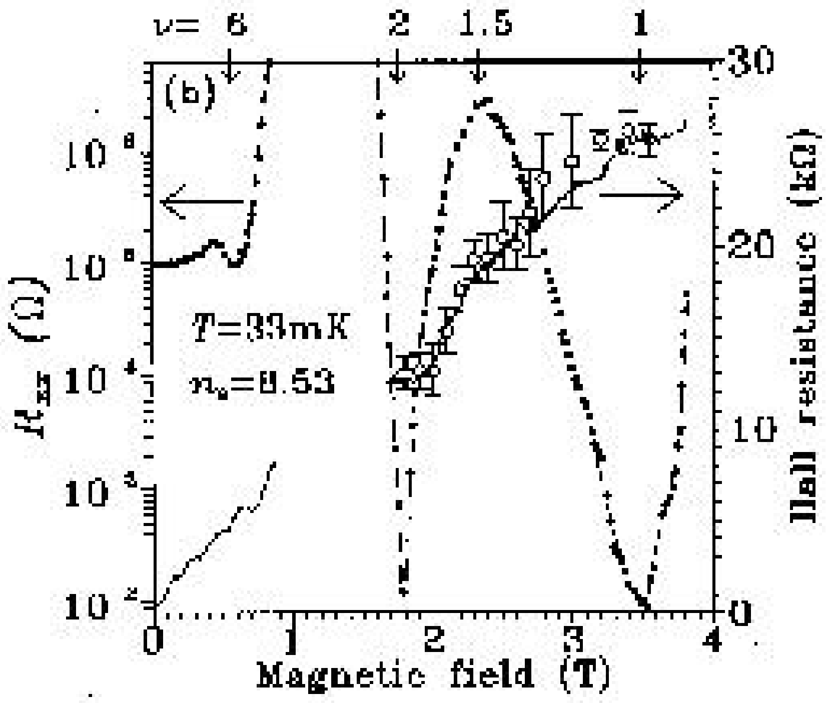


M.D'Iorio, V.Pudalov, S.G.Semenchinsky, *Phys. Lett.* A 150, 422 (1990); *PRB* **46**, 15992 (1992)

S.Kravchenko, J.Perenboom, V. Pudalov, *PRB* **44**, 13513 (1991)

Hall-insulator type behavior

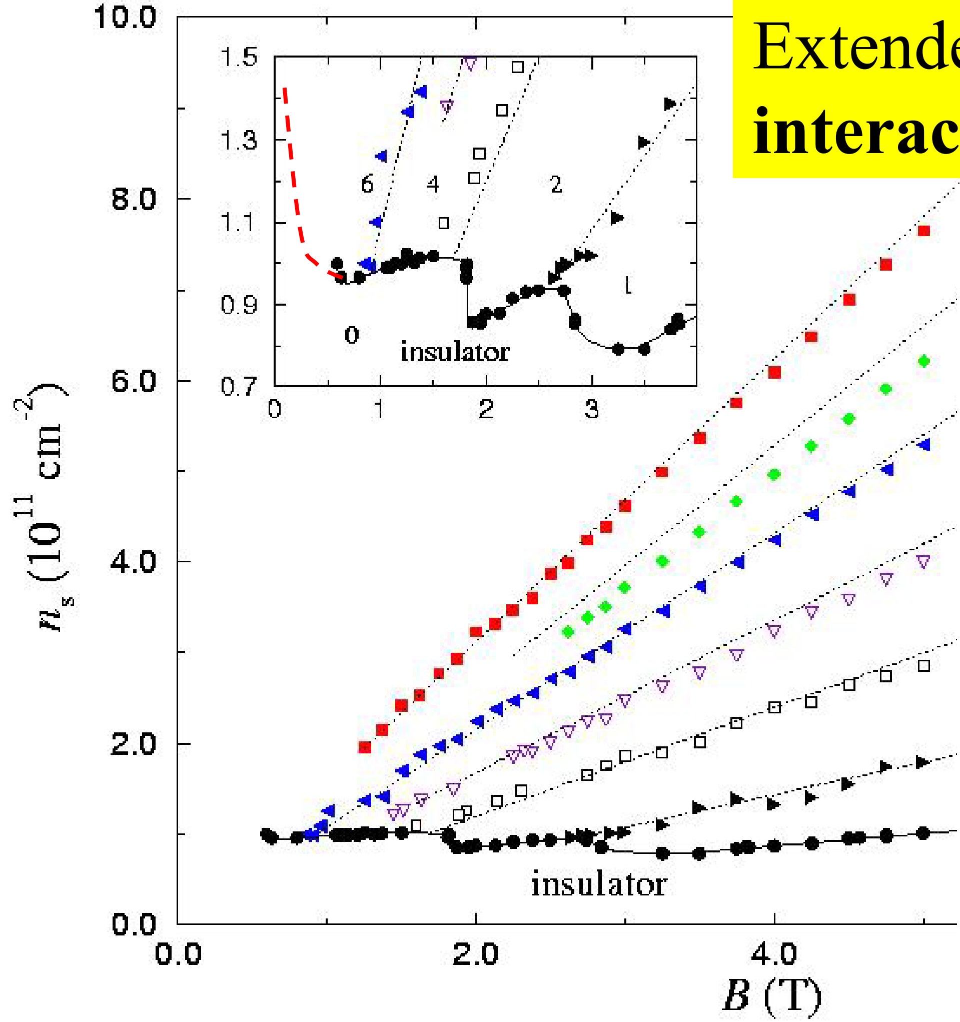


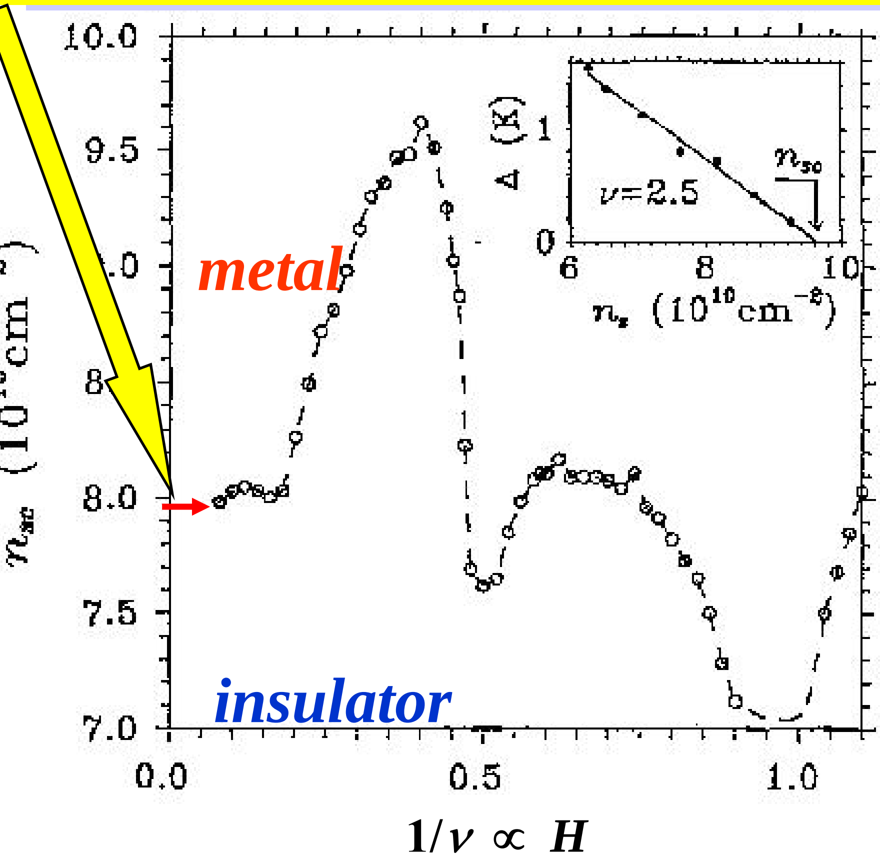


V.Pudalov, M.D’Iorio, J.Campbell, *JETP Lett.* **57**, 608 (1993)

Interacting 2D system: merging of the extended states ?

Extended states do not quit the 2D **interacting** system in the $\omega_c\tau \to 0$ limit



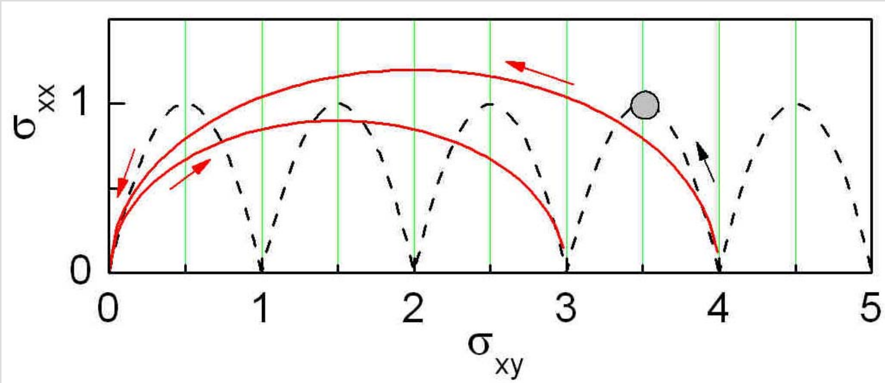


S.Kravchenko W.Mason, J.Furneaux, V.Pudalov, *PRL* **75**, 910 (1995)

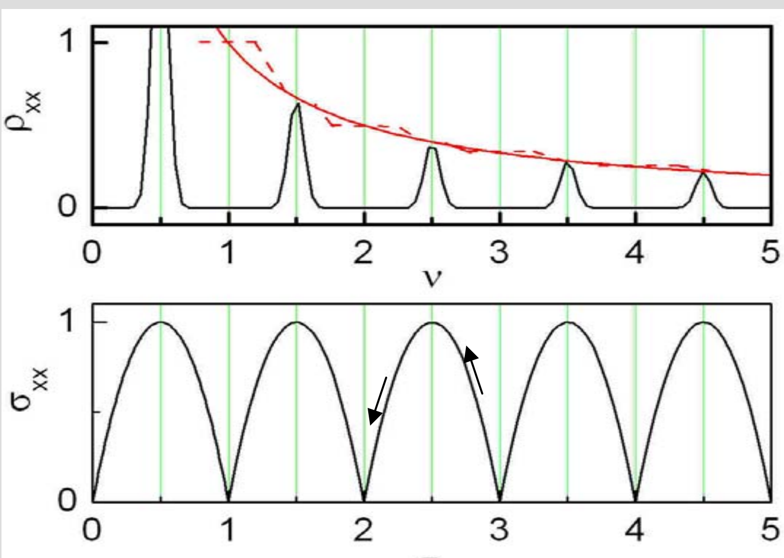
M.D'Iorio, V.Pudalov, S.Semenchinsky. *PRB* **46**, 15992 (1992)

How all this looks on the σ_{xx}-σ_{xy} phase diagram

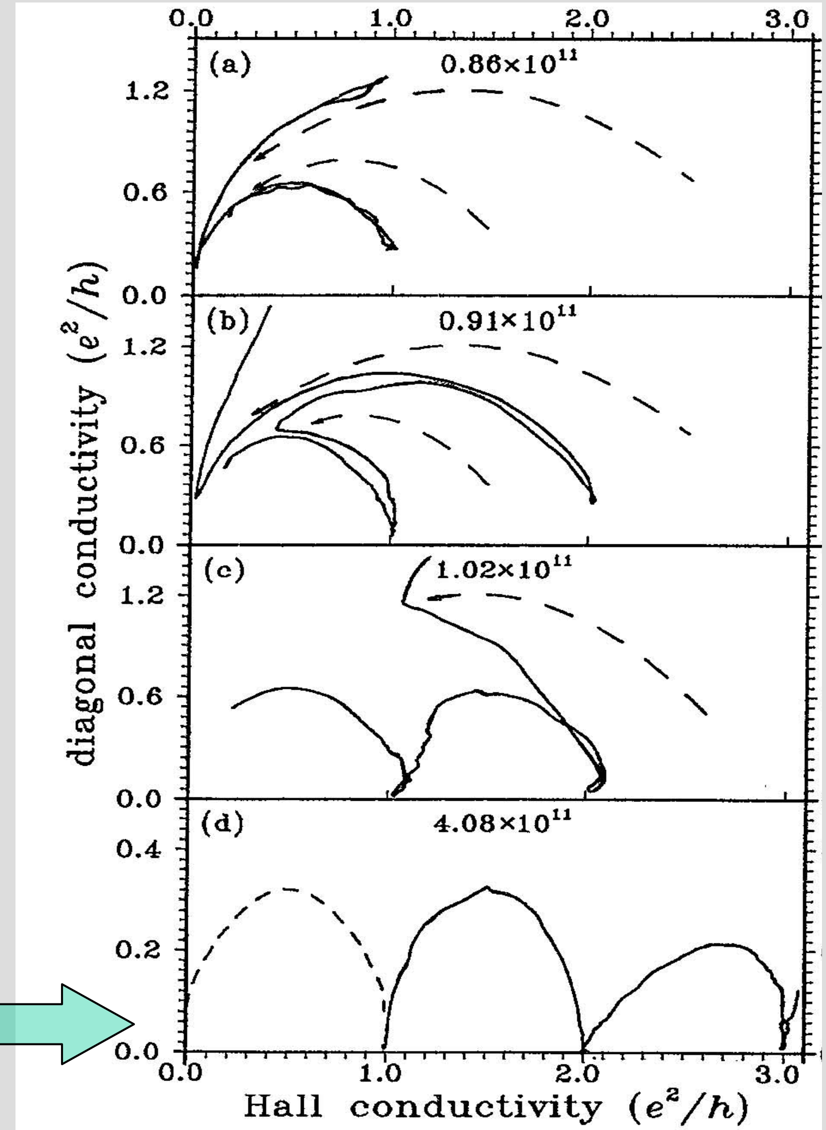
Reentrant QHE-I transitions



Ordinary QHE: plateau-plateau transitions



B **increases**



S.Kravchenko, J.Perenboom, V. Pudalov, *PRB* **44**, 13513 (1991)

Major unresolved questions:

- A true QPT (FL-WS) in a homogeneous 2D system at n_c ?
- "Non-floating" –

  is a true T=0 property or a finite T-effect?
  is an artefact, caused by misinterpretation of the trajectories of the extended states ?

or, alternatively,

- The 2D system becomes inhomogeneous in the vicinity of n_c (long-range disorder) ?
- A percolation-type M-I transition ?
- An intrinsic two-phase state and WS transition at $n = n_c$?
- An electron liquid - WS transition at $n = n_c$?35

The QHE-I reentrant transitions are natural within a picture of the electron liquid-solid phase transition

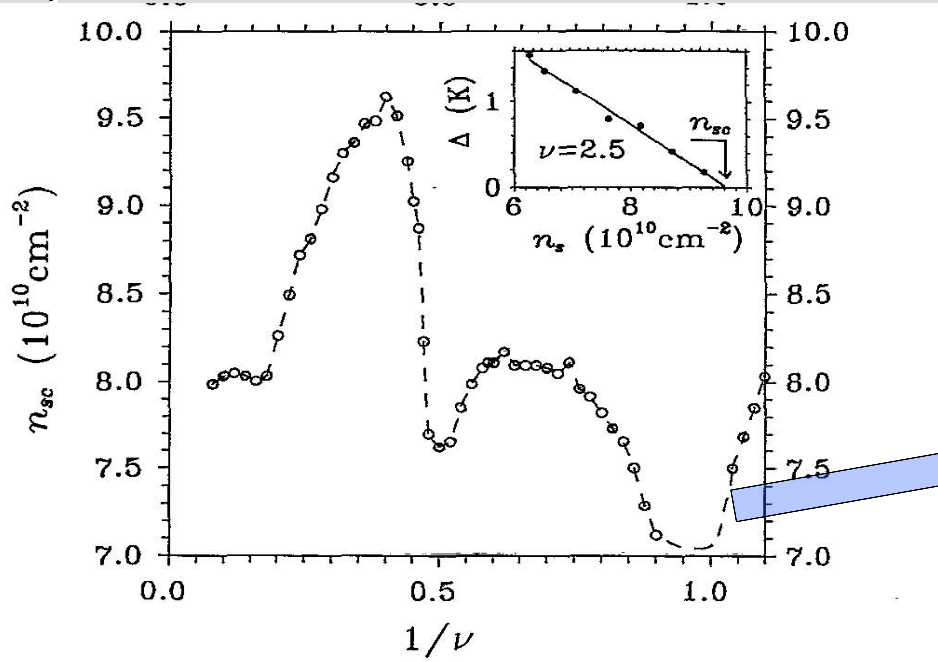
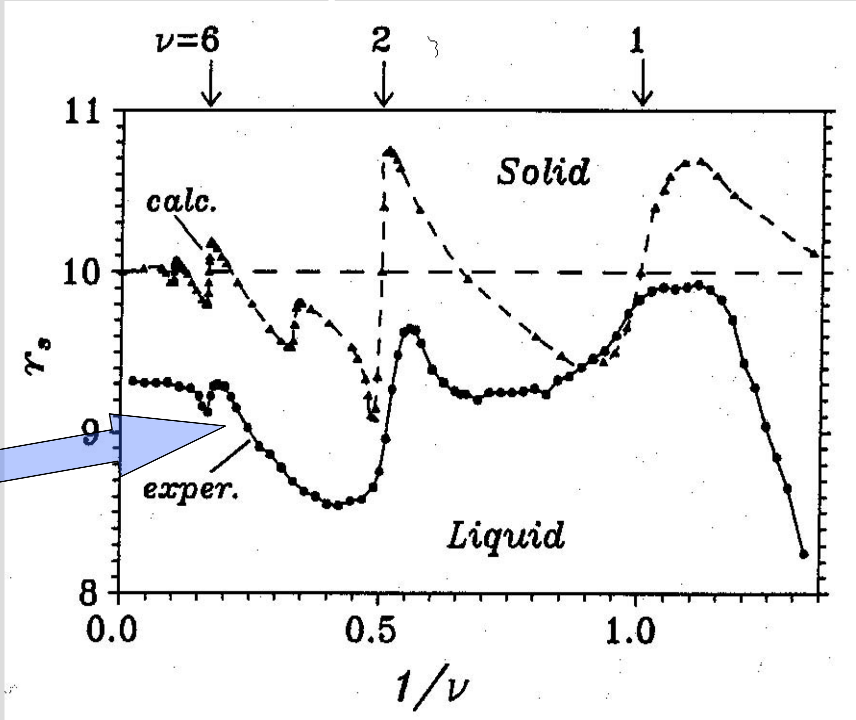


FIG. 5. Critical electron concentration n_{sc} as a function of the inverse filling factor $1/\nu$. The inset shows the definition of n_{sc} from the linear plot $\Delta(n_s)$ at fixed filling factor. Sample Si-5.



Dashed curve and triangles – estimates for quantum oscillations of the WC melting, relative the zero field value (dashed line). Dots – are the QHE-I border.

Alternative interpretation – quantum oscillations of the tunneling (hopping) within the two-phase model

Experimentally, R_{xx} minima are shifted from integer ν values to high fields (as if the density gets increased close to the insulating transition !).

This shift

• is consistent with the phase transition model, where R_{xx} minima correspond to the minima of μ.

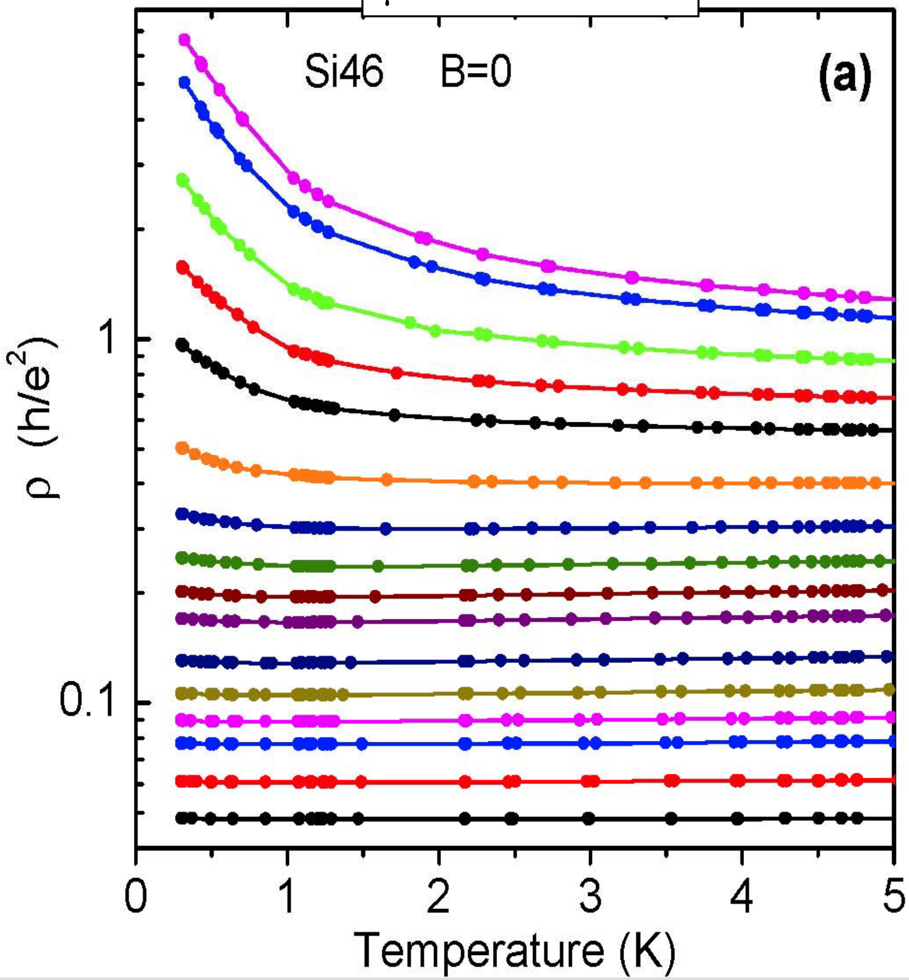
• is inconsistent with the two-phase model where R_{xx} minima correspond to the energy gaps.

For more detail, see V.P. in: “Physics of Quantum Solids of Electrons” (Chapter 4), S.-T. Chui ed., International press, NY-Singapore (1994), pp.124-178.

6. Metal-Insulator transition in 2D systems

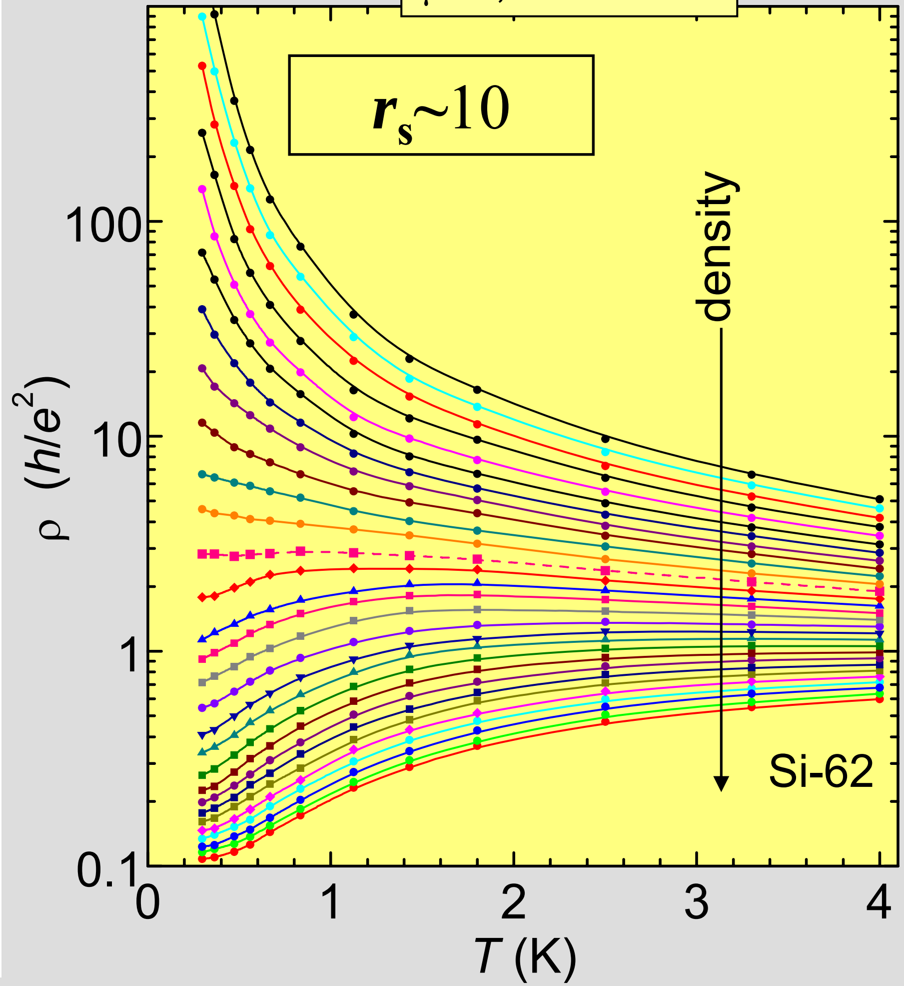
Low mobility

μ=0.15m^2/Vs



High mobility

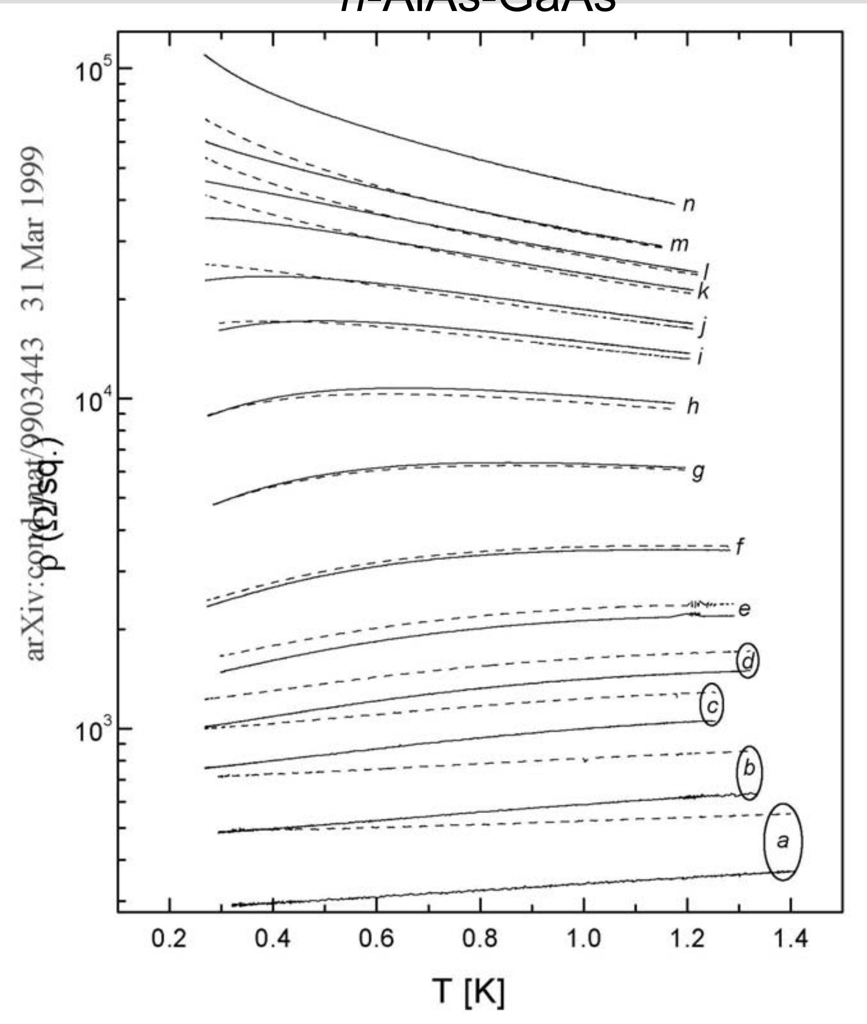
μ=4,5m^2/Vs



S.Kravchenko, G.Kravchrnko, J.Furneaux, VP, M.D'Iorio, *PRB* **50**, 8039 (1994)

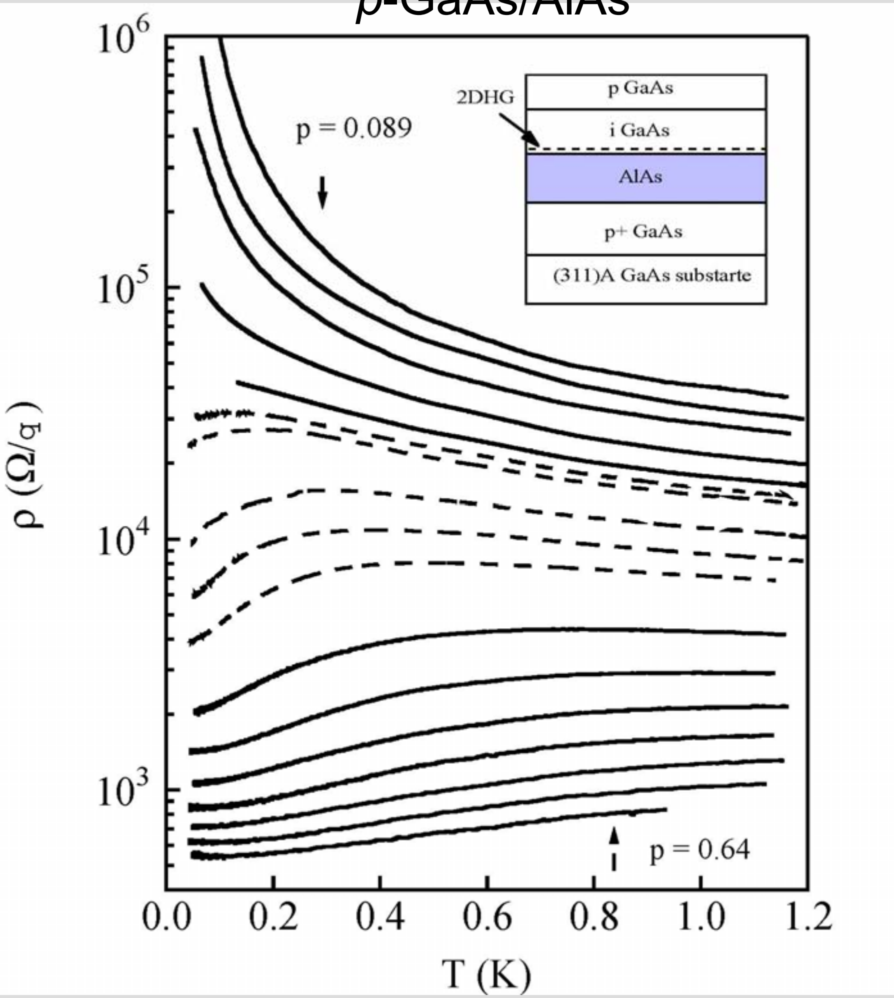
Similar ρ(*T*) behavior is observed in many other low density and high mobility 2D systems, such as *p*-GaAs, *n*-GaAs, *p*-Si/SiGe, *n*-Si/SiGe, *n*-SOI, p-AlAs/GaAs, etc.

n-AlAs-GaAs



Papadakis, Shayegan, *PRB* (1998)

p-GaAs/AlAs



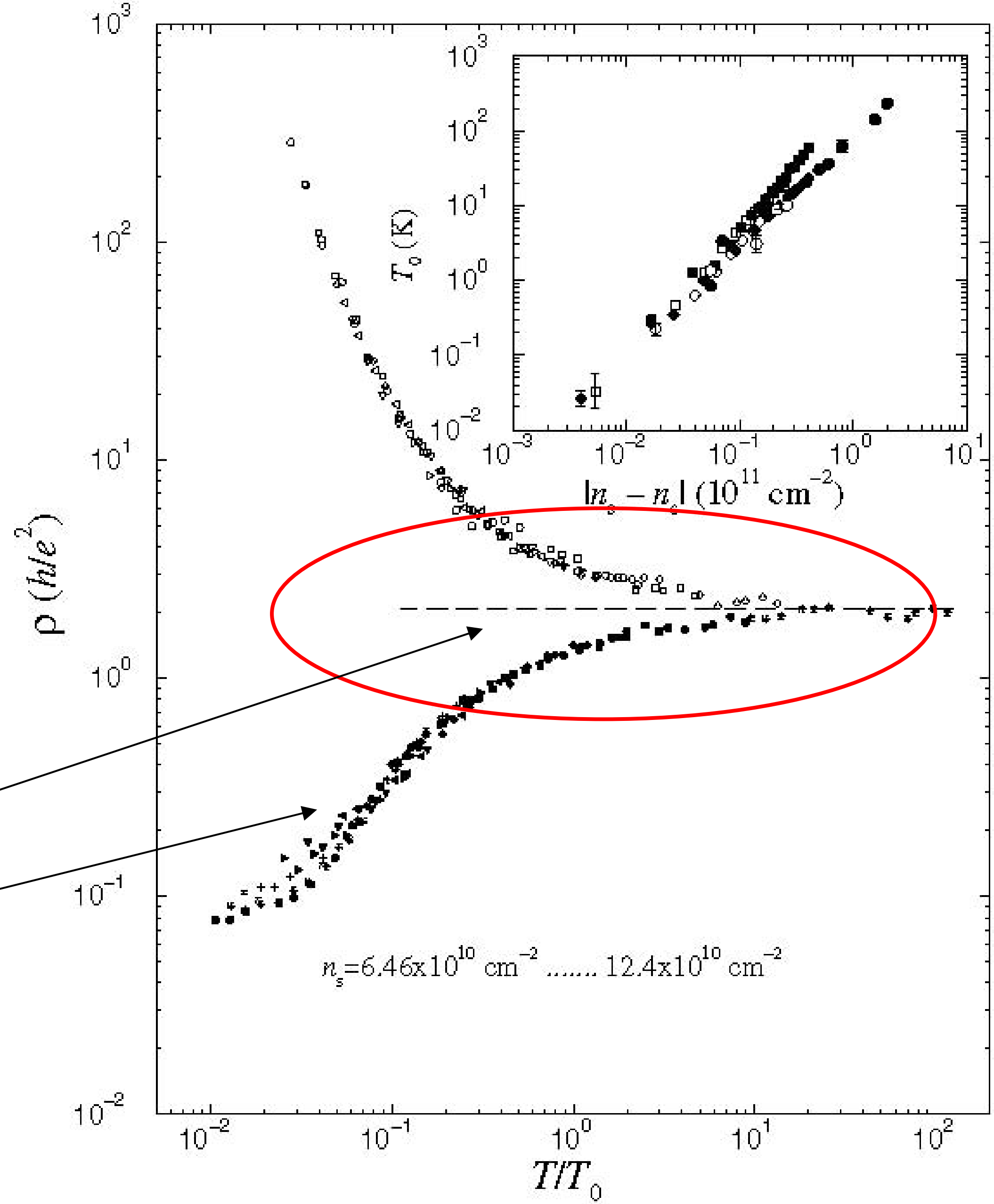
Y.Hanein et al. *PRL* (1998)

Attributes of the seeming critical phenomena (QPT)

•Mirror-reflection symmetry:
$\rho(\Delta n, T)/\rho_c = \rho_c/\rho(-\Delta n, T)$

•Scaling
$\rho/\rho_c = f\,[T/T_0(n)]$

•Critical behavior
$T_0 \propto |n-n_c|^{-z\nu}$



Symmetry: holds here

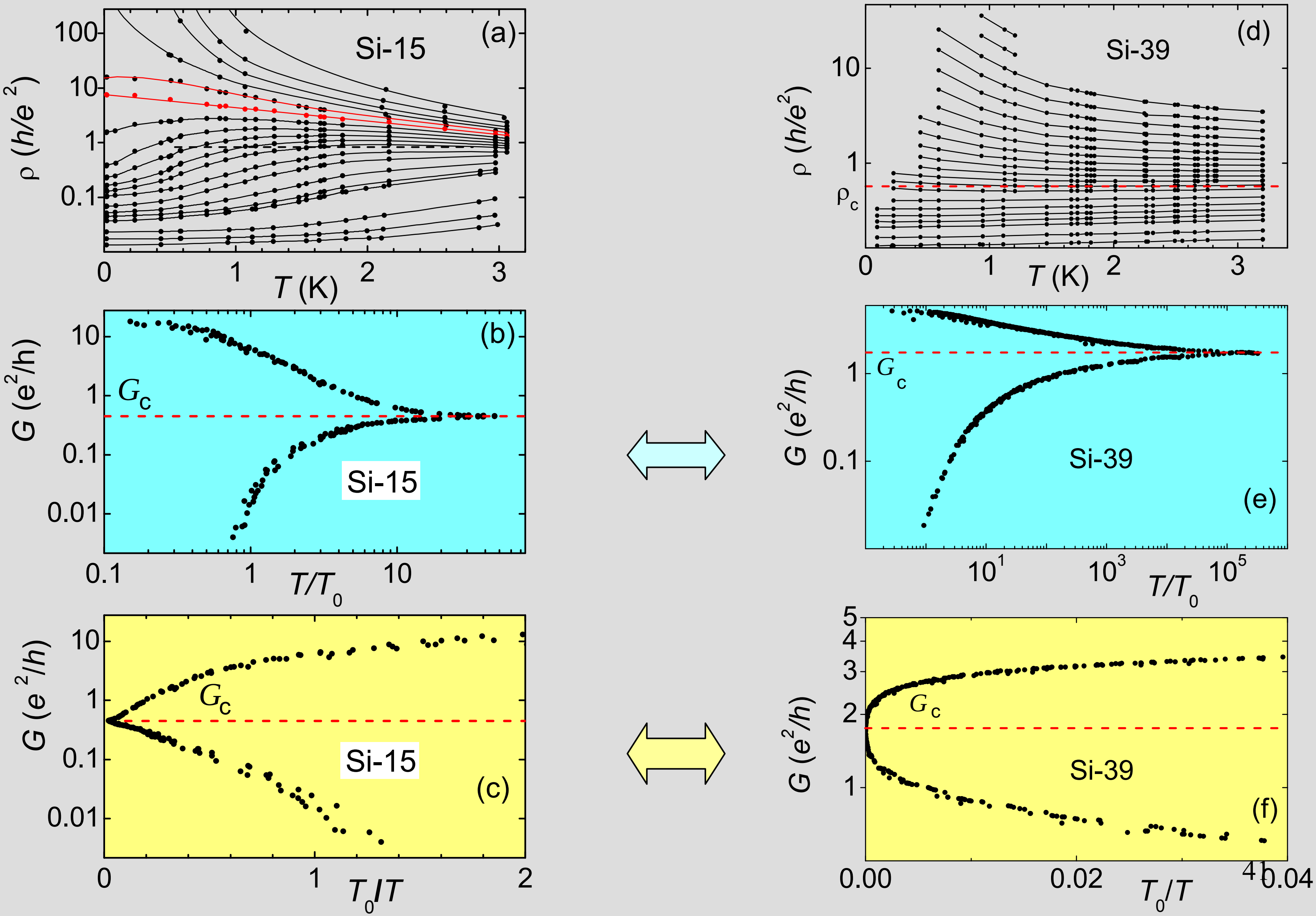
and fails outside

S.Kravchenko, W.Mason, G.Bowker, J.Furneaux, VP, M.D'Iorio, *PRB* (1995)

Misinterpretation of the data scaling

High μ sample

Low μ sample



There is no metallic state and M-I transition for non-interacting electrons in 2D

- ✓ ~~Spin-orbit interaction ?~~

  Not renormalized

- ✓ ~~Electron-phonon interaction ?~~

  Too low temperature and too weak *e-ph* coupling

Electron-electron interaction

Problems of the data (mis)interpretation (1995-00)

In analogy with the 1-parameter scaling:

If "MIT" is a QPT, it is expected:

- ρ_c to be universal,
- scaling persists to the lowest T
- "separatrix" is horizontal, $\rho_c \neq f(T)$
- z, ν are universal

Experimentally, *however,*

- $\rho_c = 0.5 \div 5$ is sample dependent,
- $z\nu = 0.9 \div 2$ is sample dependent,
- reflection symmetry fails at low T and at high T>2K

$\rho_{ins} = \rho_c \exp(T_0/T)^{p1}$ $(p_1 = 0.5 \div 1)$

$\rho_{met} = \rho_c \exp(-T_0/T)^{p2} + \rho_0$ $(p_2 = 0.5 \div 1)$

- separatrix is T-dependent

The failure of the OPST approach is not surprising: *interactions*

How to proceed in the 2-parameter problem ?

Which parameters should be universal ?

Definitions of the critical density, critical resistivity etc. ?

Non-Fermi liquid proposals (1998-2002)

- Fermion condensation (dispersion instability).
  Ходель, Шагинян, Зверев и др

- Marginal Fermi liquid state.
  Varma, Littlewood, Abrahams, et al.

- Two-component (Fermi liquid + Wigner solid) system.
  Spivak, Kivelson

- e-pairing.
  P.Phillips, Dalidovich.

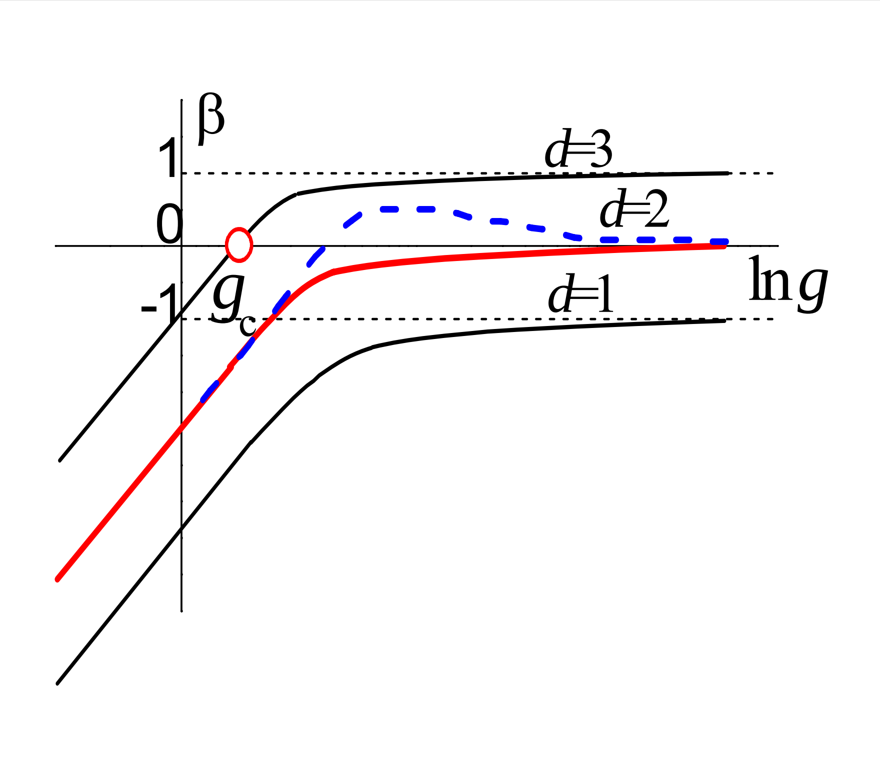
Semiclassical proposals

Percolation. Meir

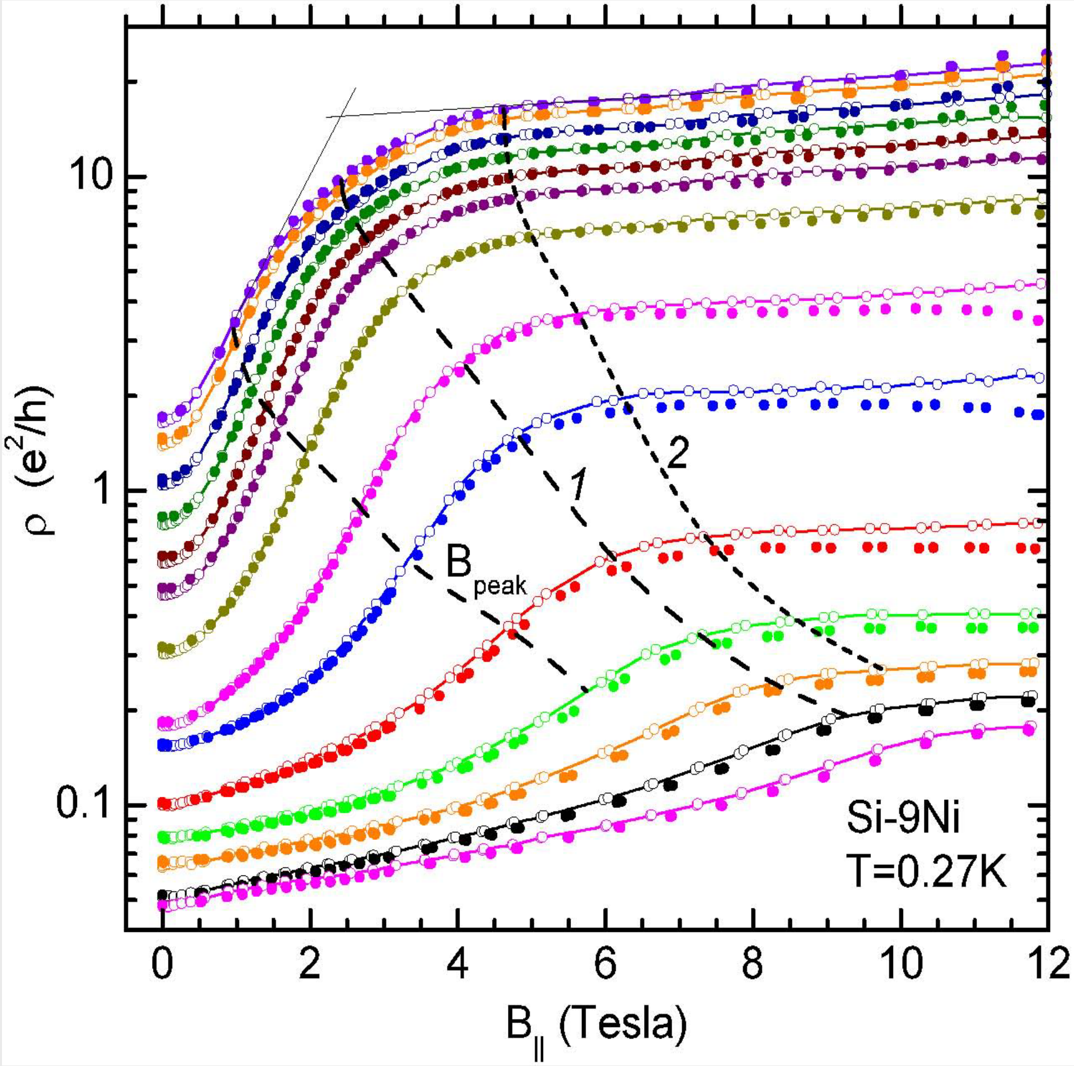
Interface traps. Altshuler, Maslov

Thermo-electric power. Cheremisin.

5. A few words about **S-O** interaction for *n*-SiMOS



S-O interaction for *n*-SiMOS



1) ***A priory***, it is expected to be weak, $\propto 1/E_g$

2) $\rho(\boldsymbol{B}_{||})$ is almost isotropic (***B*** in the 2D plane)

$\boldsymbol{j} \parallel \boldsymbol{B}$

$\boldsymbol{j} \perp \boldsymbol{B}$

VP, G.Brunthaler, A.Prinz, and G.Bauer. *PRL* **88**, 076401 (2002)

Magnetoresistance anisotropy, $\rho(j \| B) - \rho(j \perp B)$

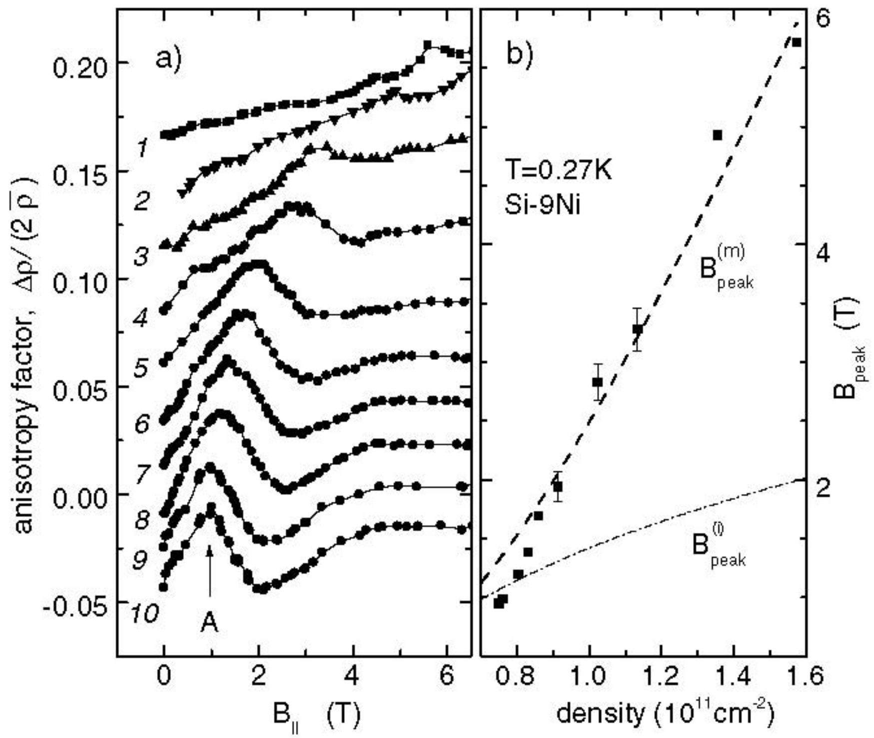


FIG. 3. (a) Anisotropy of the MR vs parallel magnetic field. For clarity, the curves are shifted vertically by 0.02 relative to each other. Arrow "A" shows the density-dependent peak. The densities in units of 10^{11} cm^{-2} are (from bottom to top) 0.748, 0.759, 0.78, 0.80, 0.835, 0.858, 0.913, 1.023, 1.133, and 1.353. (b) Magnetic field position of the peak A in the anisotropy of the MR vs electron density. Solid symbols are the measured data, the dashed curve represents the theoretical calculations of $B^{(m)}_{peak}$ [14], and the dash-dotted curve is for $B^{(i)}_{peak}$ [13]. Both

The peak

- is seen in the hopping regime only,
- shifts up in field with density,
- indicates contribution of the spin-orbit and Zeeman

$B_{peak} = \alpha k_F / g\mu B$ (Raikh 2000)

$B_{peak} = E_F / g\mu B$ (Chen, 2001)

V.Pudalov, G.Brunthaler, A.Prinz, G.Bauer, *PRL* **88**, 076401 (2002)

6. Back to *e-e* interaction in Si-MOS structures

Note1:

Within the framework of the e-e correlations, the role of the high mobility in the 2D MIT becomes transparent

The high mobility:

- Increases τ and, hence, the amplitude of interaction corrections ($\propto T\tau$);
- Translates down the critical density range (decreases the density of impurities n_i)
- Increases the magnitude of interaction effects ($\propto F_0^\sigma T\tau$).

❑ Some figures for the high-μ samples at the MIT

At the metal-insulator transition (Ioffe-Regel):

$$l_{tr} \sim \lambda_F \quad \Rightarrow \quad E_F \propto 1/\tau_{tr} \propto 1/\mu$$

$$E_F \sim n$$

The higher mobility, the **lower** density can be reached **remaining** in the **metallic** phase

MOSFETs: $n = 10^{11}\, cm^{-2}$

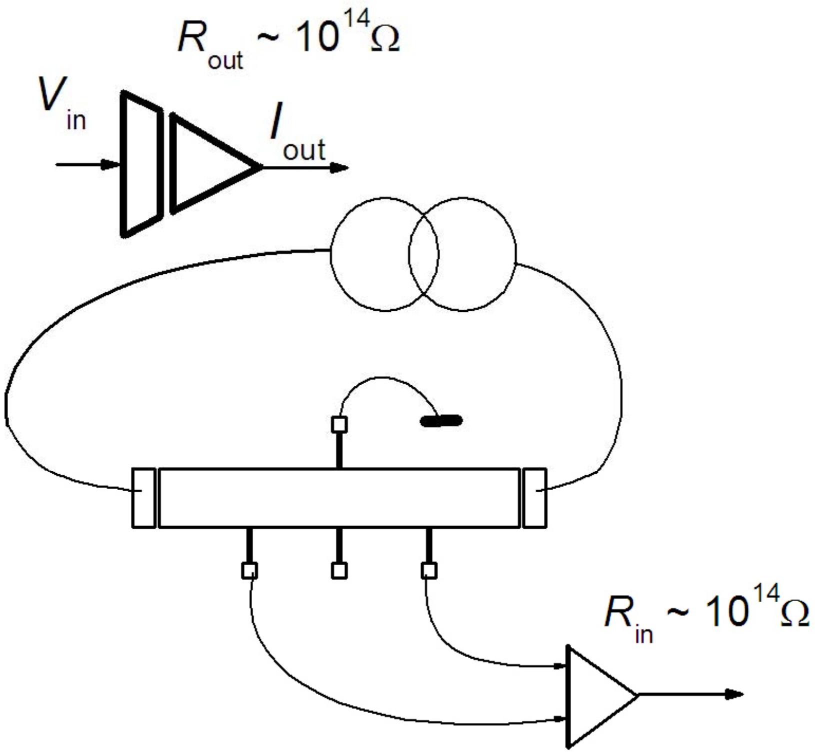
$$r_s \equiv \frac{E_{ee}}{E_F} \sim 10$$

$$E_F = \frac{\pi \hbar^2 n}{2m} \approx 6K$$

$$E_{ee} = \frac{e^2}{\varepsilon}(\pi n)^{1/2} \approx 70K$$

NB: 2 valleys !

7. Insulating State (low electron densities)



Set-up for transport measurements with almost insulating samples

Definitions:

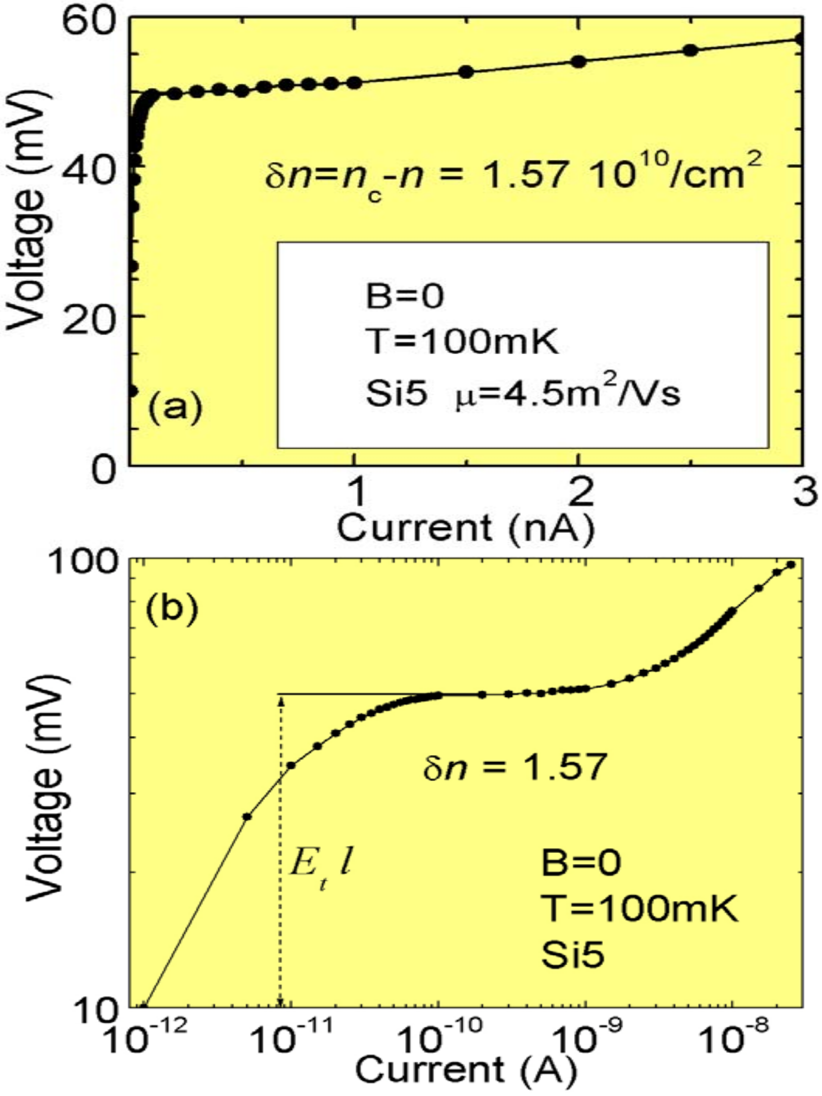
Insulator $\Rightarrow$ **Solid Phase**

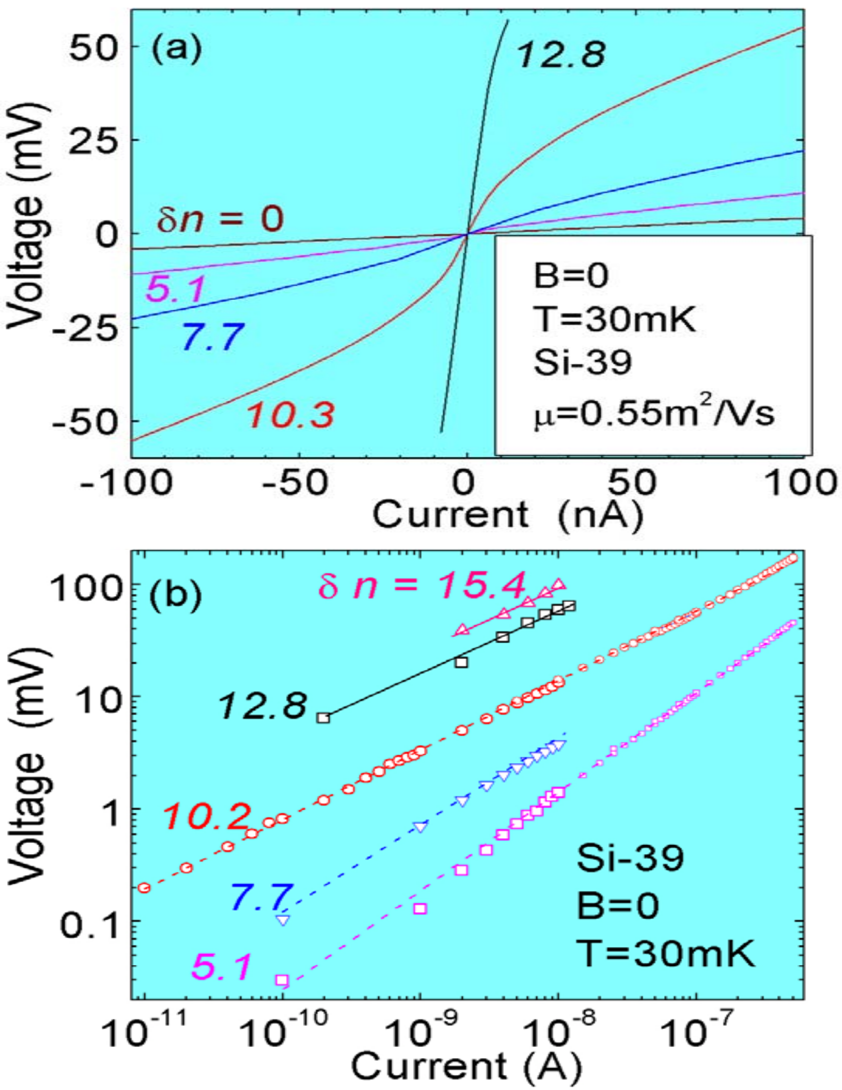
Metallic conduction $\Rightarrow$ **Liquid phase**

Caution: All measurements are done at $T \neq 0$!

Nonlinear conduction in Si-MOS samples at mK-*T*s:

high-mobility **low- mobility**

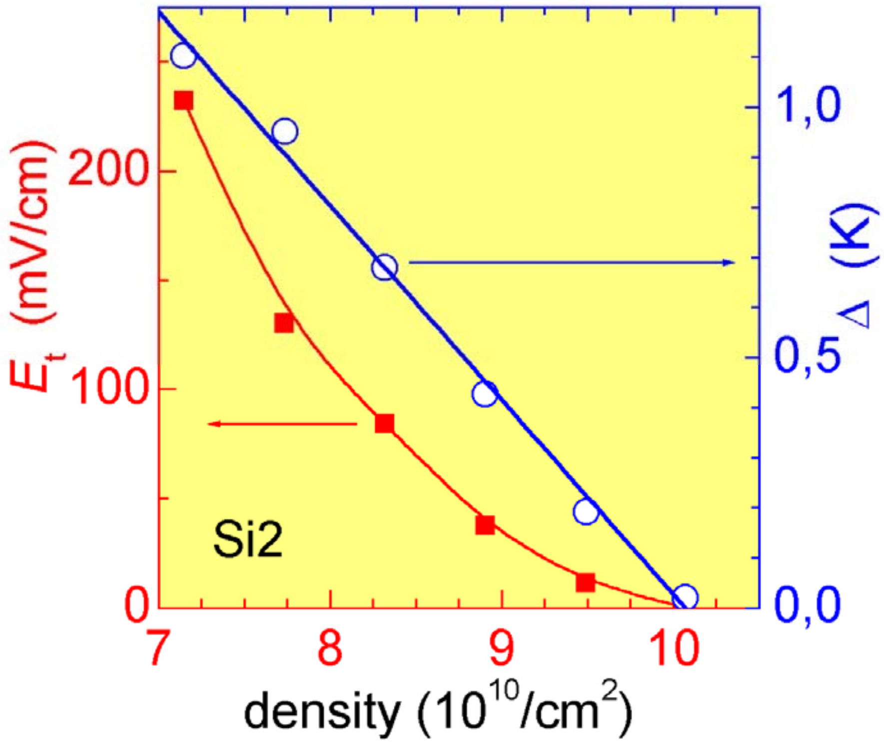


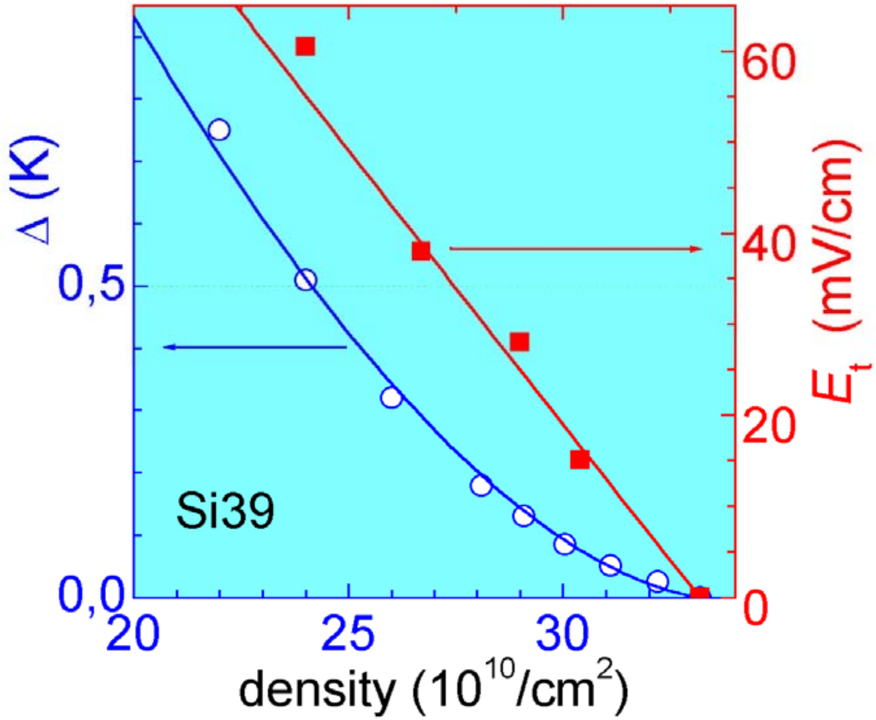


VP, *J. Physique IV* (2002)

Activation energy and threshold electric field

high mobility **low mobility**





Subthreshold conduction $\sigma \propto \exp(-\Delta/T)$

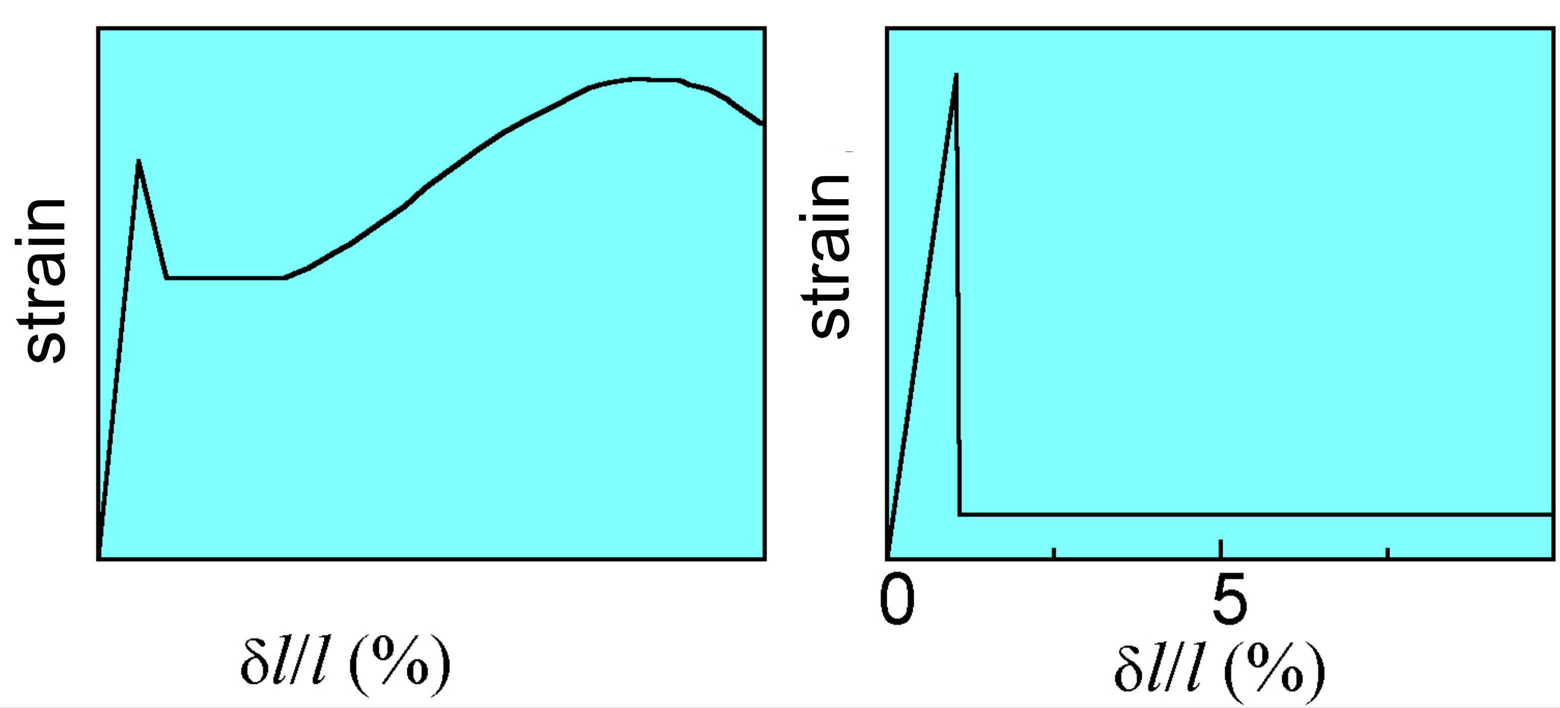
Length scale $\xi \sim \Delta/eE_t$

divergent at $n = n_c$ **non-divergent**

VP, M.D'Iorio, S.V.Kravchenko, and J.Campbell, *PRL* 70, 1866 (1993);
VP, *J. de Phys. IV*, 2002

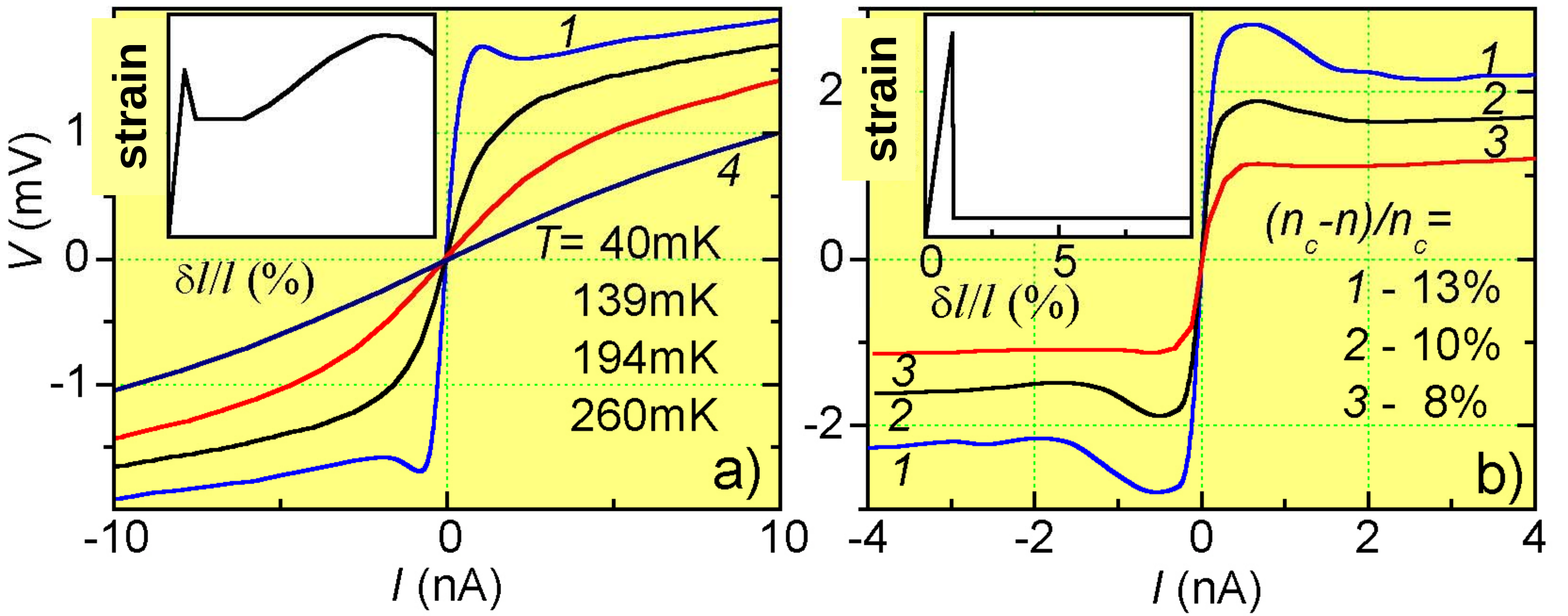
"Yield" drop effect – distinctive feature of crystals

(A.M. Cottrell, *Mechanical Properties of Matter*)



Physical cause: depinning of dislocations

Field drop effect: dislocations depinning ?



Stress	⇔	**Electric field**
Elongation	⇔	**Current**

VP, *J.Phys. IV (France)*, **12** (9), 331 (2002)

In a pure 2D system, quantum crystallization (WC) is predicted at r_s=37 Tanatar, Ceperley *PRB* (1989)

Exper: The collective transport sets at $r_s \approx 10$

Strong disorder pushes the M-I crossover to higher density, making *e-e* interaction irrelevant

Weak disorder stabilizes the pinned WC
(a short-range order)

Eguiluz, Maradudin, Elliott, *PRB* (1983).
Chui, Tanatar *PRL* (1995). Chui, Tanatar *PRB* (1997).
Thakur, Neilson, *PRB* (1996).

Valley multiplicity stabilizes WC

Summary *for insulator transport*

In (the vicinity) the 2D insulating state at r_s ~10

- Transport at $n<n_c$ ($n \sim n_c$) in the least disordered Si-MOS has a collective character (governed by interaction effects)
- Model of the pinned Wigner solid (or pinned CDW-state) fits qualitatively the existing data
- These effects, however, are observed at $r_s \sim 10$, (20) rather than 37
- There is a striking difference between **the disorder-** and **interaction**-governed insulating states

Comparison of parameters for 2DE liquids in *n*- (100)SiMOS, *n*-GaAs, *p*-GaAs, and *n*-LHe^4

Parameter	*n*-(100) Si-MOS	*n*- GaAs	*p*- GaAs	electrons above L-He^4
density, L-phase (cm^{-2})	7×10^{10}	2×10^{10}	9×10^{9}	10^{8}
band mass, m^*/m_e	0.2	0.068	0.37	1
Dielectric constant	7.7	13	13	1
Effective Bohr radius	20Å	110 Å	20 Å	0.5
Wigner-Zeitz radius	210Å	390Å	590 Å	6000 Å
interaction energy	70K	40K	60K	20K
Fermi energy (bare)	5K	4K	0.35K	0.5K
Zero-point energy	6K	18K	0.4K	0.0014K
Classical melting	0.85K	0.5K	0.3K	0.021K
Quantum melting (cm^{-2})	7×10^{9}	3×10^{8}	9×10^{9}	5×10^{12}

8. *e-e* interaction effects in the 2D metallic state (at high densities)

- Oscillations of the Fermi energy
- Enhancement of the dH-vA magnitude
- Negative compressibility
- Quantum oscillations of the Landau level broadening

8.1. Quantum oscillations of the Fermi energy: $E_F(n)$

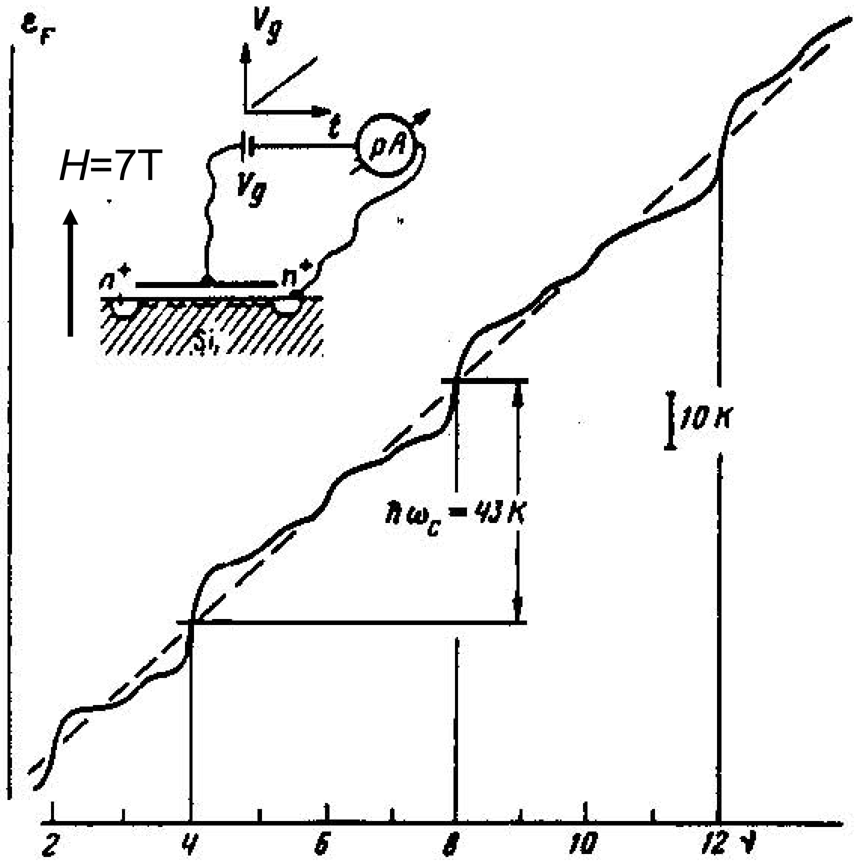


FIG. 1. Fermi energy versus the electron density (in units of $n_H = 1.7 \times 10^{11}$ cm^{-2}) at $H = 70$ kOe and $T = 1.3$ K. The inset shows the measurement layout.

678 JETP Lett., Vol. 44, No. 11, 10 December 1986 V. M. Pudalov and S. G. Semenchinskiĭ 678

8.2. Enhancement of the quantum oscillations of E_F

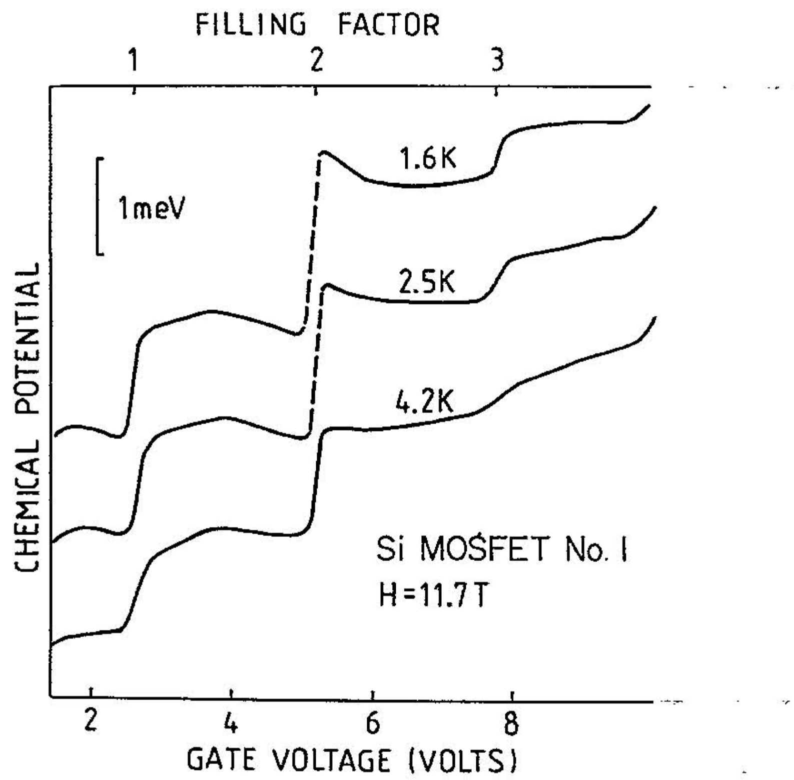


FIG. 1. The chemical potential of Si MOSFET no. 1 as a function of $V_g \propto \nu$ at three temperatures. The curves are shifted vertically for clarity.

Note:

- $d\mu/dn$ becomes negative !
- the jump magnitude $\Delta\mu$ is enhanced ($\times 2$) .

S.V.Kravchenko, D.A.Rinberg, S.G.Semenchinsky, V.M.Pudalov, Phys. Rev. B **42**, 3741 (1990).

8.3. Negative compressibility and chemical potential

$\mu(H)$ experimental

$\mu(H)$ calculated for the non-interacting 2D gas

$(dn/d\mu)^{-1}$ experimental

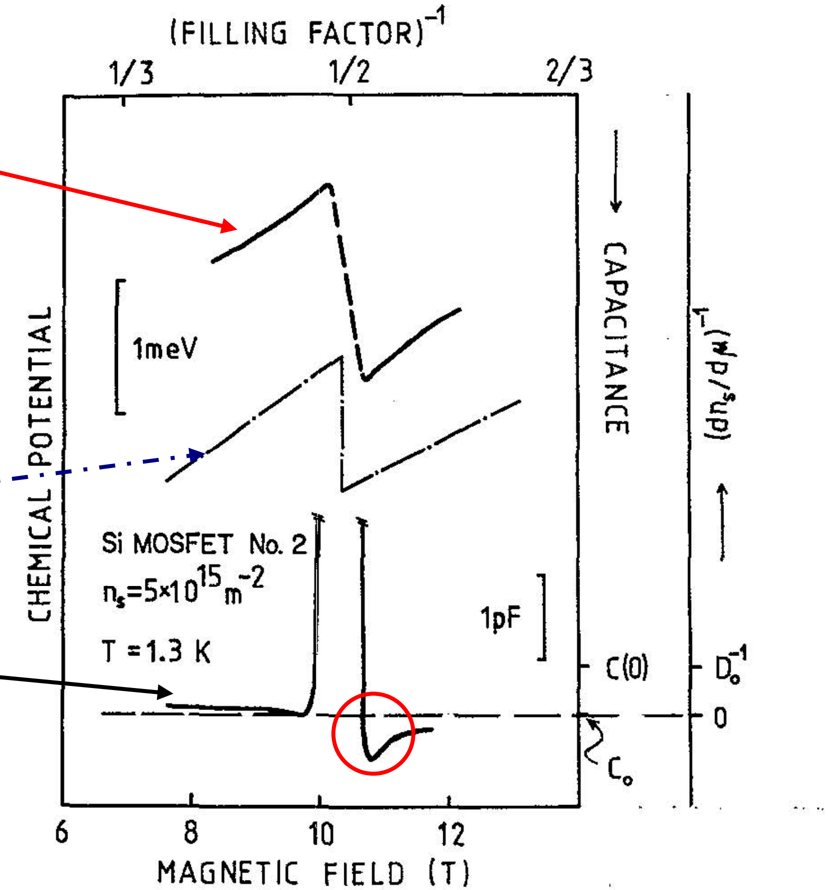


FIG. 2. The chemical potential (upper curve) and the magnetocapacitance (lower curve) as functions of $H \propto \nu^{-1}$. The dotted-dashed curve shows the calculated $\mu(H)$ dependence for a noninteracting gas at $T=0$ K and in the absence of disorder. The horizontal dashed line corresponds to the zero value of $(\partial n_s/\partial\mu)^{-1}$; C_0 is the capacitance at $\partial n_s/\partial\mu \rightarrow \infty$.

S.V.Kravchenko, D.A.Rinberg, S.G.Semenchinsky, V.M.Pudalov, Phys. Rev. B **42**, 3741 (1990).

Negative compressibility ("negative DOS") develops next to every integer fillings factors

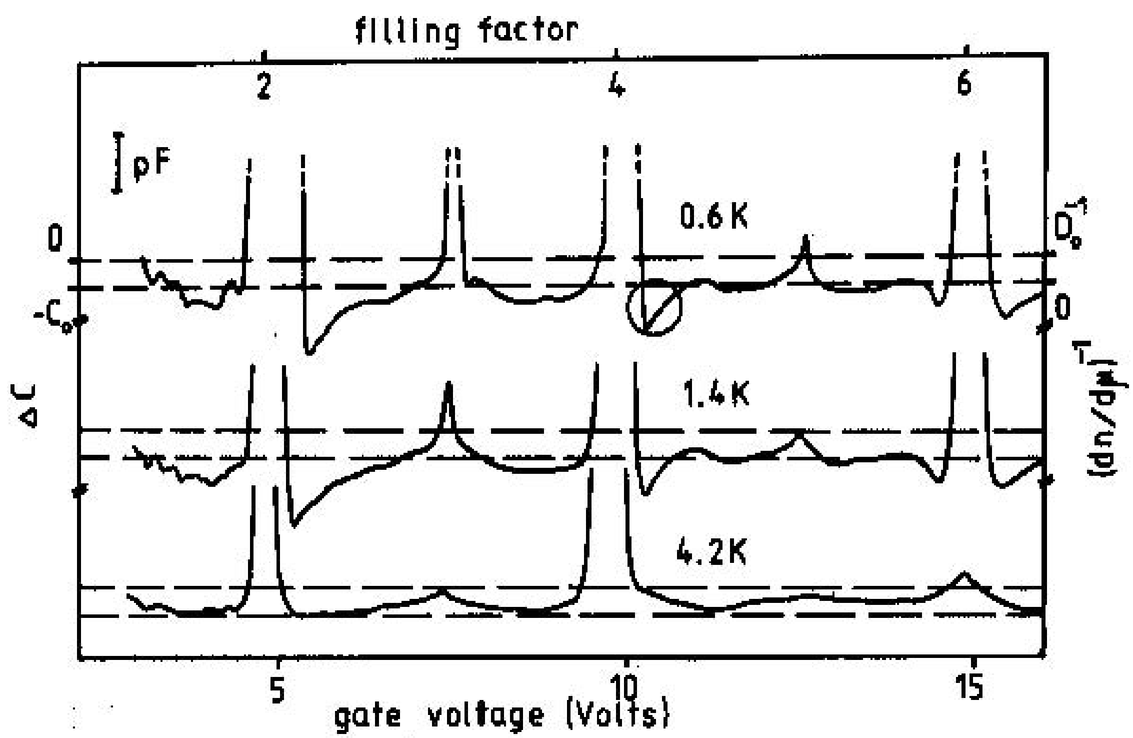


Fig. 1. ΔC and $(\partial n/\partial\mu)^{-1}$ dependences on $V_g \propto \nu$ for Si 1-33 at three temperatures and $H = 11.7$ T; $U_\sim = 70$ mV.

$$C^{-1} = C_0^{-1} + (e^2 S \partial n / \partial\mu)^{-1}$$

where $C_0 = C$ for $d\mu/dn=0$.

It can be easily found:

$$C_0^{-1} = (C|_{H=0})^{-1} - (e^2 S D_0)^{-1}$$

Experimental finding:
S.V.Kravchenko, V.M.Pudalov, S.G.Semenchinsky, Phys. Lett. A **141**, 71 (1989).

Theoretical prediction: A.L.Efros, Sol.St. Comm. **65,** 1281 (1988)

8.4. Quantum oscillations of broadening of the Landau levels

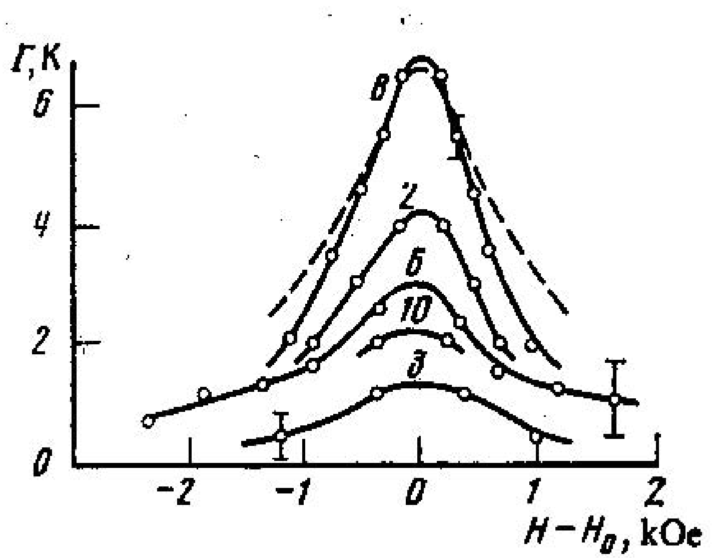


FIG. 6. Level width Γ versus the magnetic field for different ν (the numbers labeling the curves); $H_0 \simeq 80$ kOe. $T = 1.34$ K (for $\nu = 3$, $T = 0.45$ K). The dashed curve corresponds to the result of Ref. 16 for $\nu = 4$.

V.M.Pudalov, S.G.Semenchinskii, V.S.Edel'man, ZhETF, **89**, 1870 (1985). [JETP **62**, 1079(1985)].

This effect explains the mysterious "background DOS" observed in many experiments:

K. von Klitzing 337

Nobel Lecture

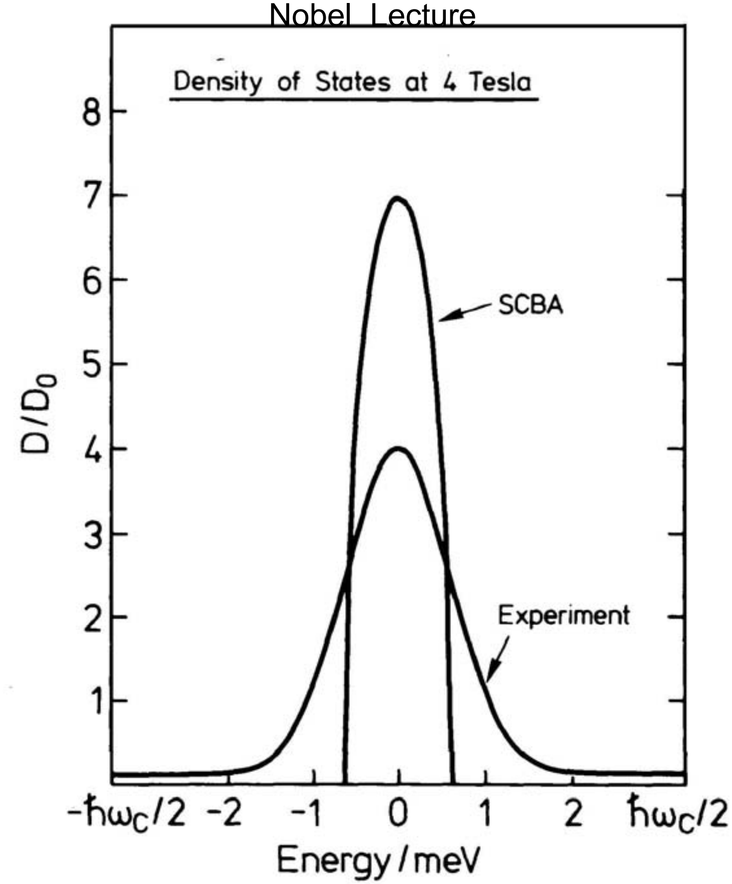


Fig. 20. Experimentally deduced density of states of a GaAs heterostructure at B = 4T compared with the calculated result based on the self-consistent Born approximation (SCBA).

Thank you for attention!

Next lecture:

- Quantitative studies of the e-e interaction
- How consistent are the data on χ^*, m* and g* ?
- Does χ^* diverge? Potential magnetic transition
- Quantum interaction corrections ⇔ experiment
- Spin polarized liquid
- Character of the e-e interaction
- Renorm group data analysis in the critical regime
- Phase diagram of the 2D interacting liquid

Физический институт имени П.Н.Лебедева

ФИАН

Electron-Electron Interaction Effects and Related Phenomena in Low-Dimensional Electron Systems
Lecture 2

Vladimir Pudalov

P.N.Lebedev Physical Institute
Russian Ac. of Sciences

ФОНД ДМИТРИЯ ЗИМИНА
ДИНАСТИЯ

Летняя школа 20-30 августа 2007г.

9. Quantitative study of the *e-e* interaction in 2D

$$\frac{g^*}{g_b} \equiv \frac{\chi^* m_b}{\chi_b m^*} = \frac{1}{(1+F_0^a)}, \quad (1)$$

$$\frac{m^*}{m_b} = 1 + \frac{1}{2}F_1^s, \quad (2)$$

$$\frac{\kappa^*}{\kappa} = \frac{m^*}{m_b}\frac{1}{(1+F_0^s)} \quad (3)$$

$$\frac{\chi^*}{\chi_b} = \frac{m^*}{m_b}\frac{1}{(1+F_0^a)}, \quad (4)$$

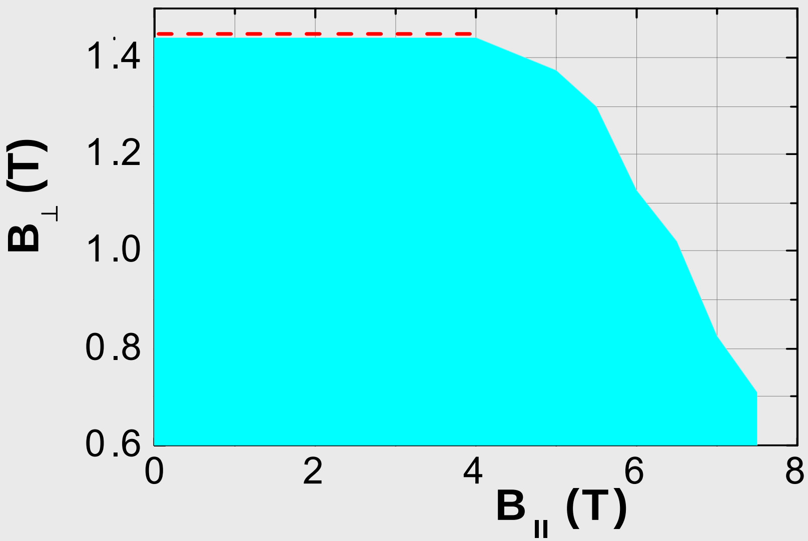
where g_b, m_b, κ_b, and χ_b are the corresponding bare values.

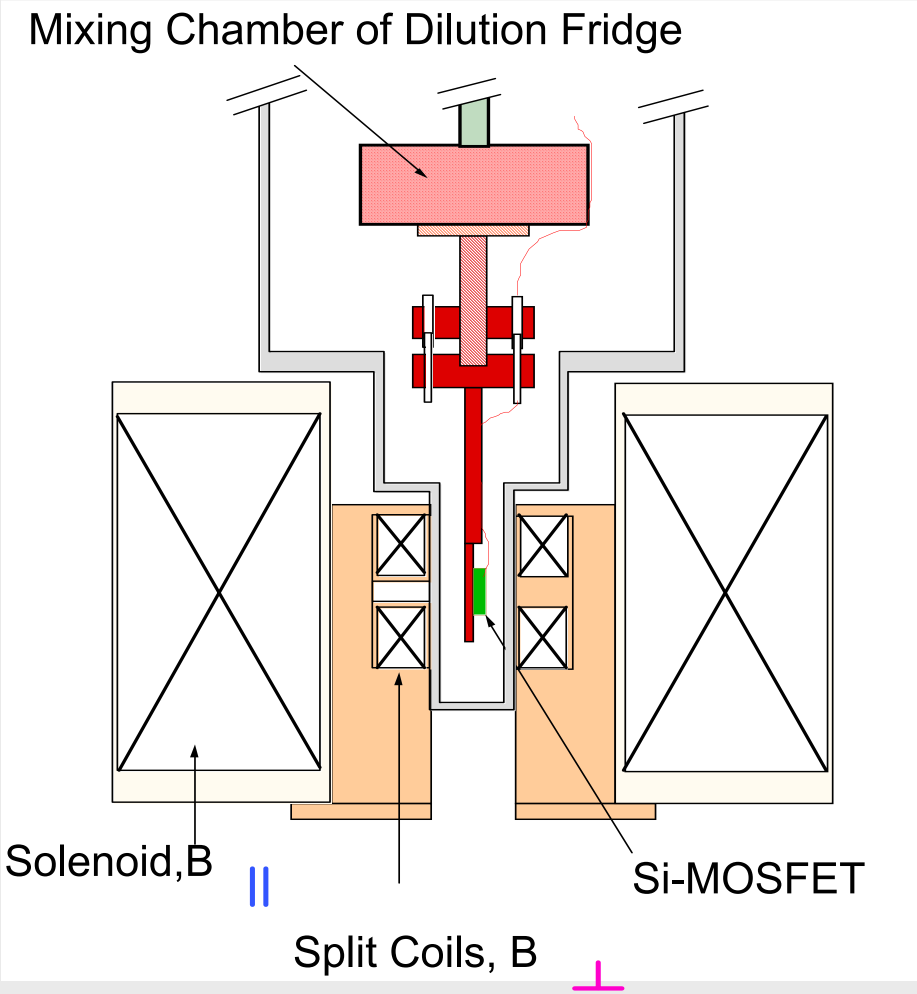
Plan:

- to use the FL-theory
- to measure renormalized electron parameters
- to determine FL-constants
- to apply this knowledge for understanding the metallic conduction and MIT

$F_i^{a,s}$ – FL-constants (harmonics) of the *e-e* interaction

9.1. Crossed Magnetic Field Technique





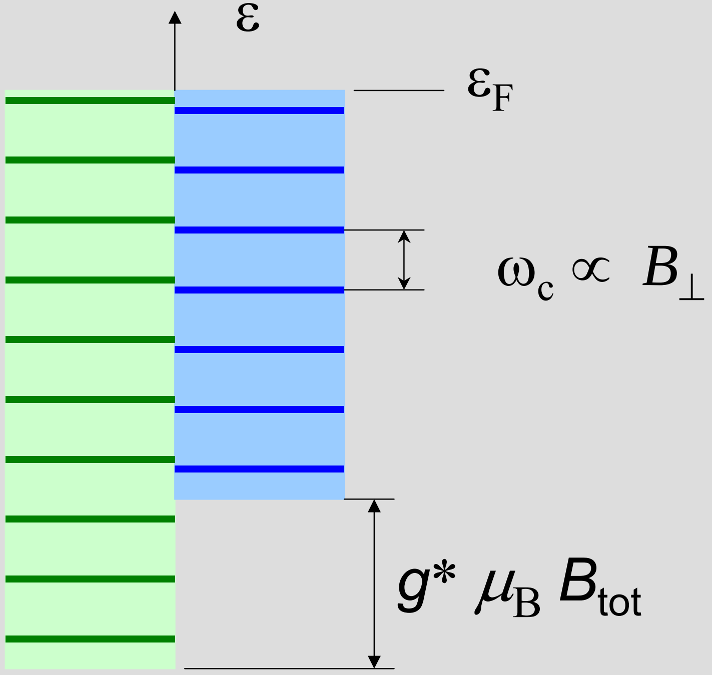
M.E.Gershenson, V.M.Pudalov, H.Kojima, et al. *Physica* E, **12**, 585 (2002).

$B_{||}$ -field sets the Zeeman splitting $g^{*}\mu B_{||}$

***$B_{\perp}$*-field** quantizes the states at Landau levels (Φ/Φ_0)

⇓

Period of beats ∝ difference in the population of ↑ и ↓ electrons:



$$n_{\uparrow} - n_{\downarrow} = \frac{\chi^{*} B_{tot}}{2\mu_B} =$$

$$= eB_{tot}\frac{g^{*}m^{*}}{h}$$

$$B_{tot} = \sqrt{B_{\perp}^{2} + B_{||}^{2}}$$

Assumption:

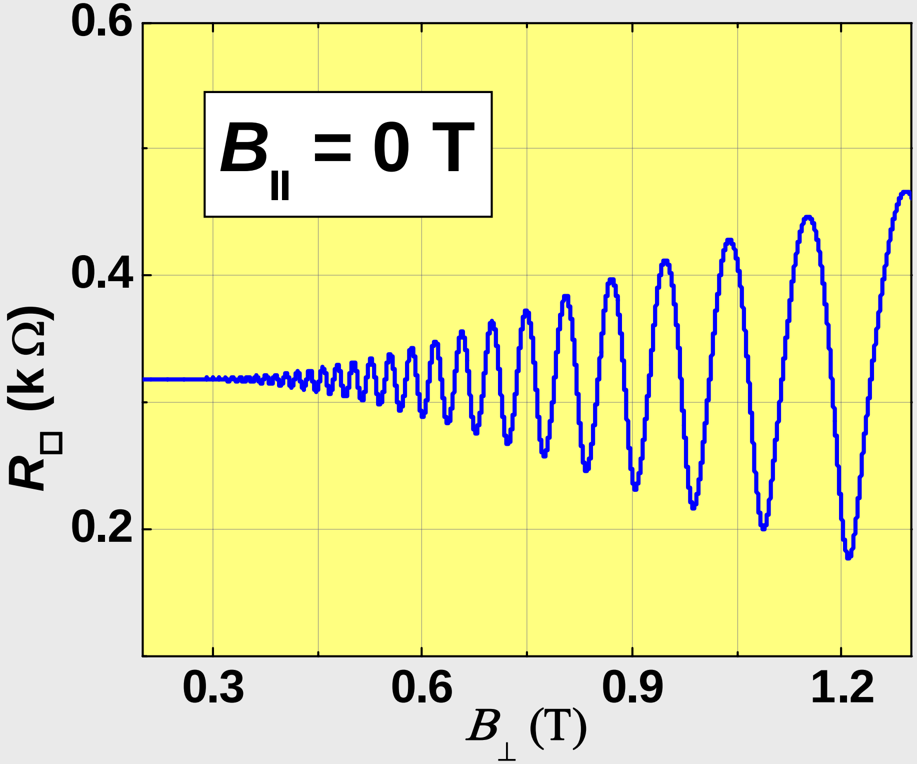
$$\hbar\omega_C << \varepsilon_F$$

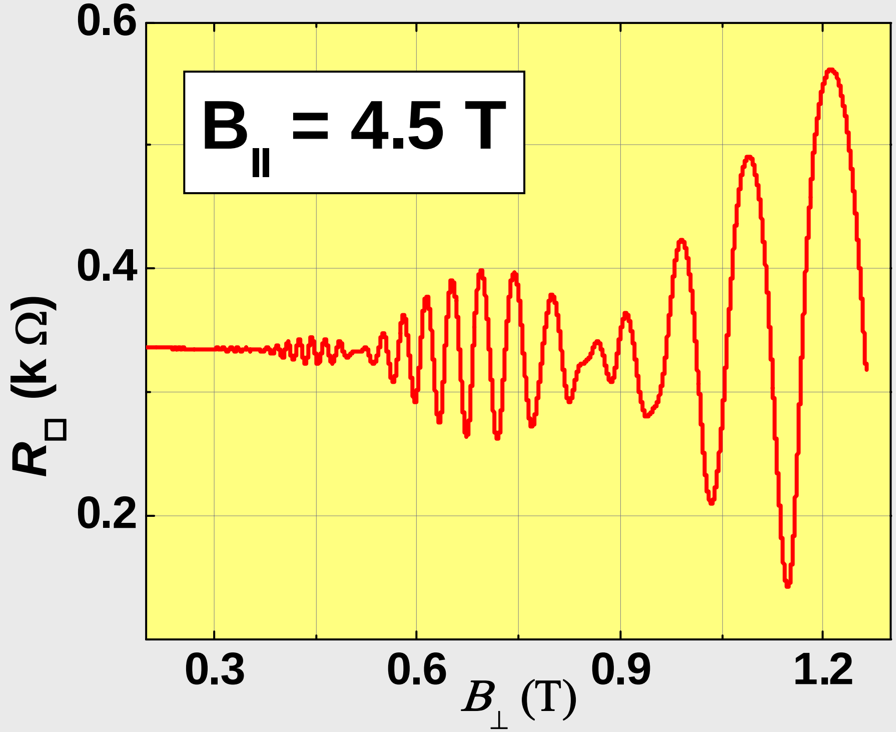
$$\frac{\Delta\rho_{xx}}{\rho_{xx}} << 1$$

Examples of SdH oscillations for zero and nonzero $B_{||}$ field

n = 10.8x10^{11} cm^{-2},

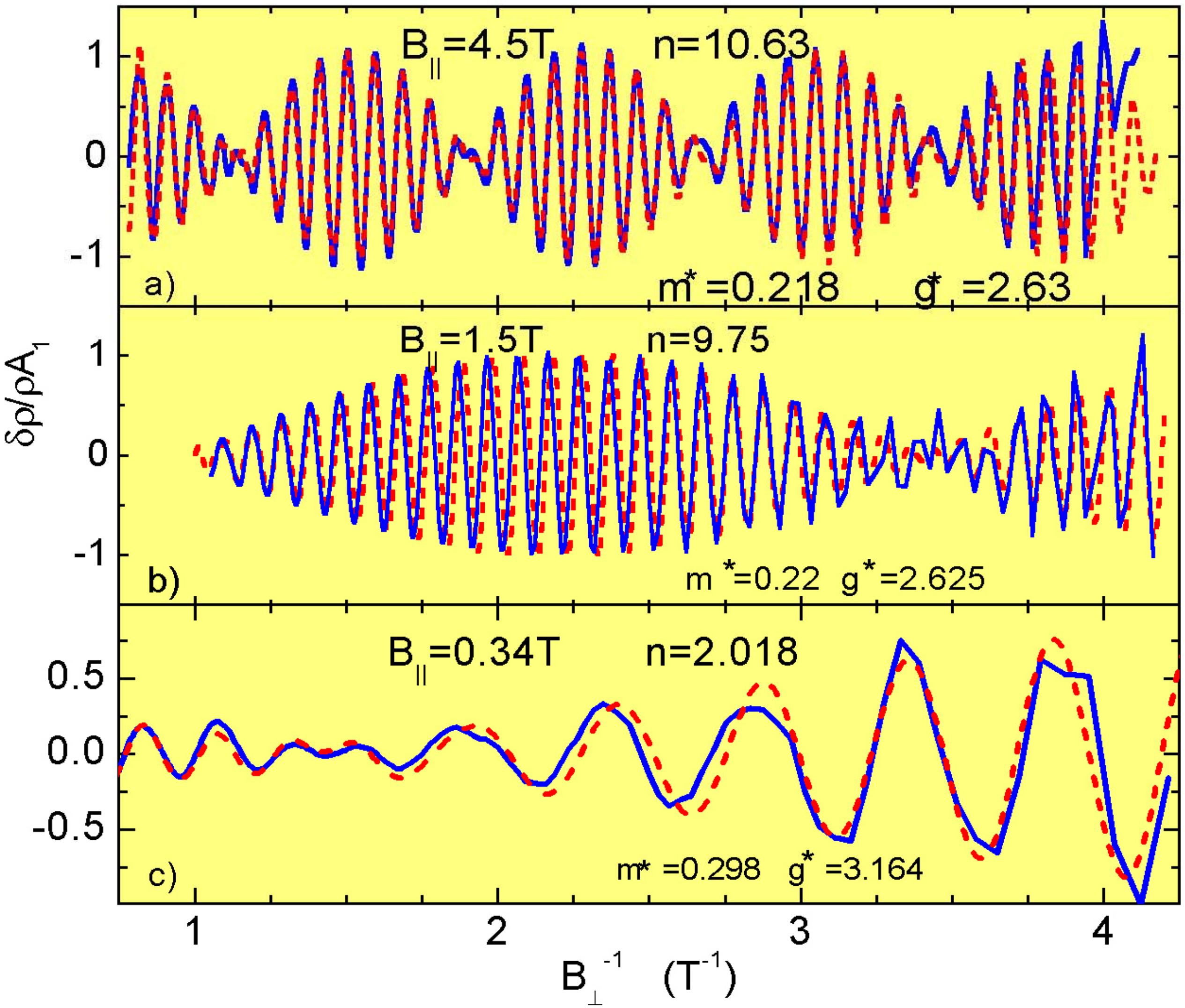
r_s = 2.5





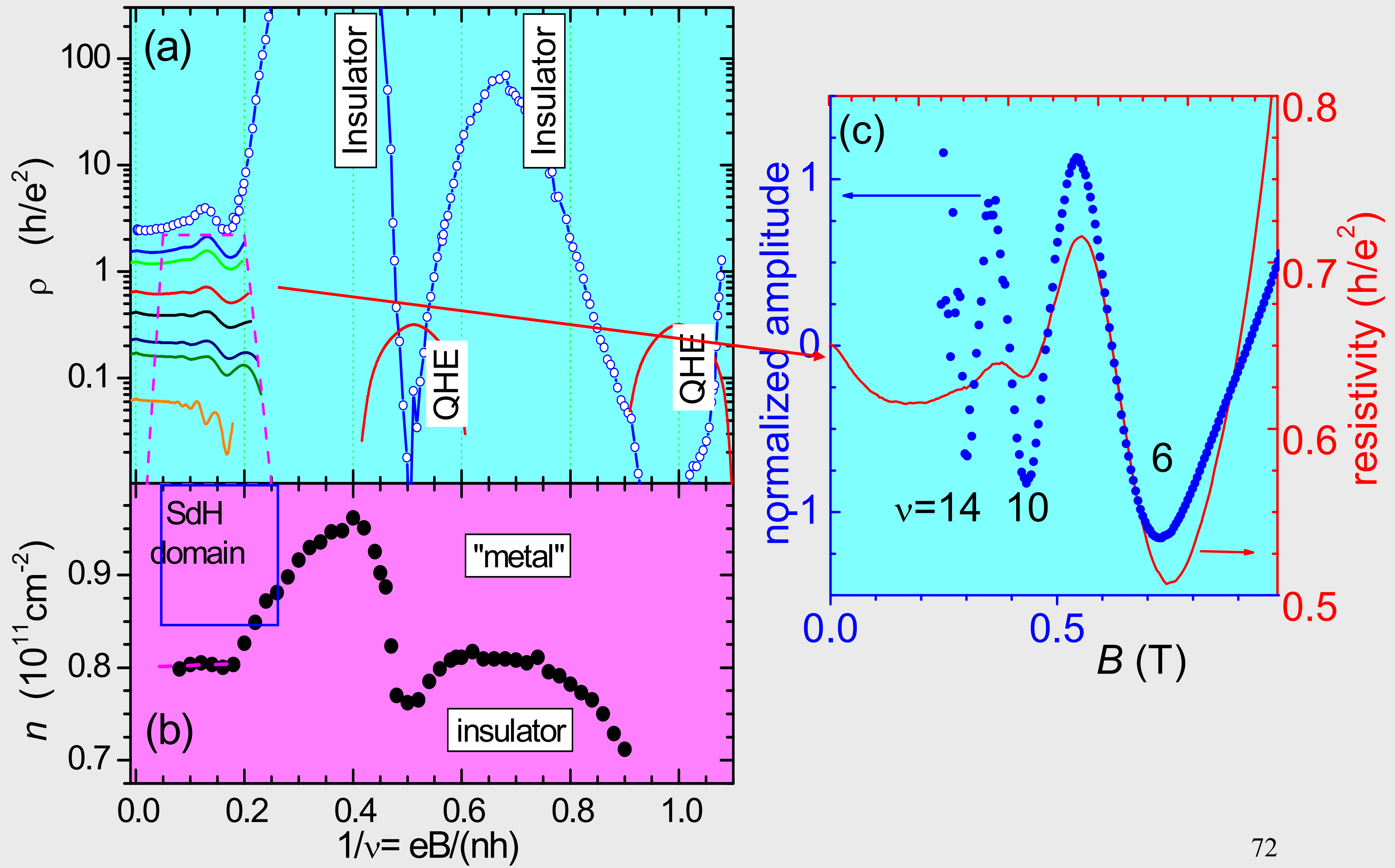
V.M.Pudalov, M.E.Gershenson, H.Kojima, et al., *Phys.Rev.Lett.* **88**, 196404 (2002)

SdH oscillations with normalized amplitude



V.M.Pudalov, M.E.Gershenson, H.Kojima, et al., *Phys.Rev.Lett.* **88**, 196404 (2002)

ρ-n domain of the SdH measurements



Effective mass m^* is obtained from the T-decay of the oscillations amplitude. **One needs a model for this !**

$$\Delta\rho/\rho \propto \exp[-2\pi^2 k(T+T_D)/h\omega_c]$$

$$\ln \delta\rho \propto m^*(T+T_D)$$

(i) Lifshits & Kosevich (55); Dingle; Isihara & Smrchka (86):

Gaseous model (LK-formula): $T_D=Const$

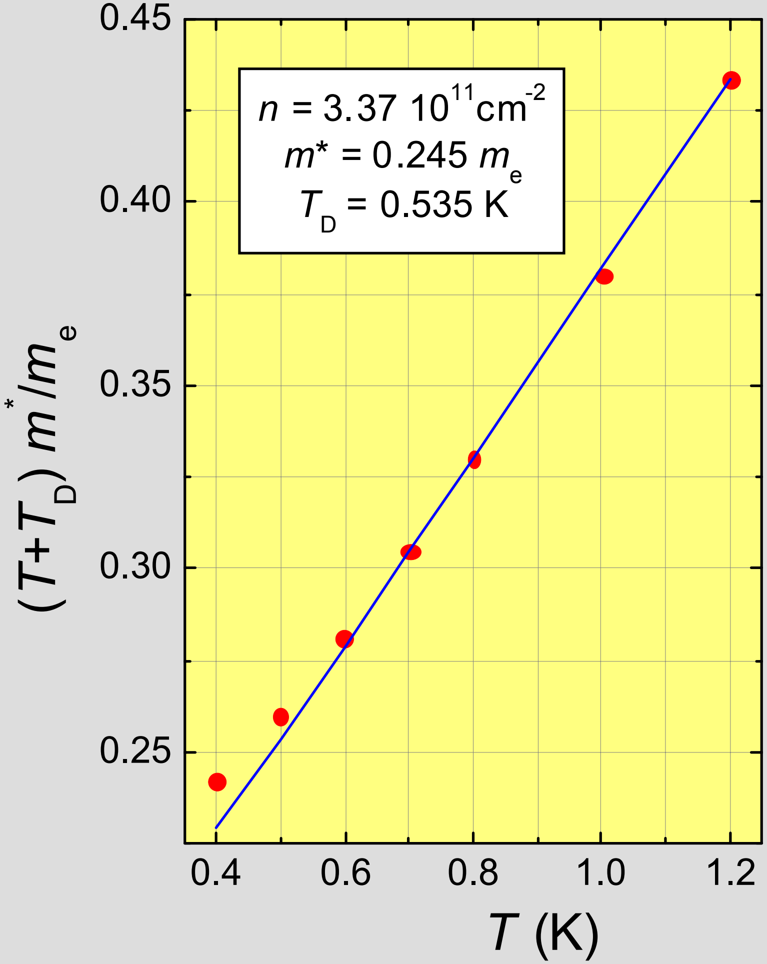
(ii) Martin, Maslov, Reizer *PRB* (2004)

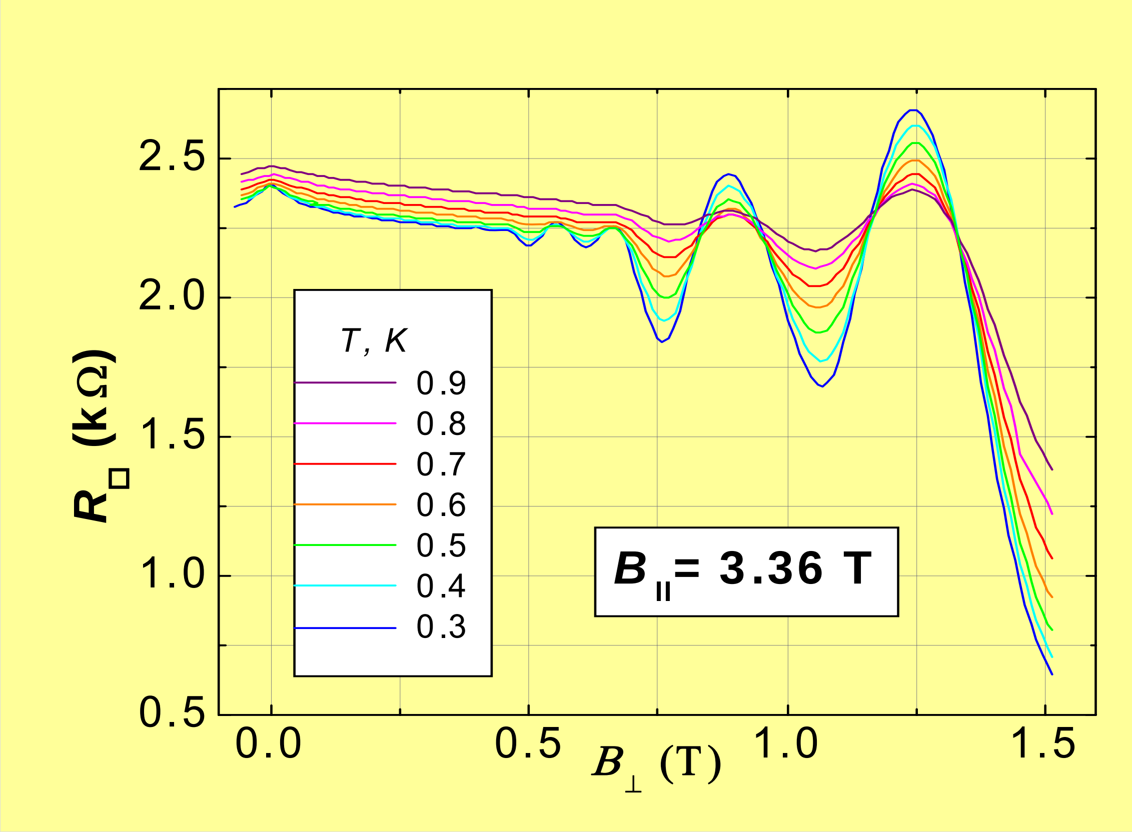
FL quantum corrections: $T_D \sim \rho(T) \sim T_{D0}(1+aT)\ln(T/E_F)$

(iii) Gorny, Mirlin *PRB* (2006):

FL quantum corrections: $m^*=m_0(1+\beta T)$

Renormalized effective mass *m** from SdH effect





Assumption: $T_d^* \propto \sigma \propto (1+\alpha T)$

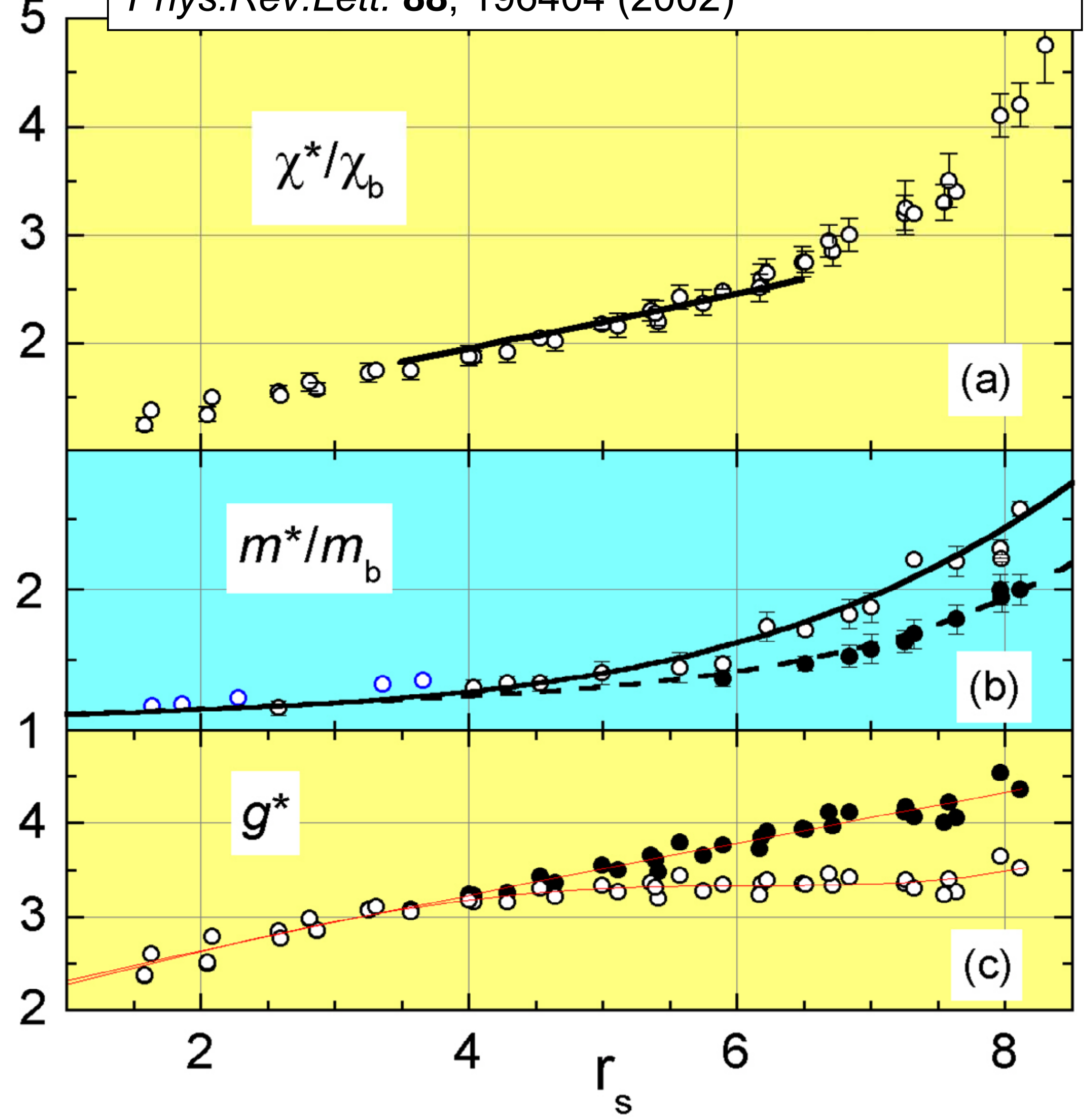
Resulting $\chi^* \sim g^* m^*$

Effective mass *m** and

*g**-factor

Two possible values for *m** (in the two assumptions):

V.M.Pudalov, M.E.Gershenson, H.Kojima, et al., *Phys.Rev.Lett.* **88**, 196404 (2002)



high *m**– for T_D being *T*-independent and **low** *m**- for $T_D \sim \rho(T)$

9.3. Do the data on renormalized χ^*, m^*, g^* represent an interaction-related physics ?

How consistent are the experimental data on

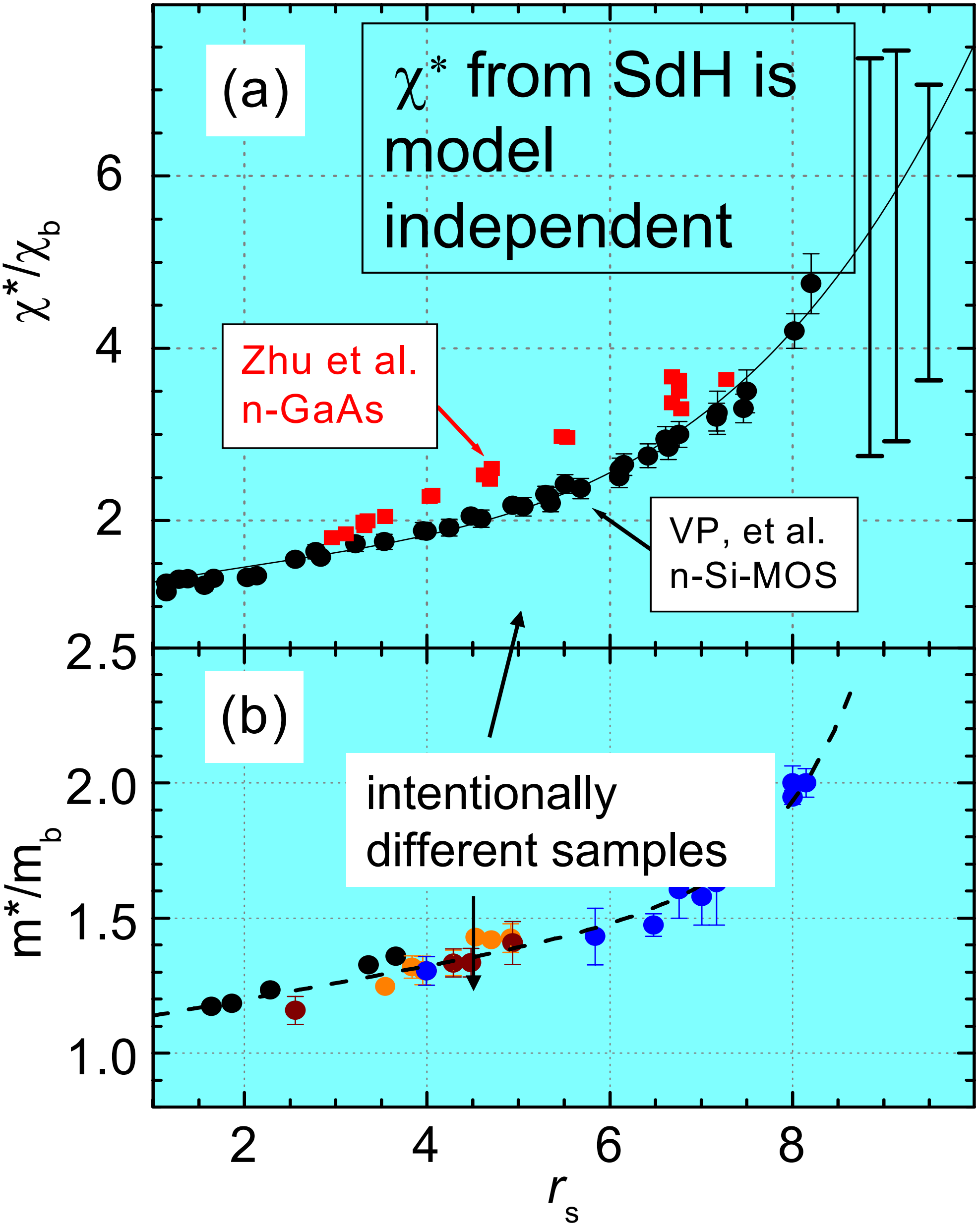
χ^*, m^*, g^*

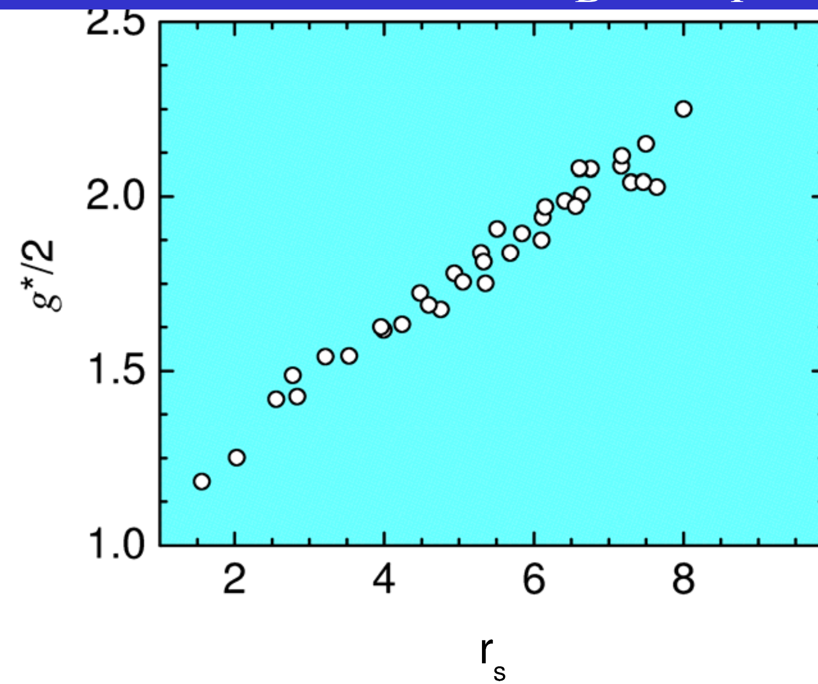
(taken by different techniques and on different materials) ?

Renormalized χ^*, m^*, and g^* versus r_s determined from SdH oscillations in crossed fields

NB: (i) $\chi^* \sim m^* g^*$

(ii) Linear regime: $g^* \mu_B << E_F$

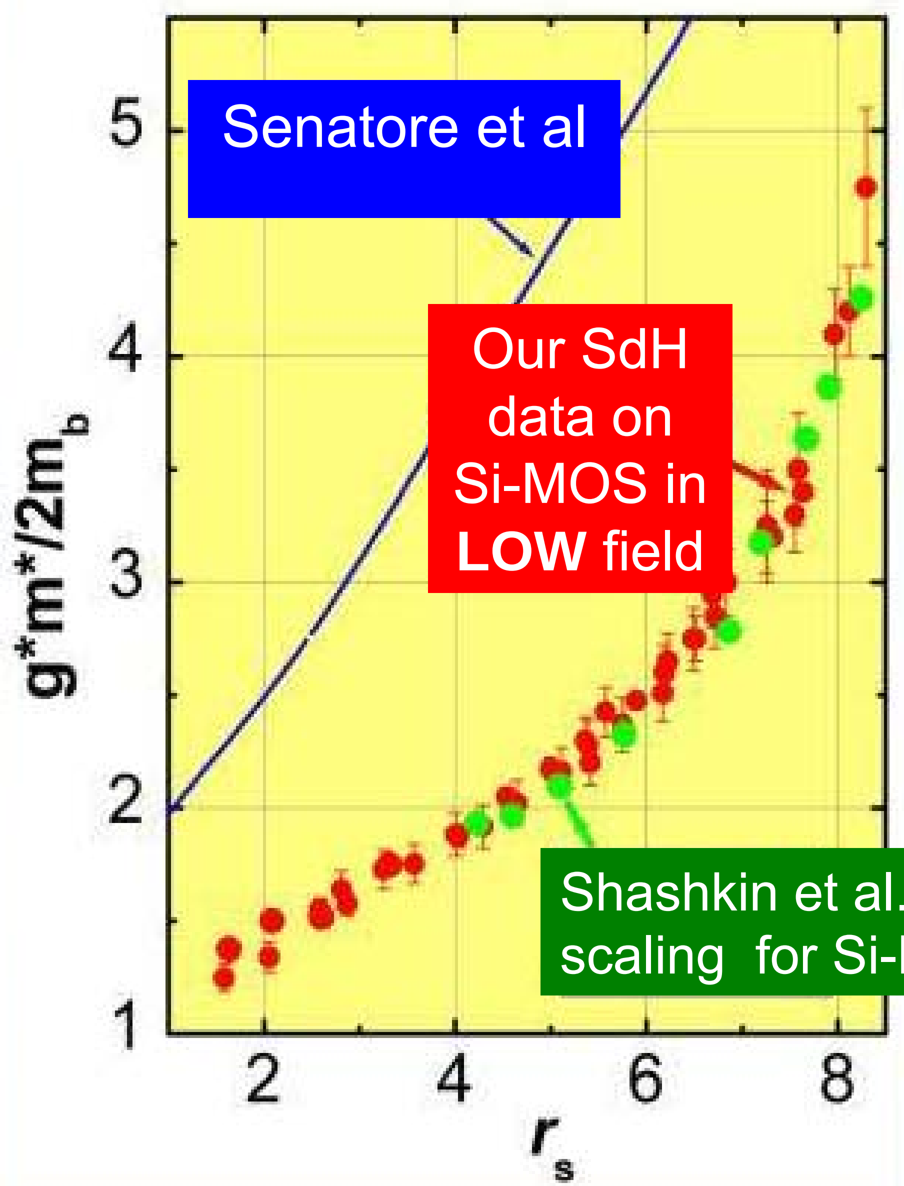


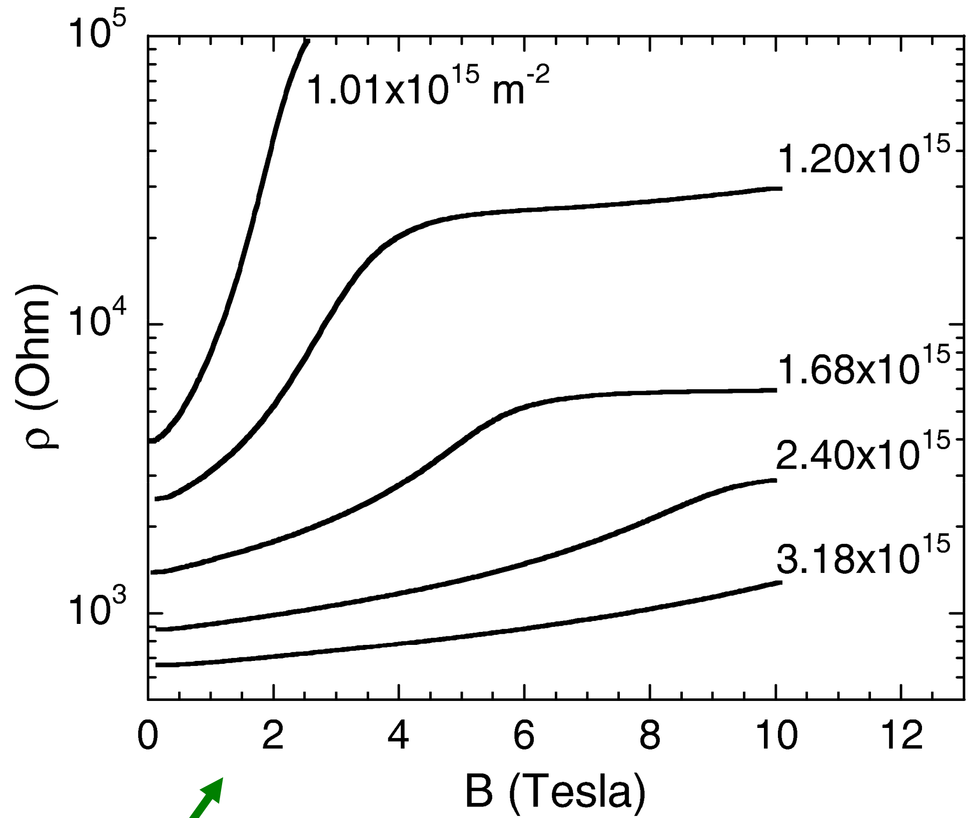


V.M.Pudalov, M.E.Gershenson, H.Kojima, et al., *Phys.Rev.Lett.* **88**, 196404 (2002)

J.Zhu, H.Stormer, L.N.Pfeiffer, et al., PRL **90**, 056805 (2003)

Renormalization of χ: overview of the data for Si-MOS in a wide range of densities



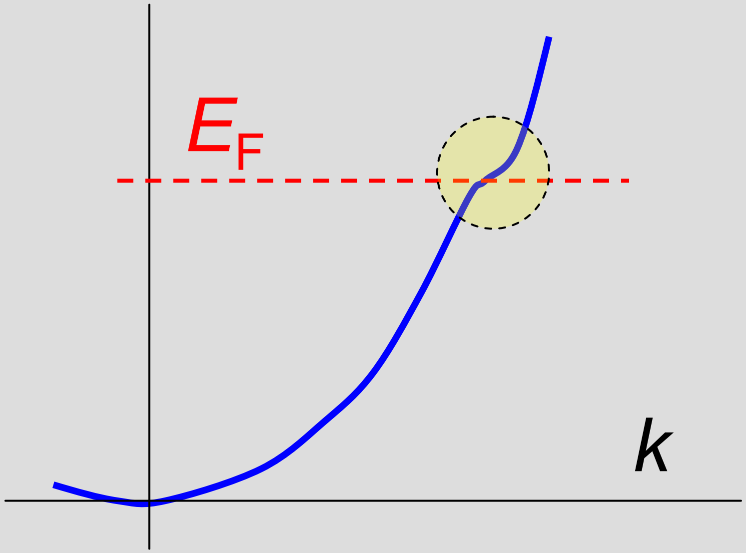


Consistency of the experimental data for $\chi^*(n)$

✓ taken at $E = E_F$ and

✓ over the whole energy interval **(0 – E_F)**

➢Electrons do interact in a wide energy interval, ~ E_F, much wider than kT



Summary of the experimental results for χ in the metallic regime (high densities):

- Renormalized χ^* is similar for the single- and double-valley systems, when **reduction in E_F** (a factor of $1/g_v$) is ignored in calculation r_s **!**

- The effect of the width of the potential well on renormalization of χ^* is weak (in low fields), though becomes noticeable in *high fields*

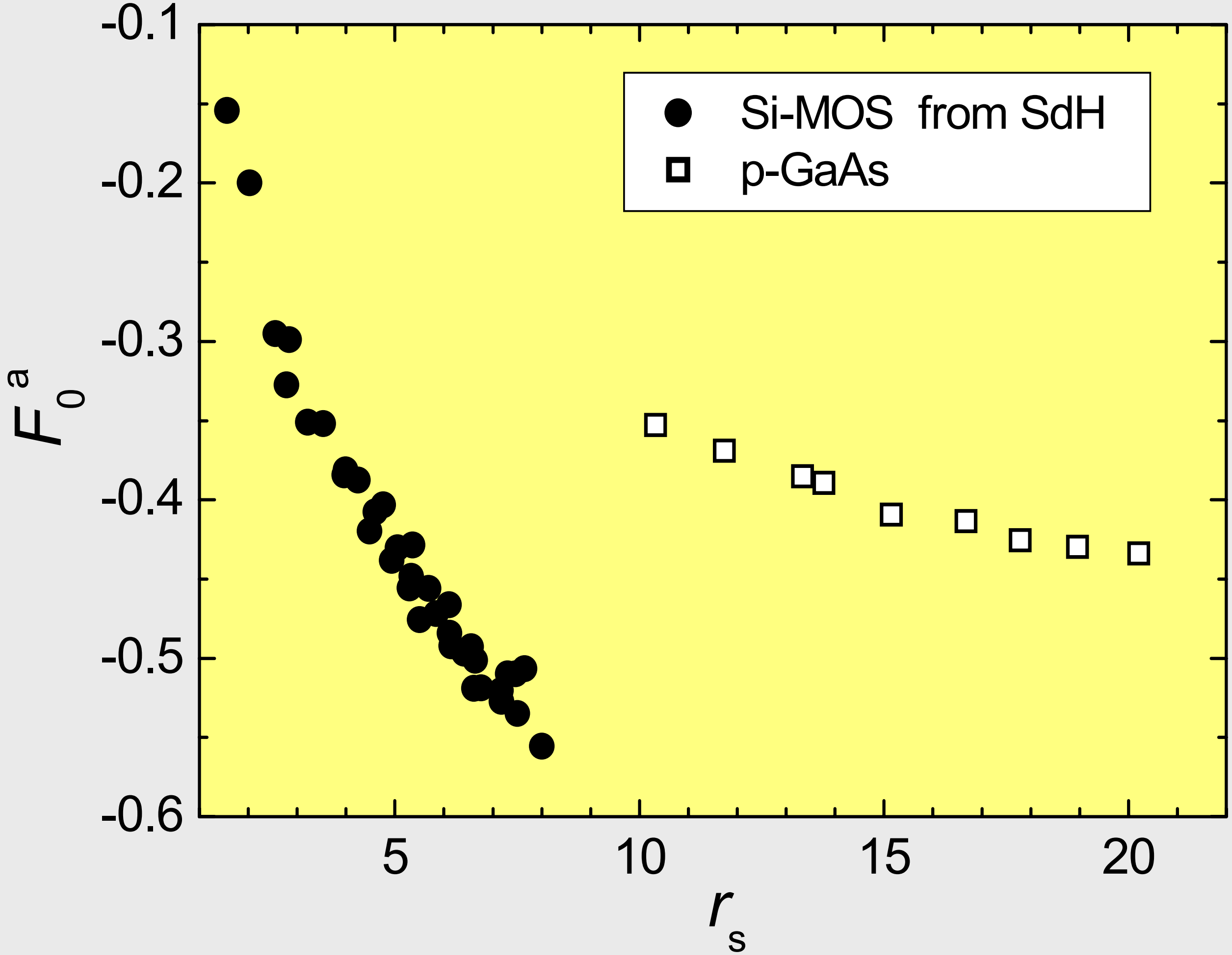
- The effect of disorder on the renormalization of χ^* at high density is negligible or, at least, weak

9.4. e-e interaction effects: a puzzle of *p*-GaAs

n*-SiMOS:** χ^* and ***m* from SdH (VP et al. *PRL*, 2002)

***p*-GaAs:** $\rho(T)$-fit with m^* from SdH amplitude

(Savchenko et al *PRL,* 2002)



Seems to be a big difference between *n*-Si-MOS- and *p*-GaAs-data

$$F_0^a = 2/g^* - 1$$

NB: valleys

There are a number of puzzling results which suggest that in *p*-GaAs, E_{ee} is much less than $e^2/\varepsilon a$

1) $\rho(T)$ drop is smaller than for other system with a simpler band structure, at the same r_s value (Savchenko et al, Tsui et al, etc)

2) Critical ρ_c value is too low

3) "Metallic" transport (non-hopping) is observed for **r_s up to 80** (Tsui et al)

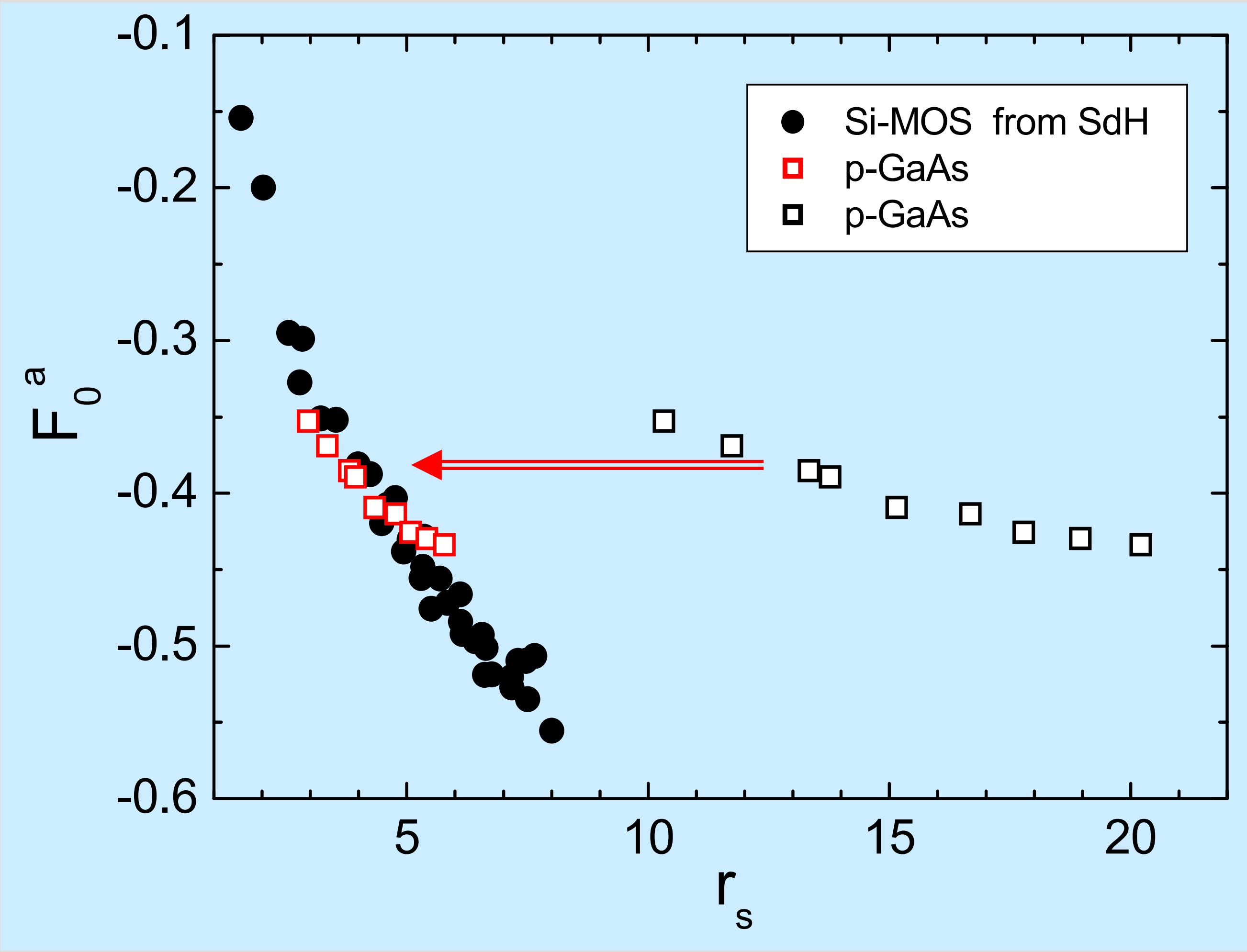
4) m^* is almost independent of r_s (Savchenko et al)

<u>Conjecture:</u> assume that due to a complexity of the band structure, the e-e interaction is screened and effective r_s is smaller than E_{ee}/E_F .

Will tray to consider r_s as a fitting parameter

Rescaling the data for *p*-GaAs: $r_s = r_s/\kappa$ with $\kappa = 3.5$

the difference may be reduced if r_s is treated differently



$r_s = E_{ee}/E_F$

with bare

$E_F \sim 1/m^*$

$m^* = f(r_s) \Leftarrow$ n-Si

$m^* \neq f(r_s) \Leftarrow$ p-GaAs

Conclusions:

1) Most of the exper. data do agree with each other and, hence, represent quantitative information on the *e*-*e* interaction. Effects of individual material properties are weak

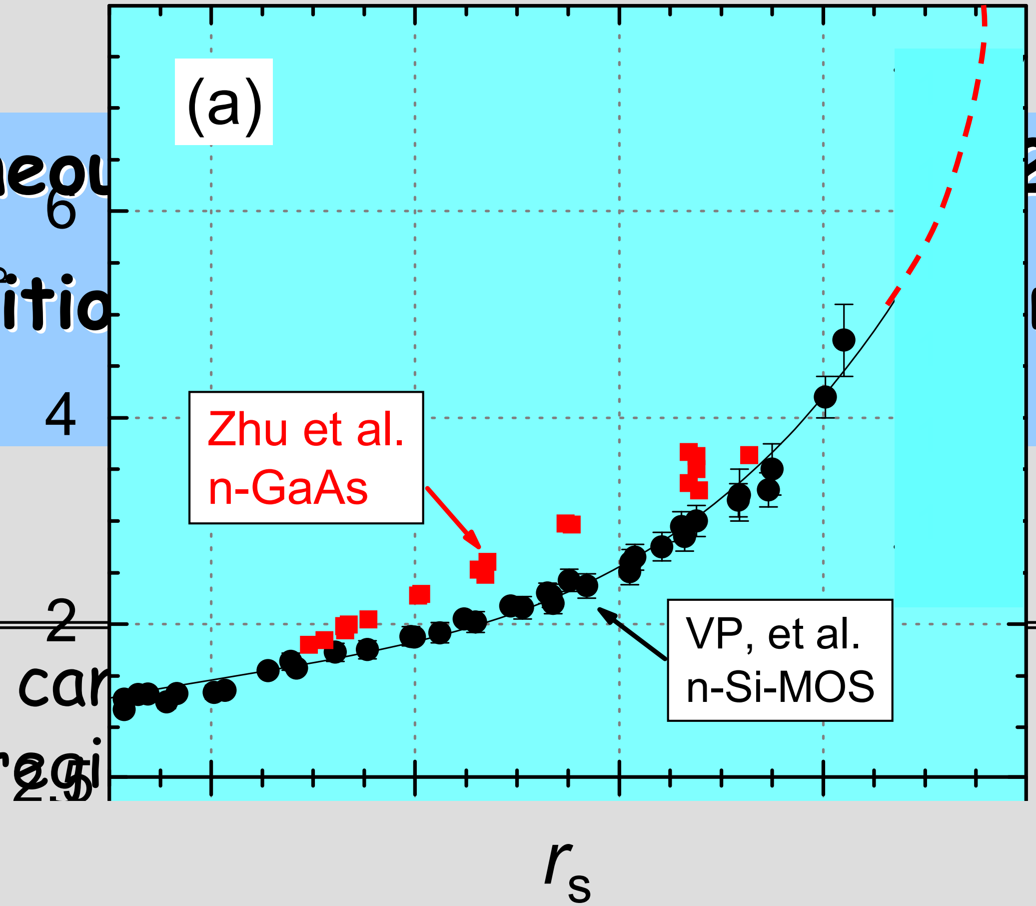
2) There is only qualitative similarity of the numerical results with data on $\chi*(r_s)$ and $m^*(r_s)$

3) Puzzling data for 2D holes in GaAs: theory of interaction (screening) for p-type systems needs to be refined

9.5. χ^* is strongly enhanced. Does it diverge ?

Spontaneou 2DE liquid?

M-I transitio magnetic transition.

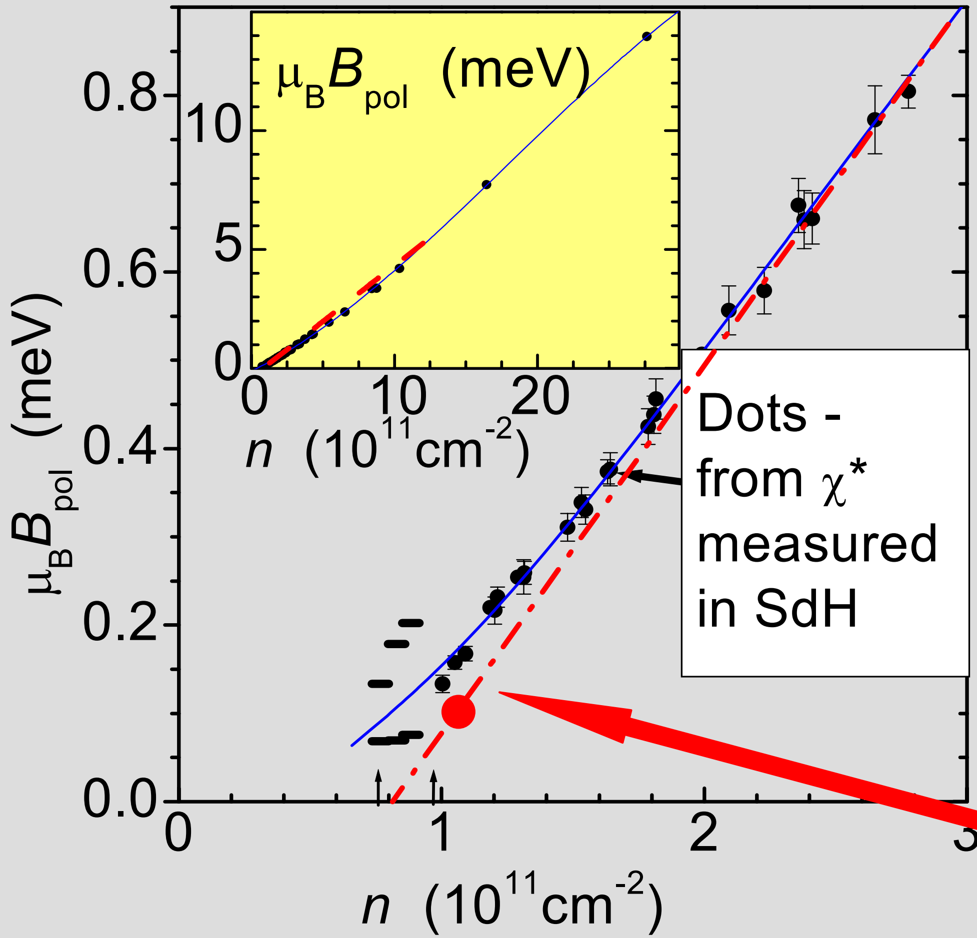


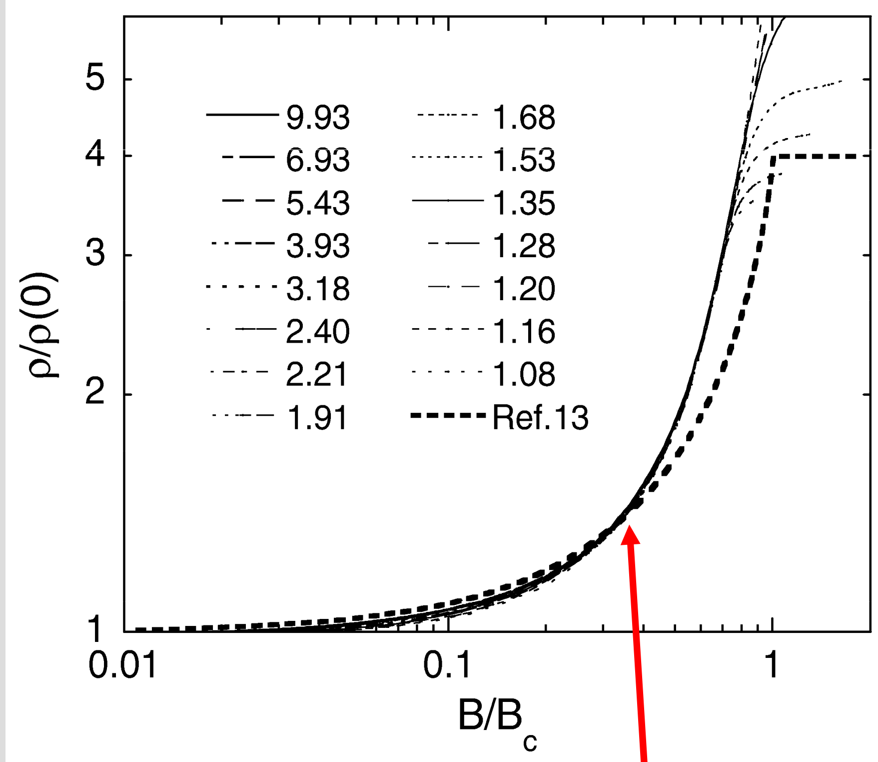
How one car in the critical regi M-I transi

Does χ^* diverge as $n \to n_c$?
or as $T \to 0$?

How the information on χ^* can be obtained at the lowest carrier density $n \approx 7.5\ 10^{10}$cm^{-2} (i.e. $r_s \approx 10$)

Low field (SdH) data vs High field data ($g^*m^*B_{pol} \sim E_F$)





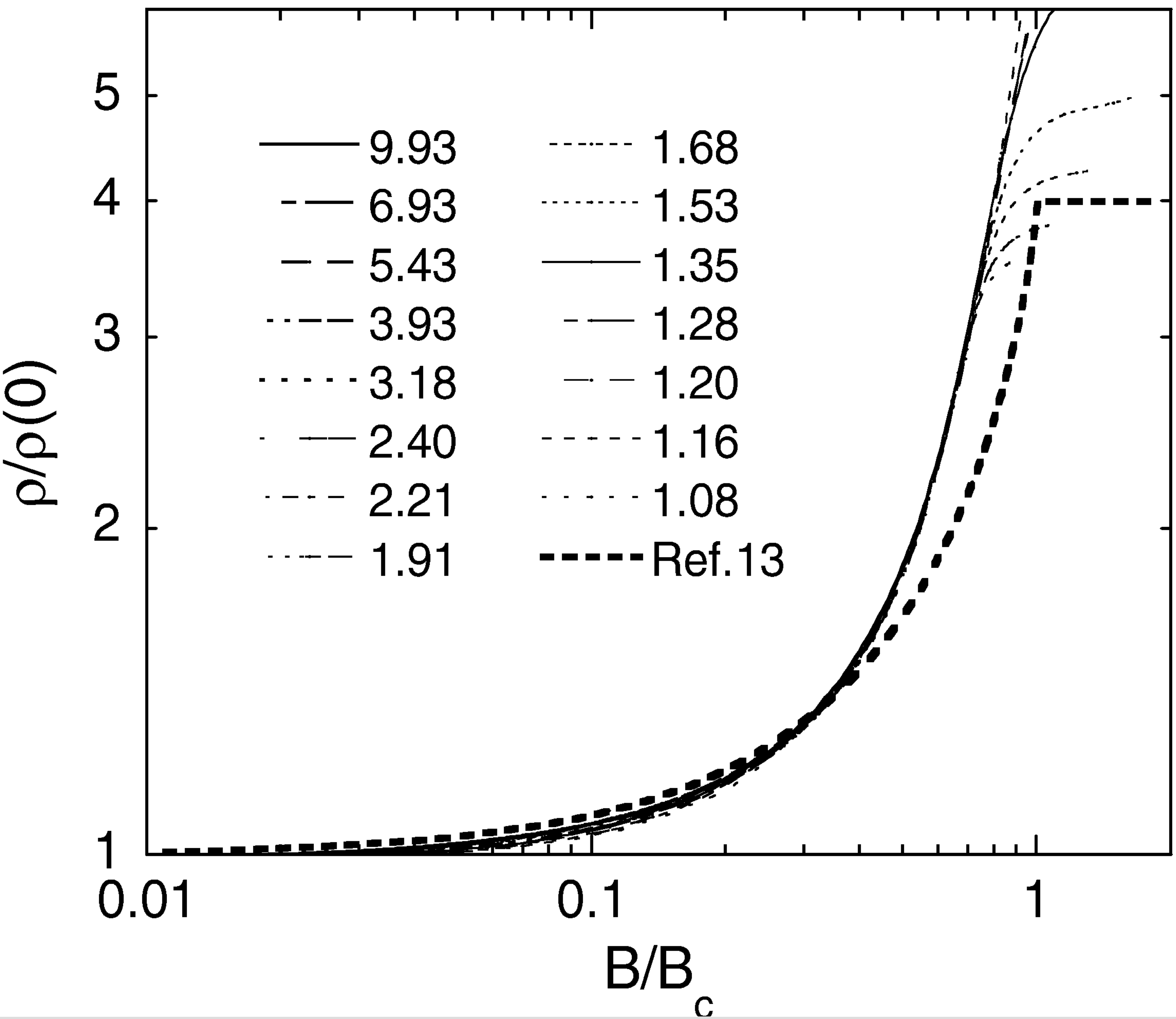
$R(B_{||})$ scaling in high fields

Shashkin et al. PRL (2001)

B_{pol} calculated from B_c and interpreted as $g^*m^*B_{pol} = 2E_F$

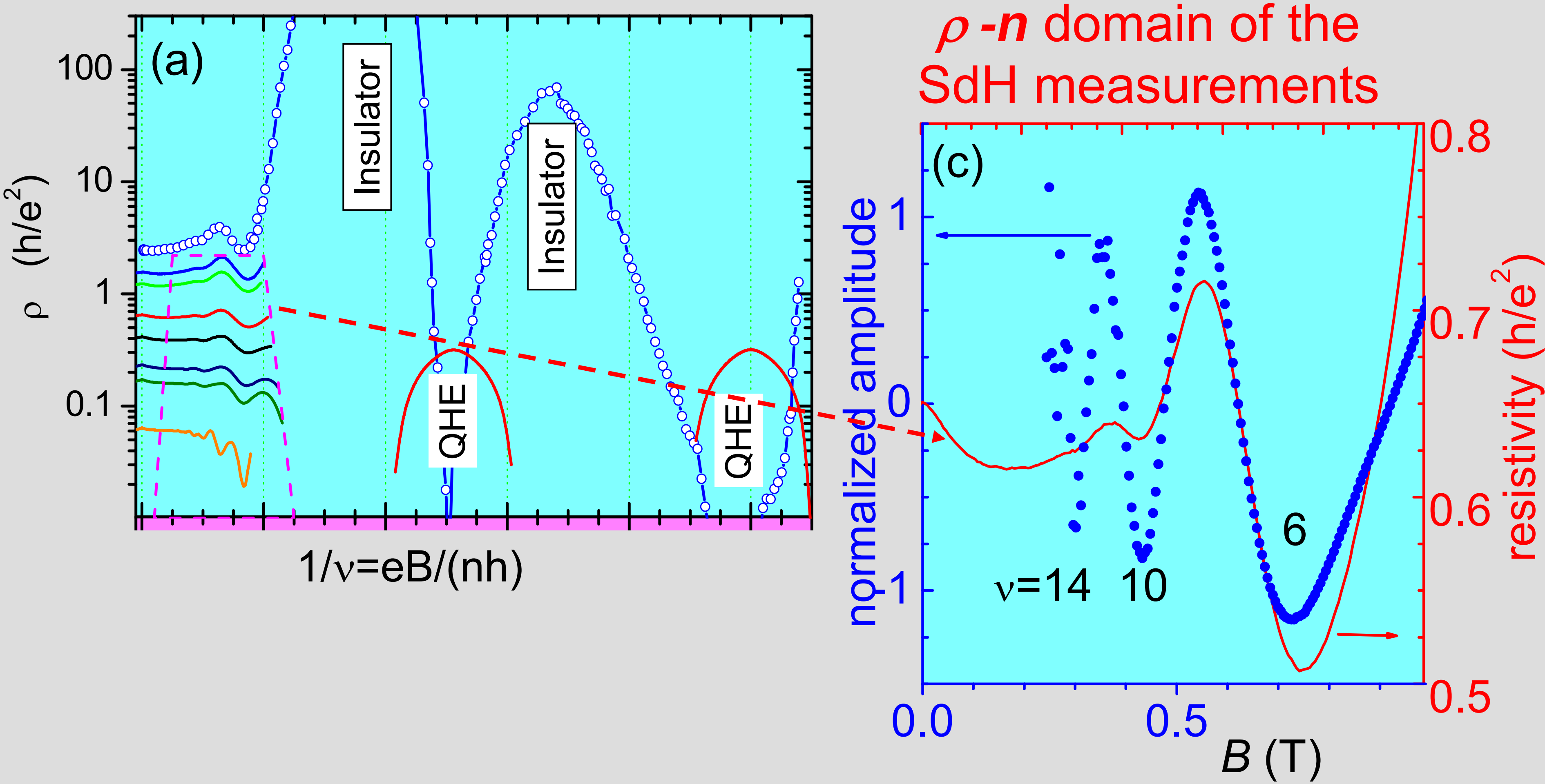
There is a minor discrepancy at low density.

Possible reasons: (i) **$\chi^*=\chi(B_{||})$,** (ii) Scaling is not perfect
(iii) a crossover from **M** to **I** phases in high $B_{||}$



ρ/ρ(0)
B/B_c
0.01
0.1
1
1
2
3
4
5
9.93
6.93
5.43
3.93
3.18
2.40
2.21
1.91
1.68
1.53
1.35
1.28
1.20
1.16
1.08
Ref.13

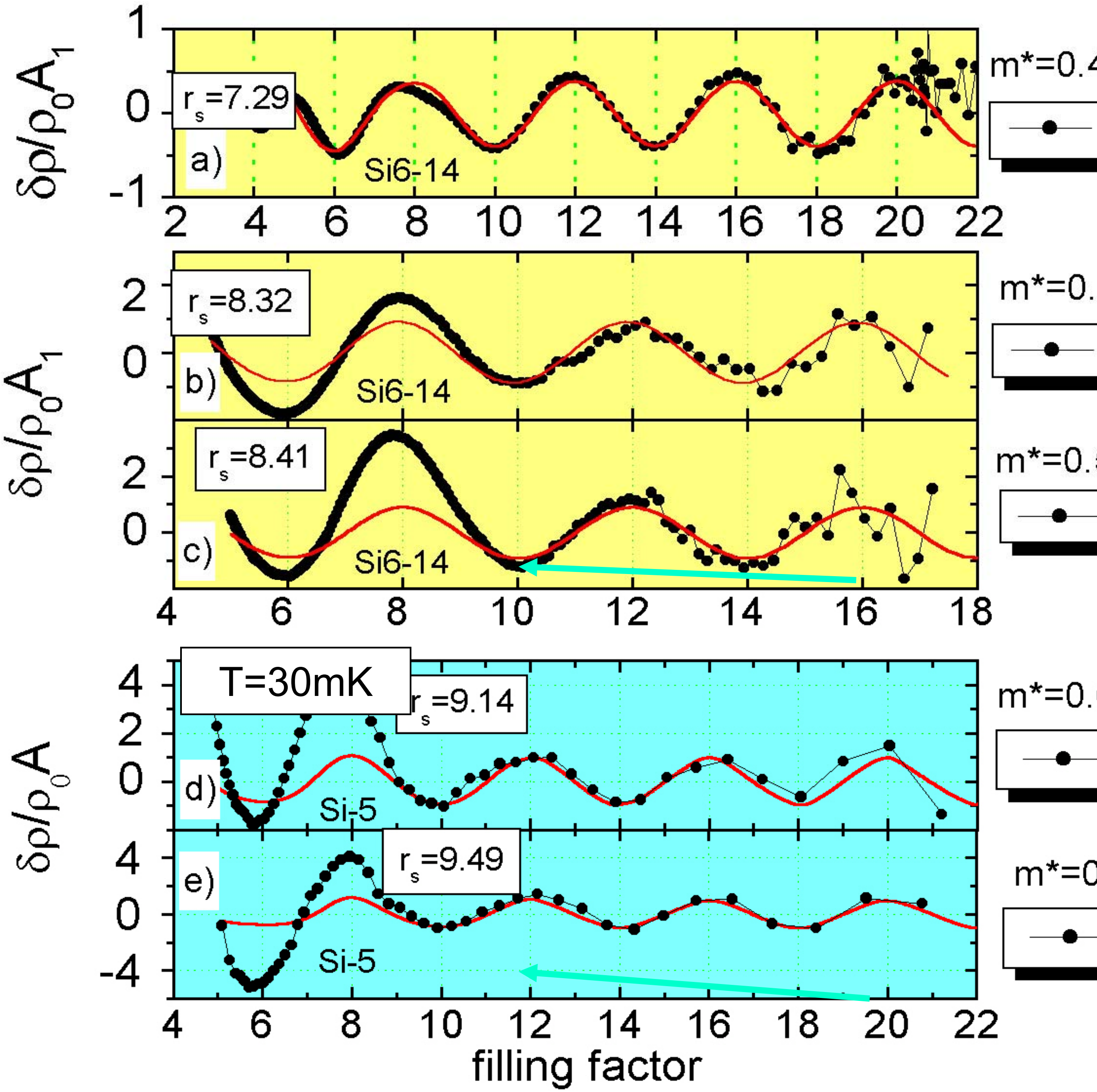
ρ-B-n phase space where SdH can be measured



VP, M.D'Iorio et al, *PRB* (1992); *Phys.Lett*. (1990).

Sample
Si6-14
$n = n_c$

Sample
Si15
$n = n_c$



VP, M.Gershenson, H.Kojima, in: Fundamental Problems of *of mesoscopic Physics,* Kluwer (2004)

(i) Periodicity

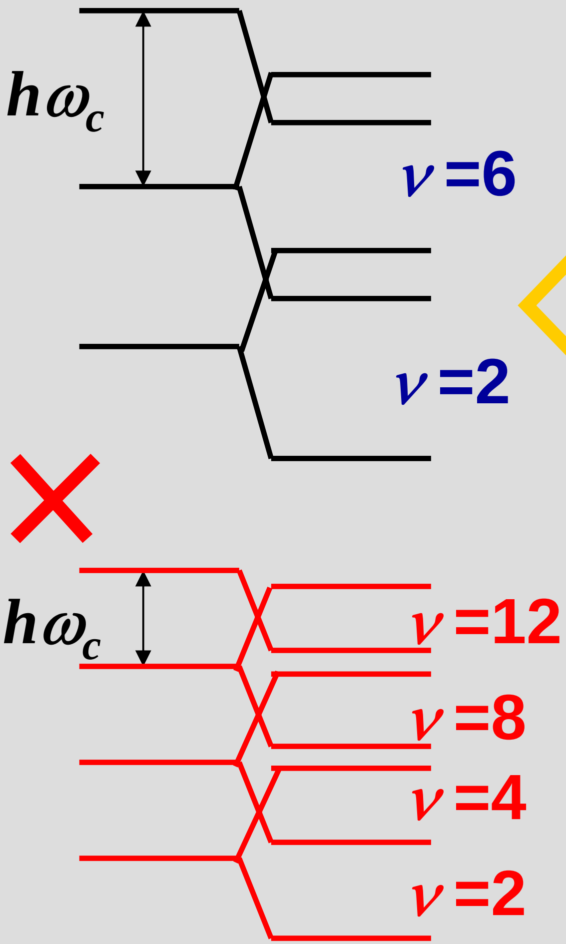
of SdH oscillations corresponds to the presence of the **two** spin subbands for all densities (4-fold degeneracy)

The **2D system is unpolarized down to $n \approx 7.5 \cdot 10^{10}$cm^{-2}**

Note: this result does not depend on temperature !

(ii) Phase

Estimation of the χ^* value from sign of the SdH oscillations



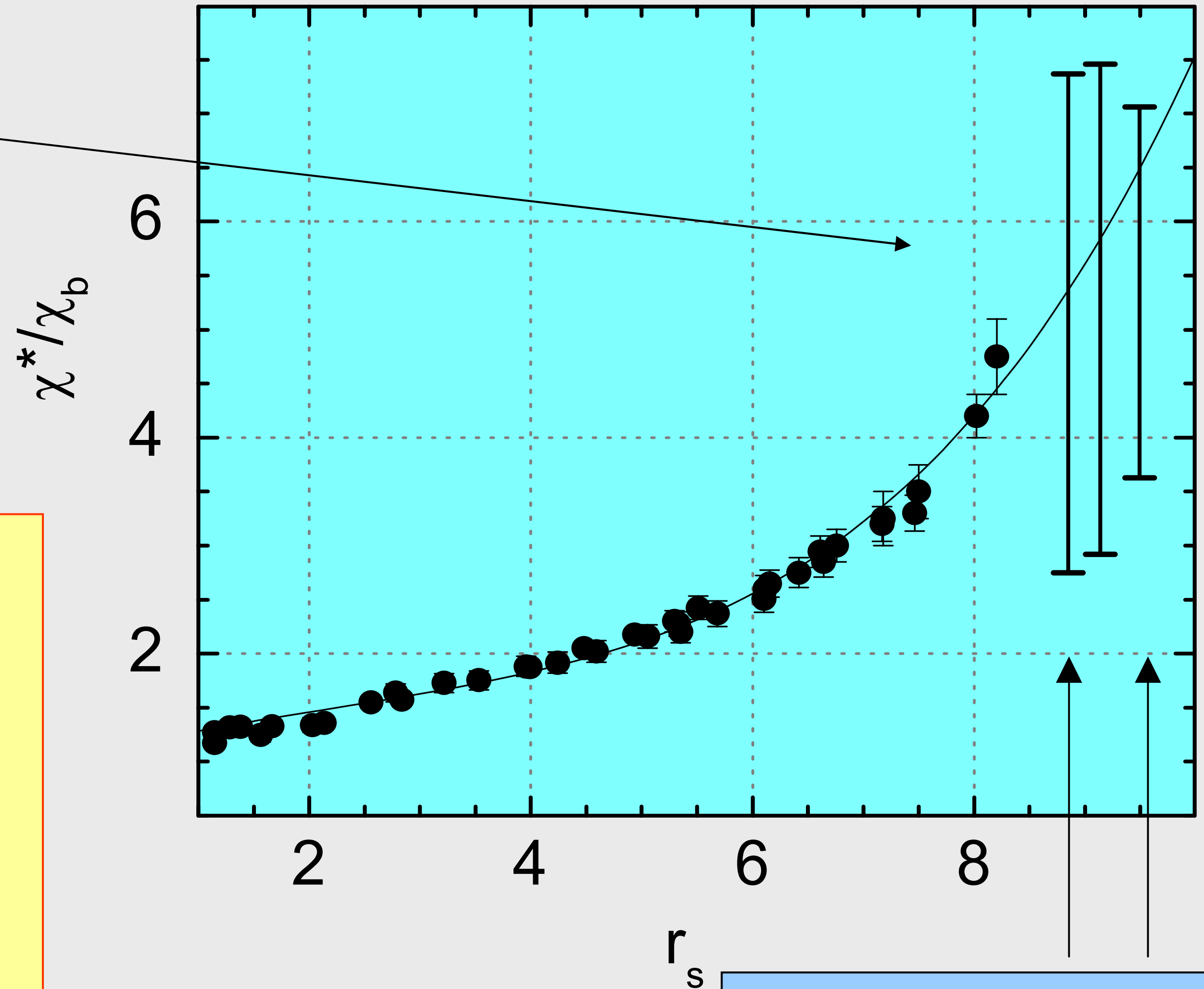
$$\cos\left(\pi \frac{E_z}{\hbar\omega_c}\right) = \cos\left(\pi \frac{\chi^*}{\chi_b} m_b\right)$$

ρ_{xx}-minima occur at ν = 2, 6, 10, etc

Hence,

$2.6 = 1/(2m_b) < \chi^*/\chi_b < 3/(2m_b) = 7.9$

The lower and upper bounds -from the period and phase of quantum oscillations



A factor of 7 growth in χ^* is now explained by:
(i) conventional FL model,
(ii) dispersion instability
(iii) CDW model,
(iv) two-phase model

n_c-values for two high-μ samples

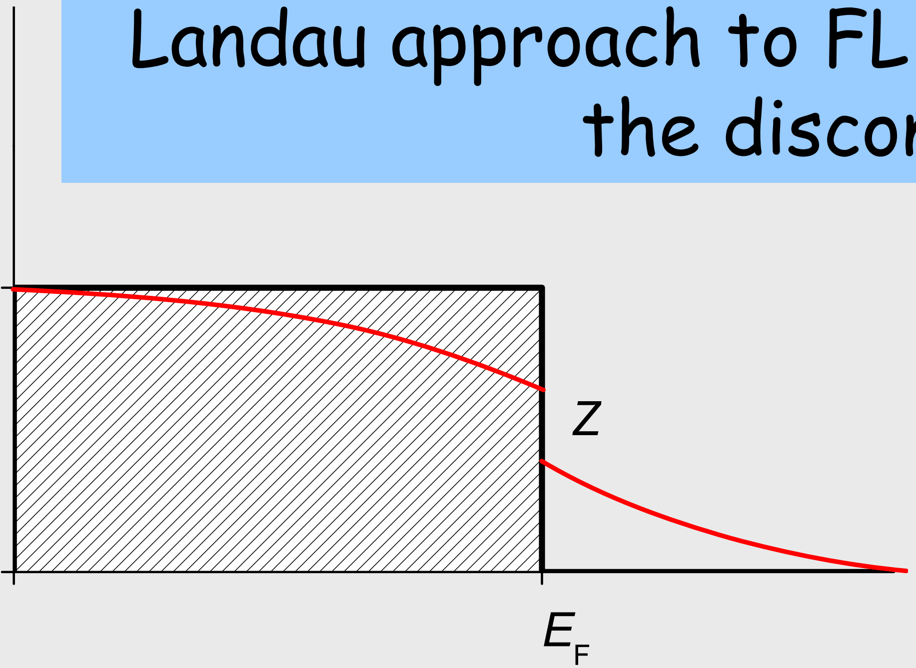
VP, M.Gershenson, H.Kojima, in: Fundamental Problems of *of mesoscopic Physics,* Kluwer (2004)

Period and **phase** of the quantum oscillations $\Rightarrow$
two spin-subbands are filled for $n > n_c$
(4×degenerate)

2D system remains <u>unpolarized</u> down to
$n \approx 7.7\times10^{10}\text{cm}^{-2}$

Magnetic transition is not be excluded though for $n < 5\times10^{10}\text{cm}^{-2}$
(in the electron "solid" phase)

9.6. Non-FL models.
Landau approach to FL assumes the existence of the discontinuity at E_F



It fails, e.g. for:

- Marginal Fermi-liquid (Varma, Littlewood, Abrahams, et al.), $Z \to 0$ logarithmically,

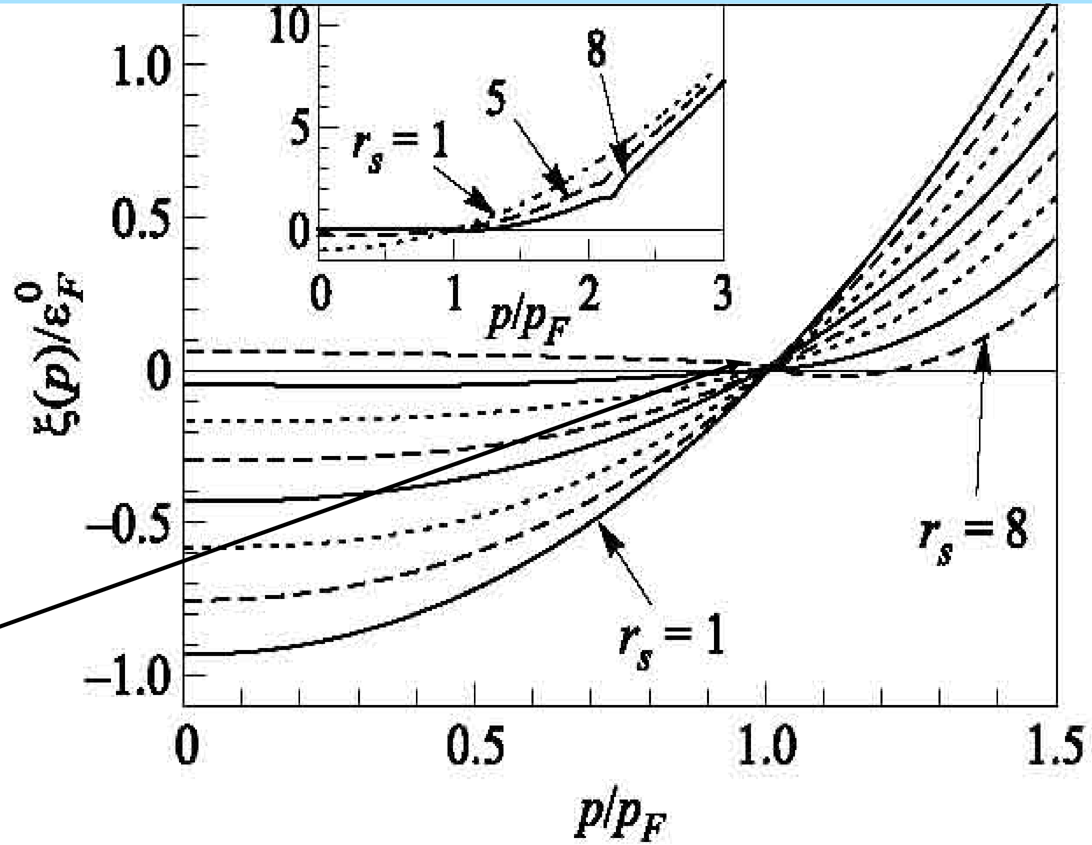
- "Fermion condensation" model ("dispersion instability") (Khodel, Shaginyan, M.Zverev, Das Sarma, et al.).

Weakness of the $\chi(T)$ dependence does not support the "dispersion instability" model

The "Fermion condensation" model predicts:
m^* or g^* to diverge $\propto 1/(n-n_c)^{3/8}$

$\chi^*(T) \propto T^{-2/3}$

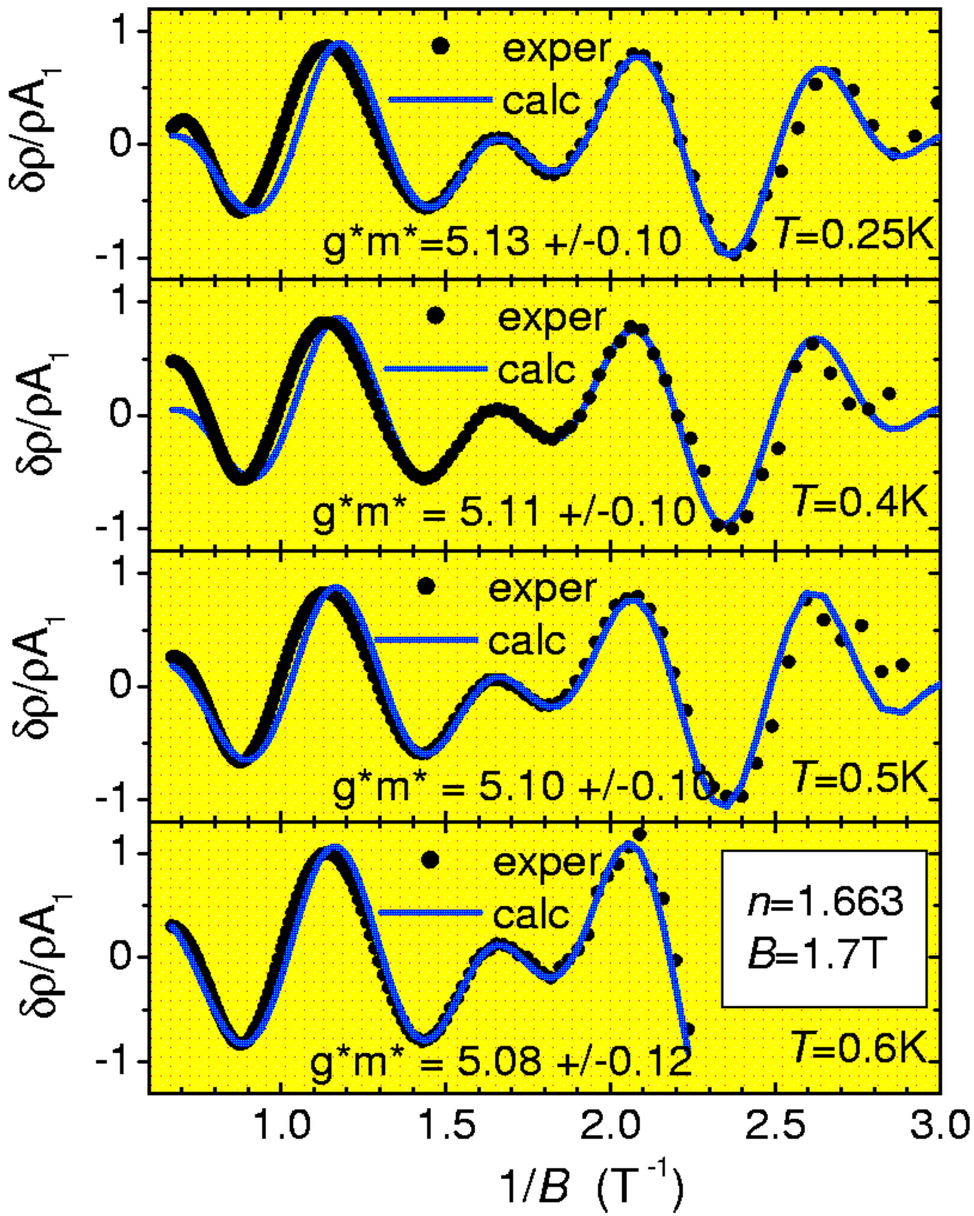
Note: an instability for $r_s \approx 7$



Single-particle spectra calculated for r_s= 1, 2, …8.

After:
Khodel, Shaginyan, JETP Lett. **51**(9), (1990).
Galitski, Khodel, cond-mat/0308203;
Baldo et al. J.Phys. Cond-mat, **16, 6431** (2004)

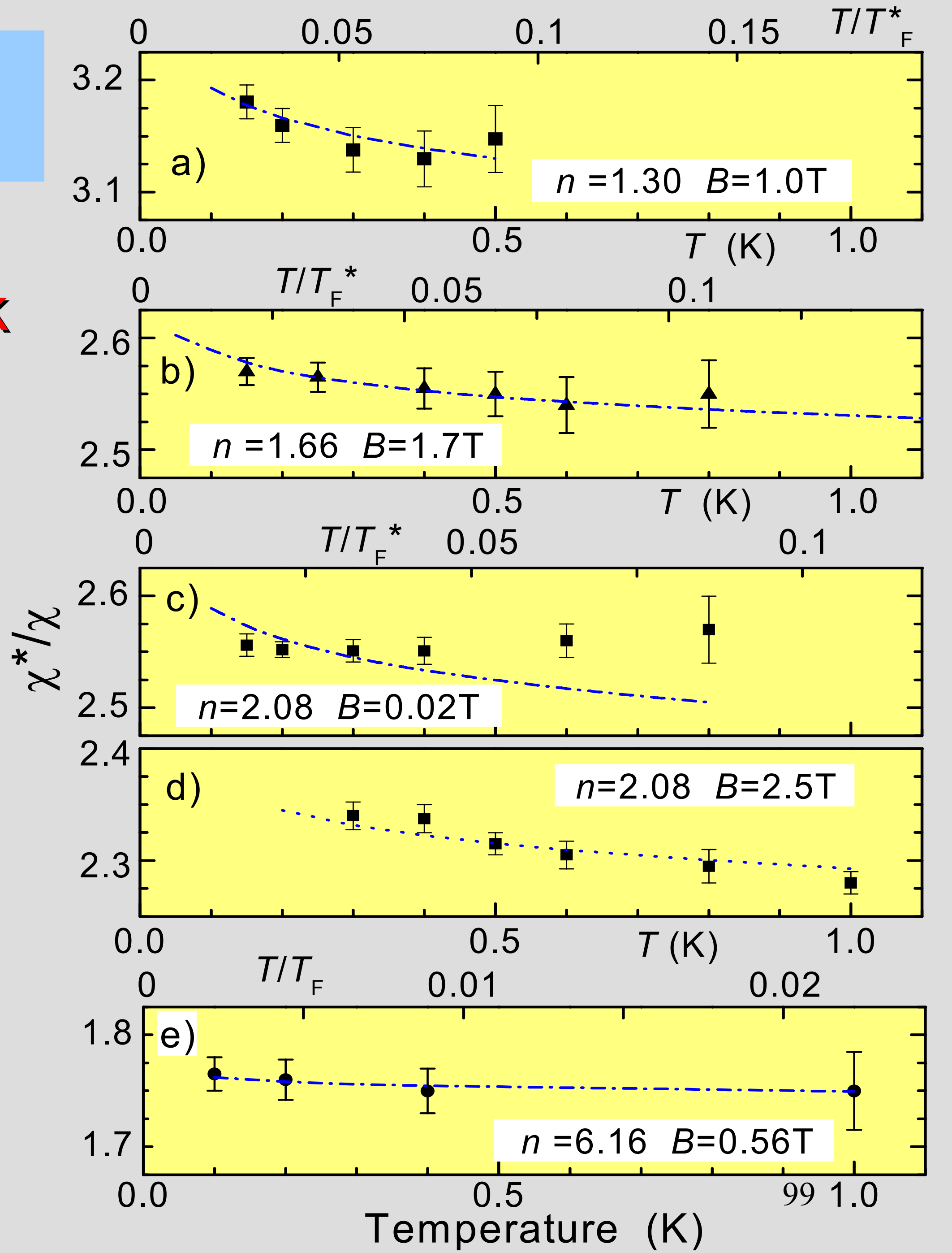
If χ* remains finite at finite *T, does it diverge as T→0*



Does χ^* diverge as T decreases ?

✓ $\chi(T)$-dependence is weak

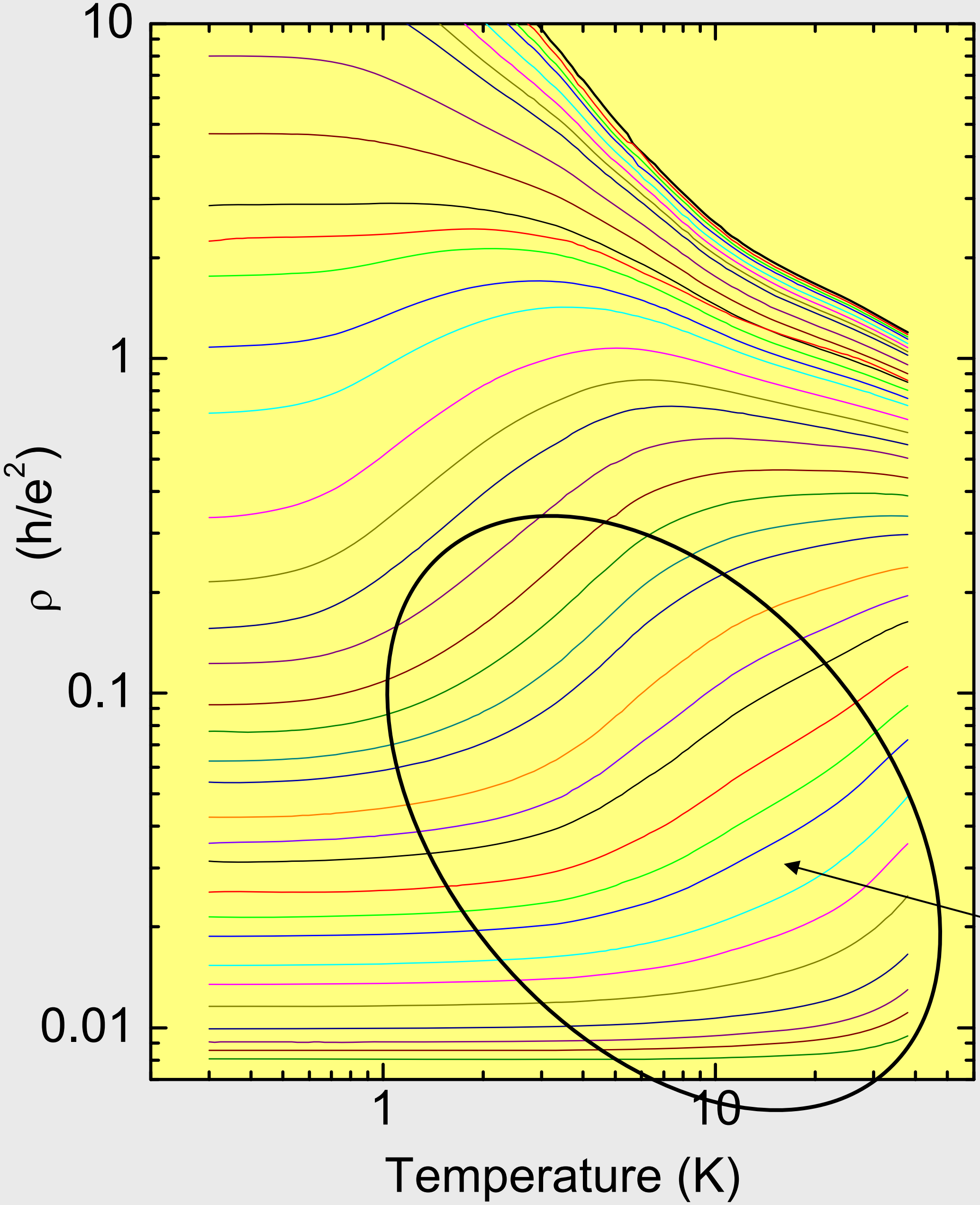
✓ $d\chi/dT > 0$ for $T\tau \ll 1$



10. *e-e* -Interaction Effects in 2D Transport

10.1. At high density, away from the M-I transition

ρ(*T*) in a wide *T*-range



high density

high conductivity,

ballistic regime

***Tτ* >>1**

Theory of quantum interaction corrections (1980....)

A.G.Aronov

Anatoly Larkin, 1932-2005

L.P.Gorkov

B.Altshuler

I.Aleiner

D.Khmelnitskii

Recent important progress

In theory (Zala, Narozhny, Aleiner, 2001): quantum corrections to σ calculated beyond the diffusive regime in terms of the τ and FL coupling constants

In experiment (Okamoto et al. 1999; Pudalov et al. 2002; Zhu et al 2003): FL-coupling constants F_0^a and F_1^s versus density determined from measurements of SdH oscillations

These advances allow to understand the metallic-like conduction in the regime $\rho \ll h/e^2$

Savchenko et al. *PRL* (2002); Kravchenko et al *PRB* (2002); Vitkalov et al. *PRB* (2003); VP et al. PRL (2003), etc.

Coherent e-e interaction (Zala, Narozhny, Aleiner, 2001)

Quantum corrections to the Drude conductivity :

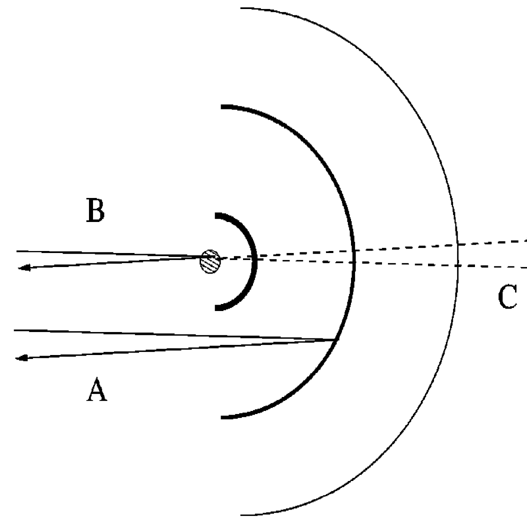
$$\sigma\left(T, B_{\parallel}\right) - \sigma_D = \delta\sigma_{\mathrm{C}} + 15\delta\sigma_{\mathrm{T}} + \delta\sigma_{\mathrm{loc}}\left(T\right).$$

Here (in units of e^2/h) :

$$\delta\sigma_{\mathrm{C}} \approx \left(T\tau\right)\left[1 - \frac{3}{8} f\left(T\tau\right)\right]$$ **- singlet**

$$\delta\sigma_{\mathrm{T}} \approx \frac{F_0^a}{1+F_0^a}\left(T\tau\right)\left[1 - \frac{3}{8} t\left(T\tau, F_0^a\right)\right]$$ **- triplet**

$$\delta\sigma_{\mathrm{loc}} = \frac{1}{2\pi}\ln\left(\tau/\tau_{\varphi}(T)\right)$$ **- weak localization**



Due to 2-valleys in (100)-Si, there are 4x4-1= 15 triplets. $d\sigma/dT$ becomes **< 0** for F_0^a **<0 !**

How it works:

(a) ballistic regime $T\tau >> 1$

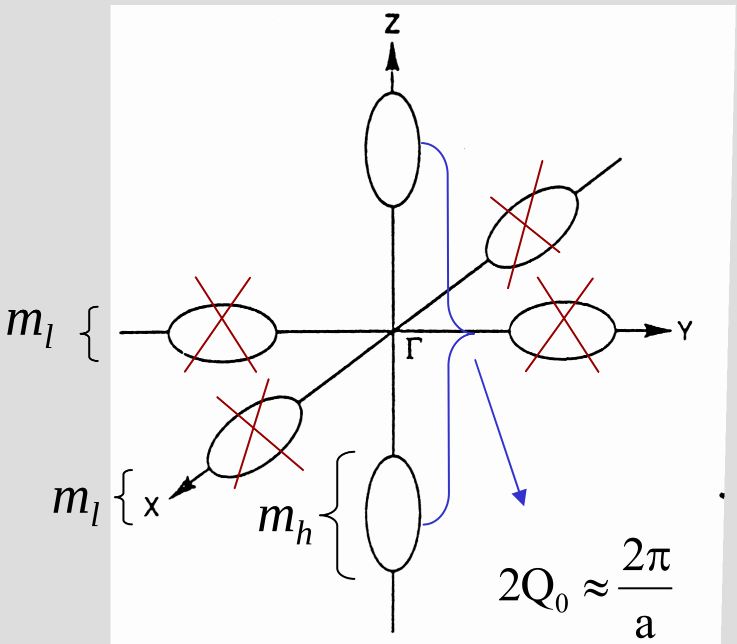
$\Delta\sigma \cong T\tau\,[1+ 15F_0^a/(1+ F_0^a)]$

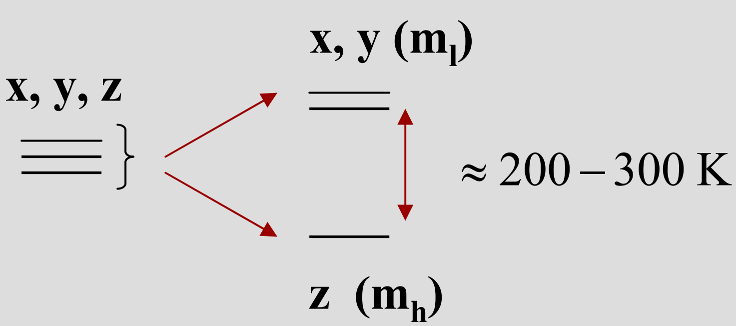
b) diffusive regime $\Delta_v < T << 1/\tau$

$\Delta\sigma \cong [1+1-15x(1-\ln(1+ F_0^a))/F_0^a]\,\ln T$

2DEG in Si-MOSFETs

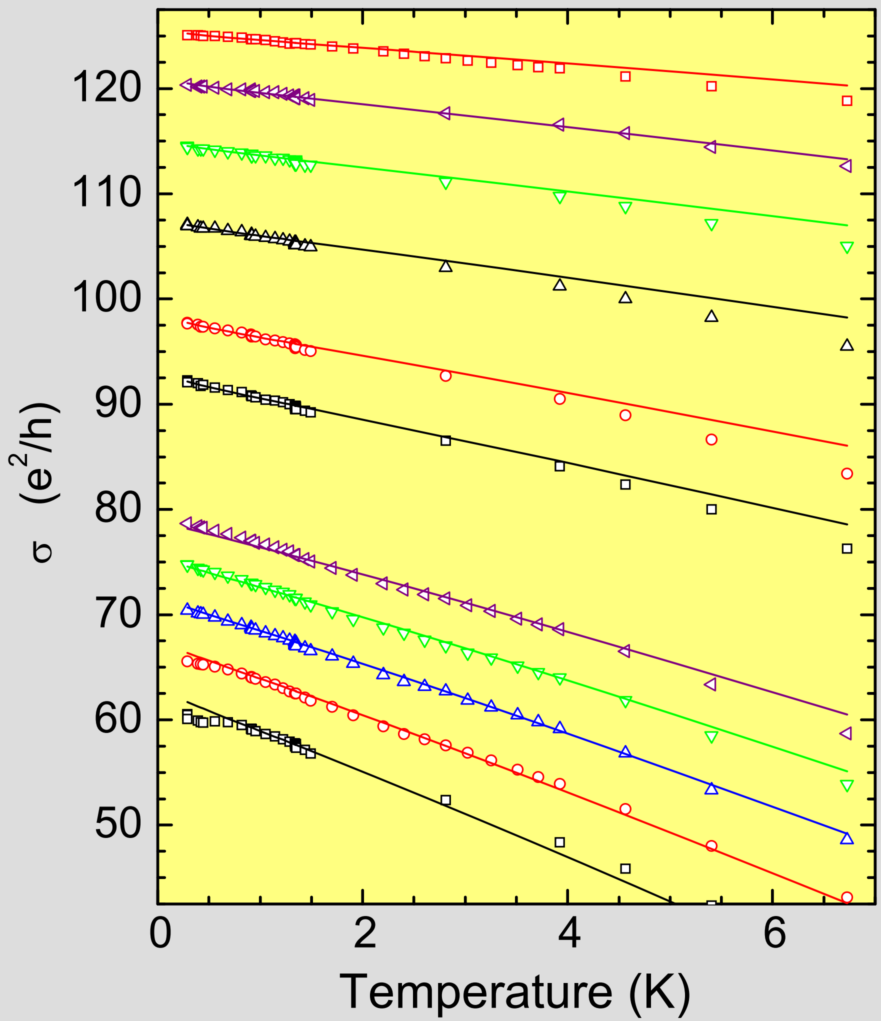
Lifting of the valley degeneracy in a (001) layer





Test of the measured g^* and m^* in the description of the transport (ballistic regime of interaction, $T\tau >> 1$)

Comparison with no adjustable parameters:



$$\delta\sigma \approx \left[1 + 15\left(\frac{F_0^a}{1+F_0^a}\right)\right]\frac{e^2}{\pi\hbar}T\tau$$

singlet **triplet**

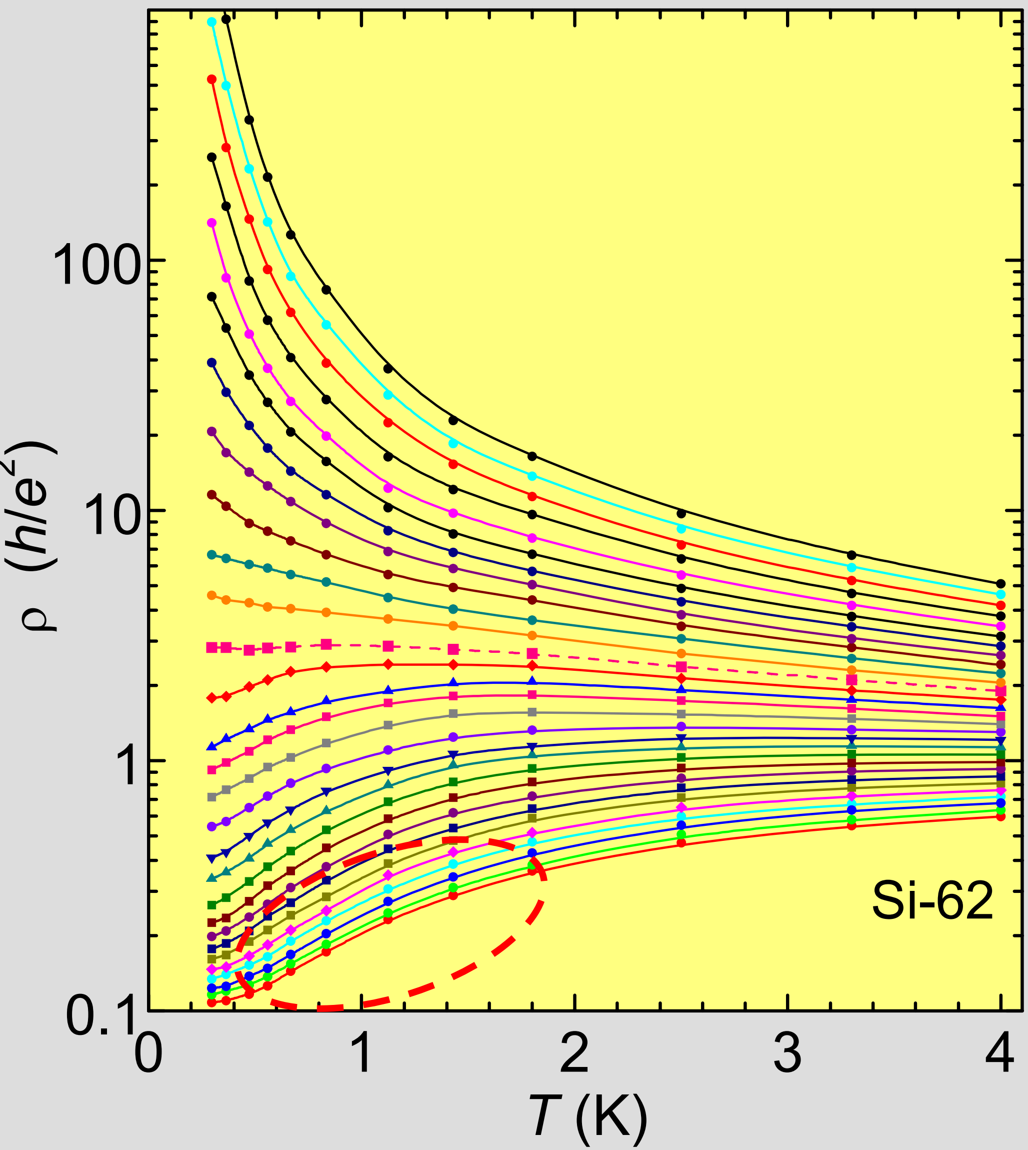
$(2n_v)^2-1$

$$-\left[15\left(1-\frac{\ln\left(1+F_0^a\right)}{F_0^a}\right)\frac{1}{2\pi}\ln\left(\frac{E_F}{k_B T}\right)\right]$$

V.Pudalov, M.Gershenson, H.Kojima, et al., *Phys.Rev.Lett.* (2003)

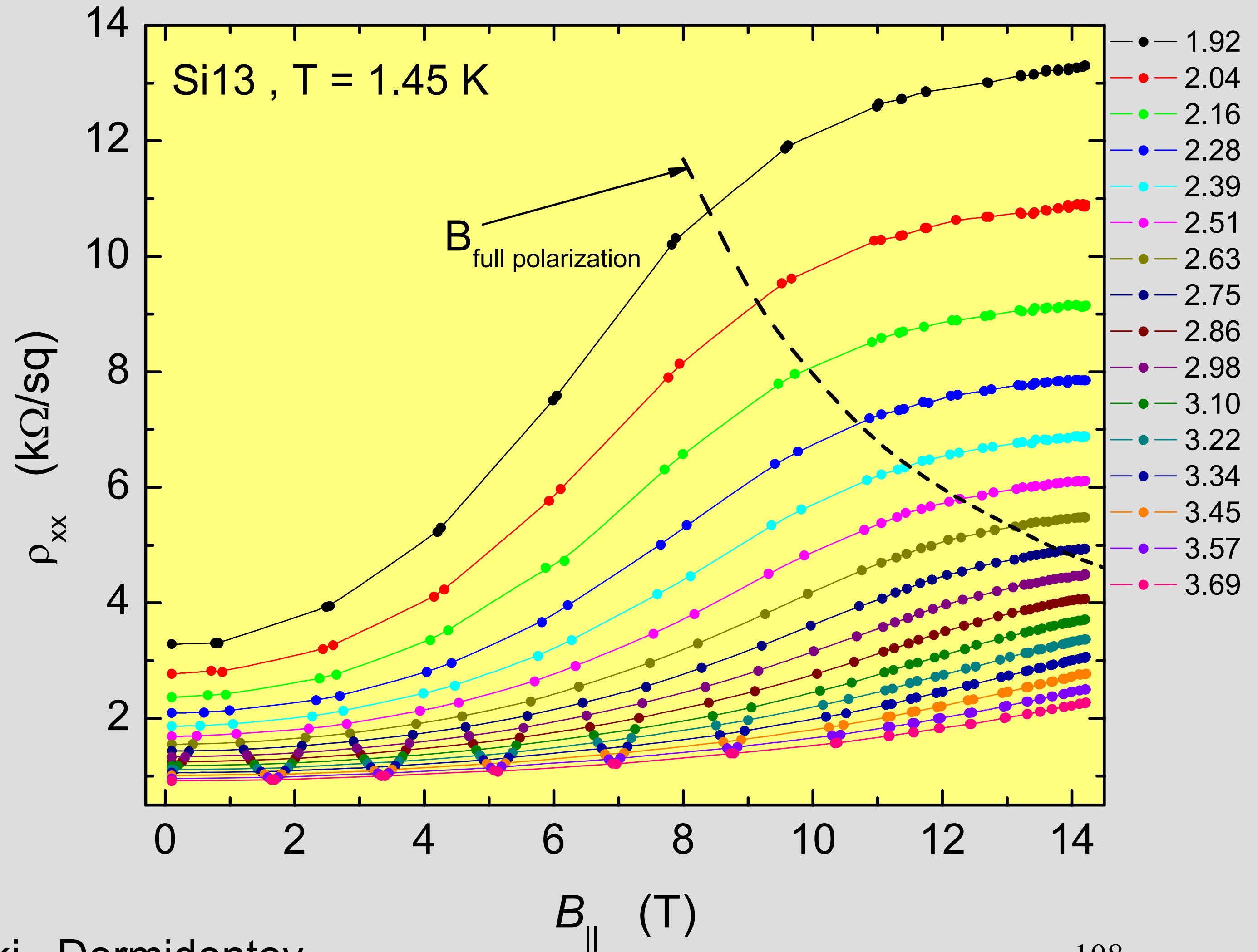
Conclusion:

at "high" T $(T\tau >> 1)$
the quasi-linear $\delta\sigma(T)$-growth is a consequence of the e-e interaction



V.P. et al. *JETP Lett.* (1998)

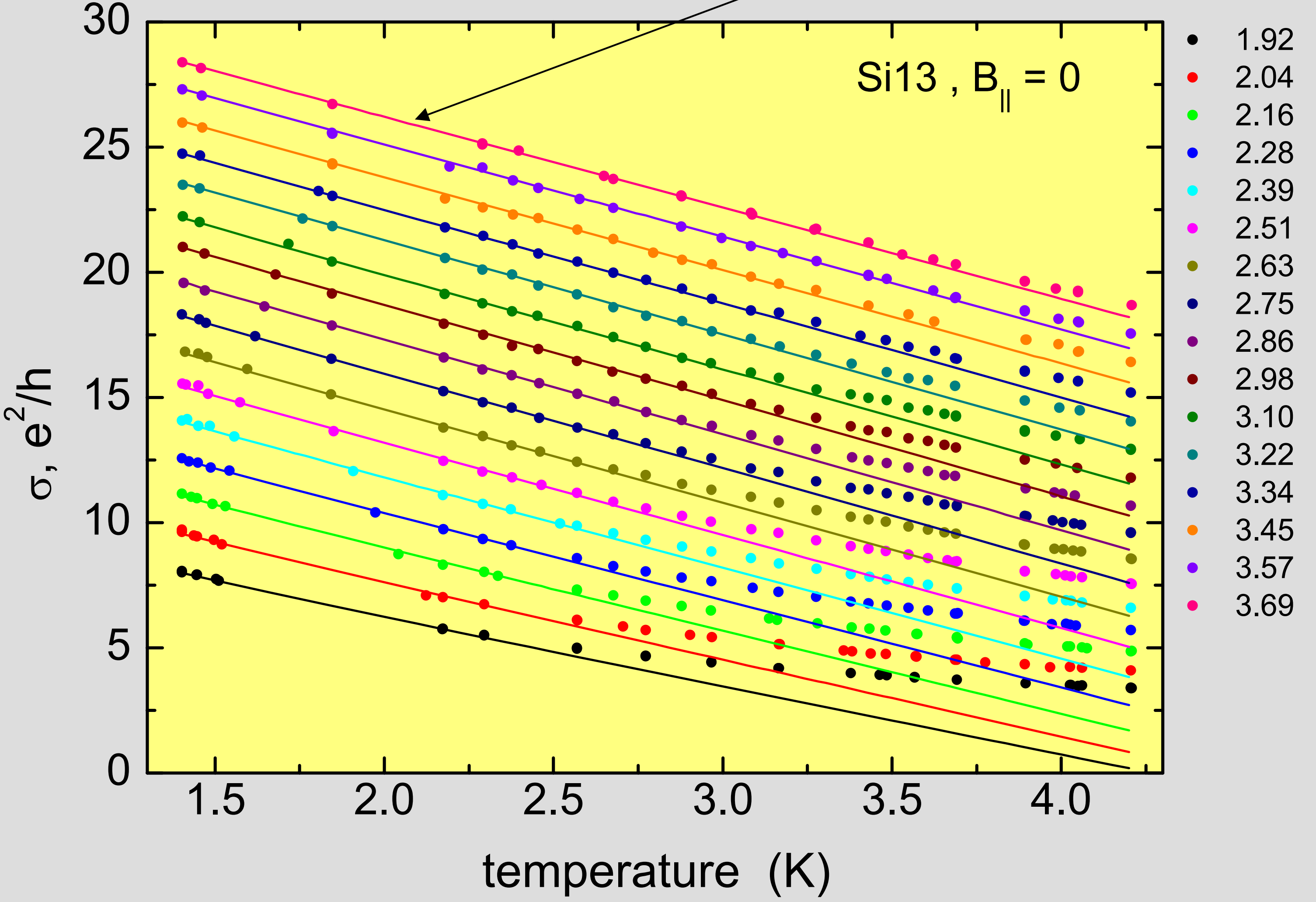
11. Fully spin-polarized liquid



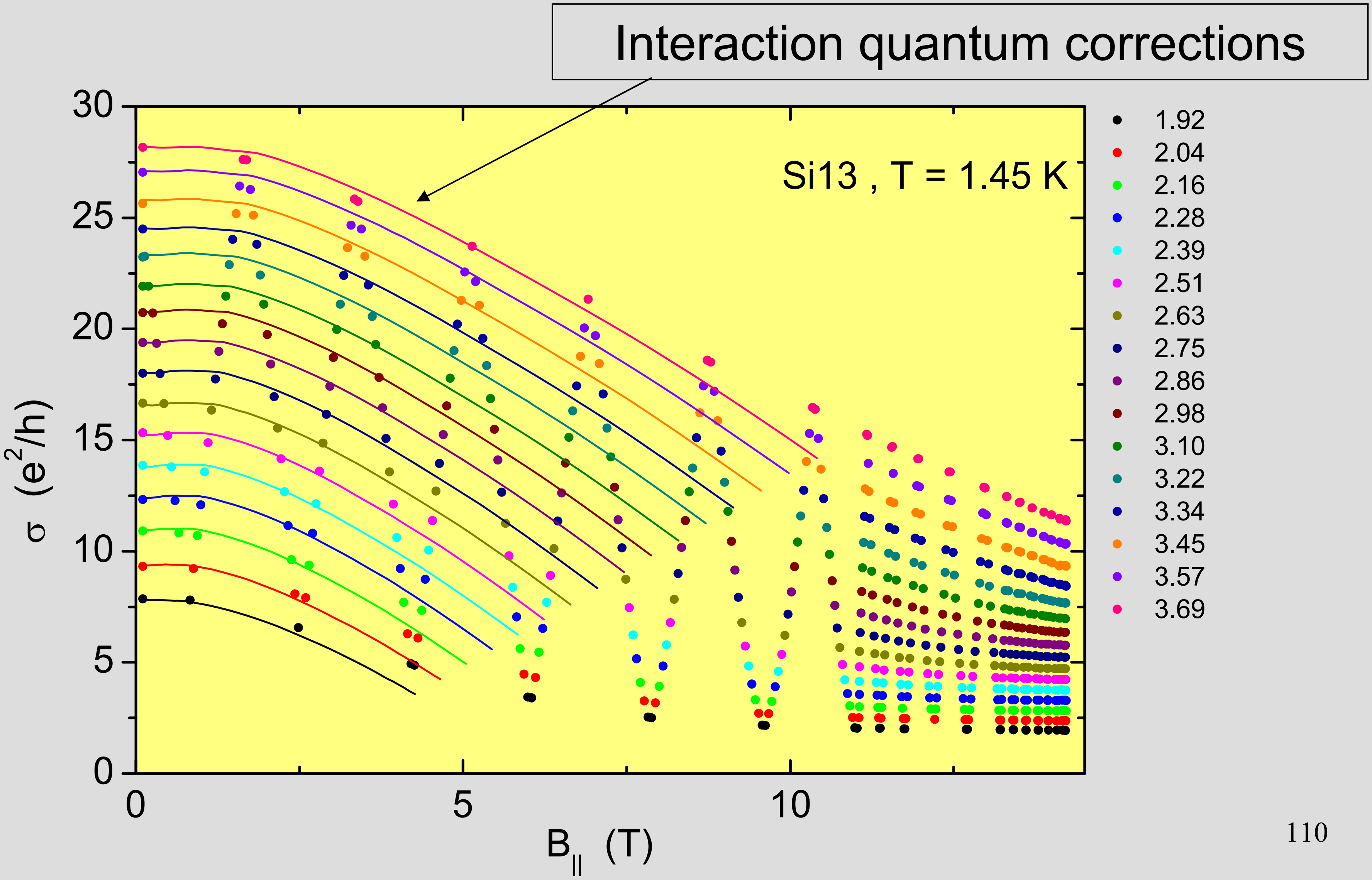
Knyazev, Omel'yanovski, Dormidontov, Pudalov JETP Lett. (2006)

B=0

$$\sigma \approx \sigma_D + T\tau\left(1 + 15\frac{F_0^a}{1+F_0^a}\right)$$



Magnetoresistance in $B_{||}$ field

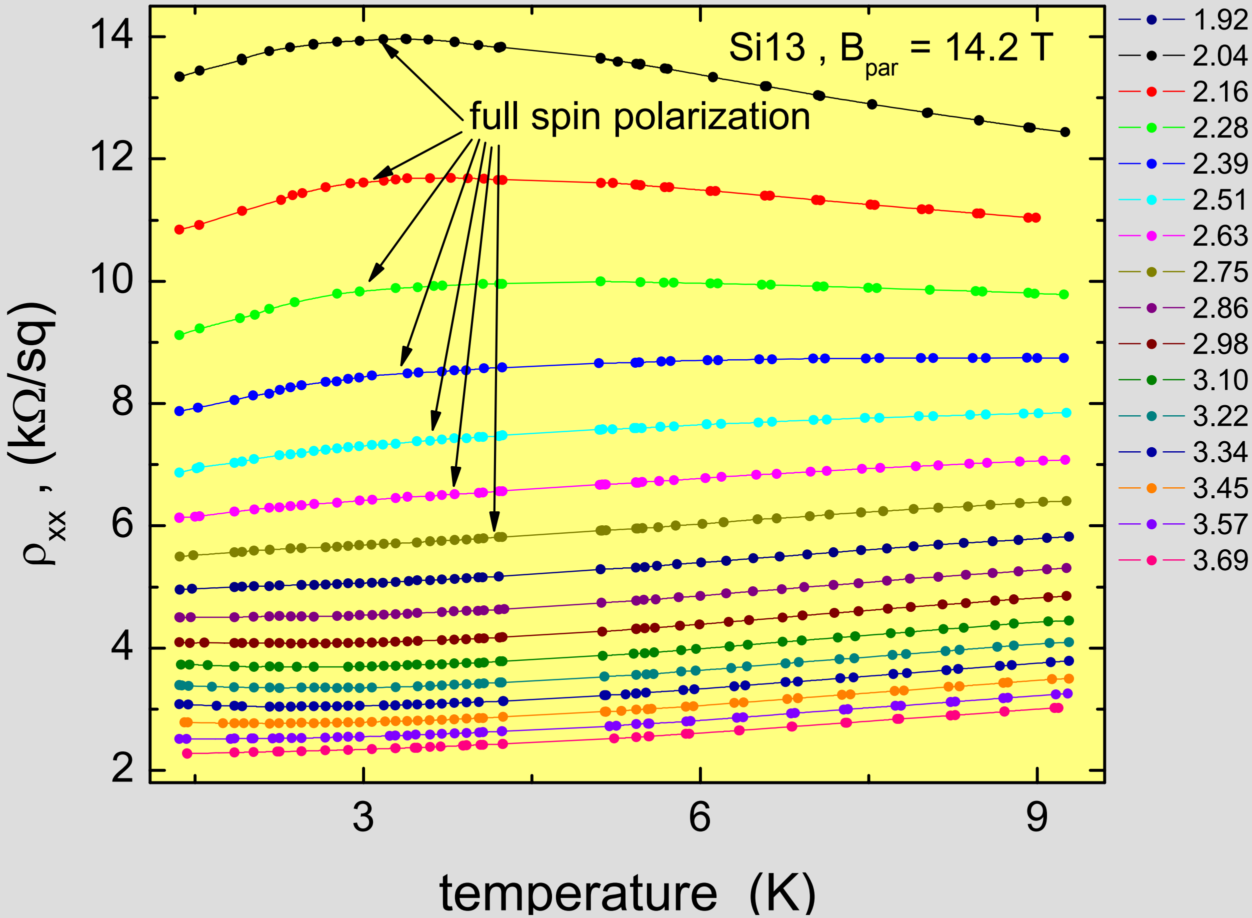


Full spin polarization

Theory predictions:

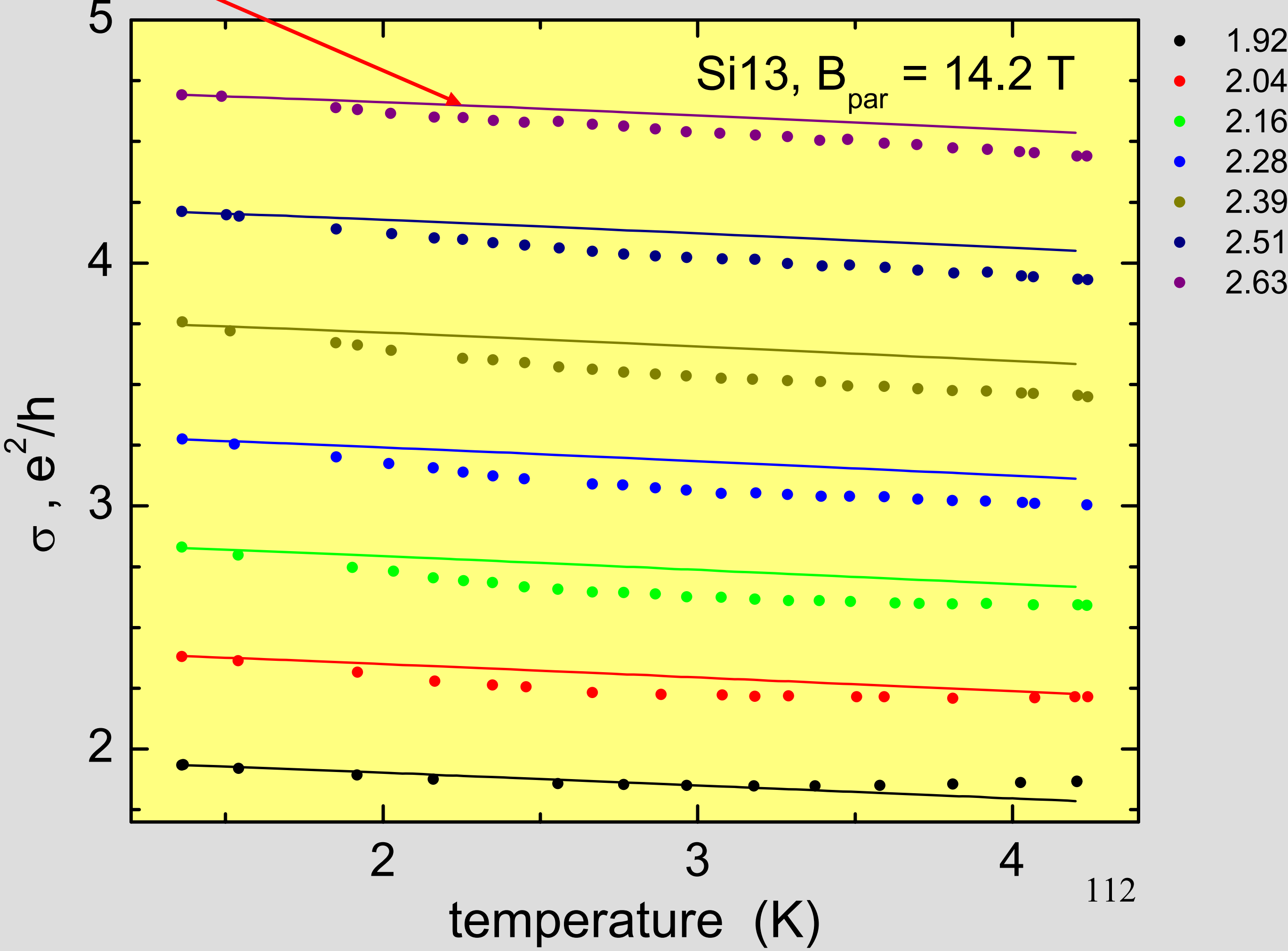
- Non FL, two phase model (Spivak, Kiveslon): dσ/dT=0
- Interaction quantum corrections

$$\frac{d\sigma}{\sigma\tau dT} \approx 1 + \frac{F_0^a}{1+F_0^a}$$



Comparison with quantum interaction corrections

$$\frac{d\sigma}{\sigma} \approx T\tau\left(1+3\frac{F_0^a}{1+F_0^a}\right)$$

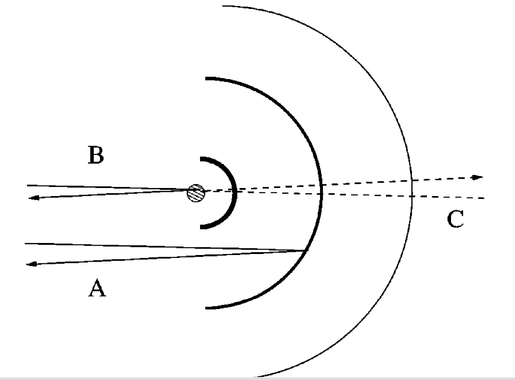


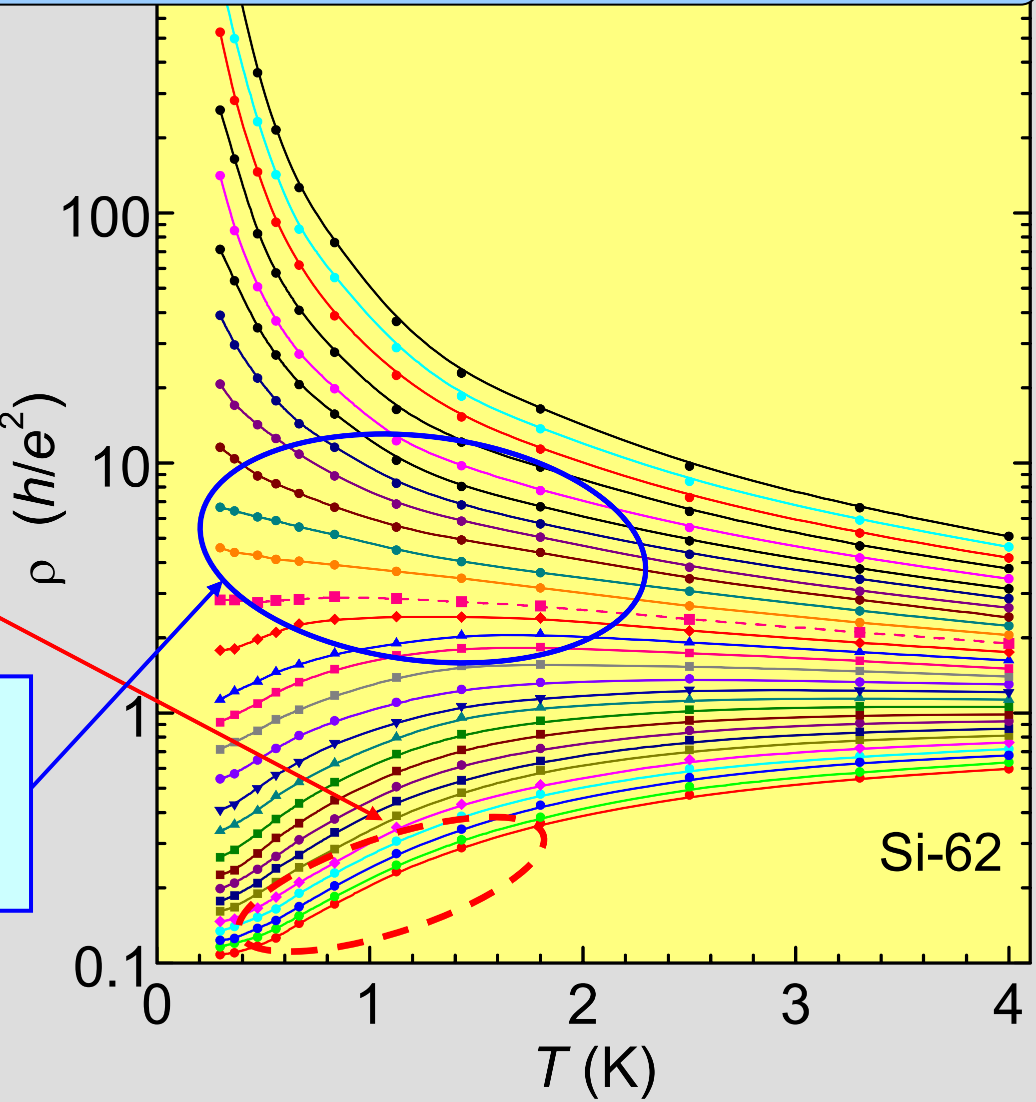
12.Critical regime: strong interactions, strong disorder

Success of the e-e interaction approach

in the high density/low disorder metallic regime

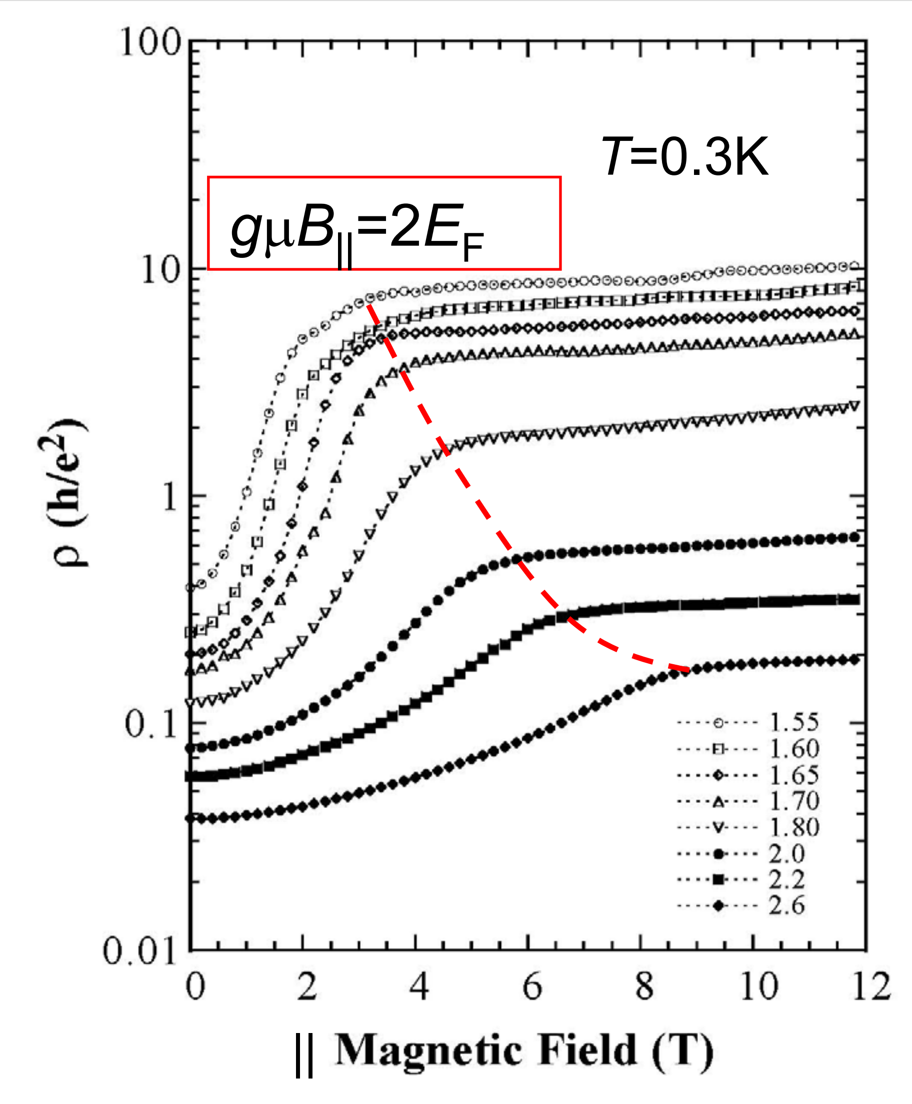
encourages one to apply the same ideas to the low-density & strong disorder regime



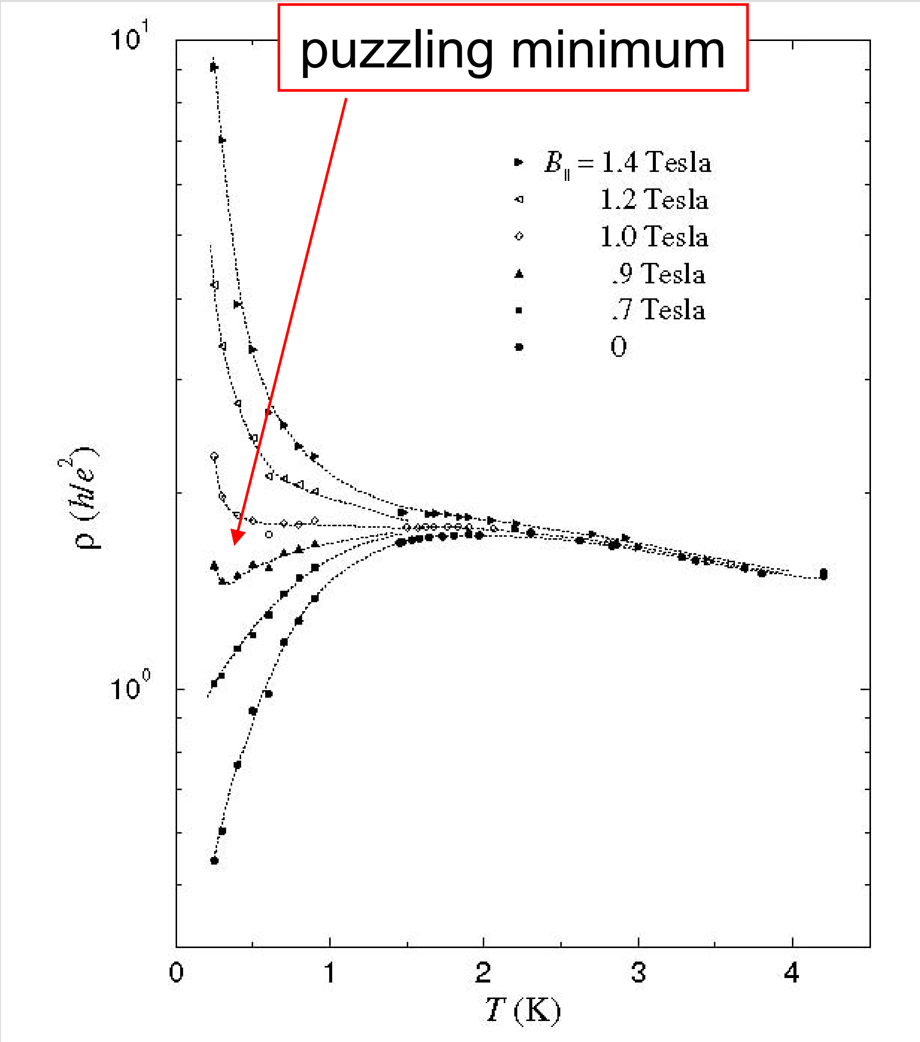


VP et al. *JETP Lett.* (1998)

In-plain magnetic field effect



VP, Brunthaler, et al. JETPL (1997)



D.Simonian, Kravchenko, VP et al. PRL (1997)

8.1. Two-parameter scaling

Finkelstein (1982), Castellani & Di Castro (84), Varma (98)

The main idea:

The length scale renormalizes both, disorder (resistance) and interaction.

The two-parameter scaling includes equations for both, disorder and interaction.

Interplay of disorder and interaction

RG-result in the 1-loop approximation

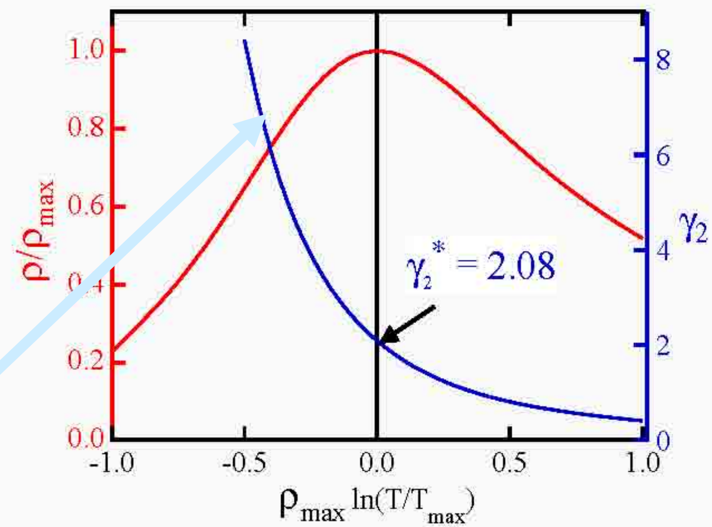
A. Finkel'stein (83)

finite ρ increases γ_2

while, γ_2 reduces ρ

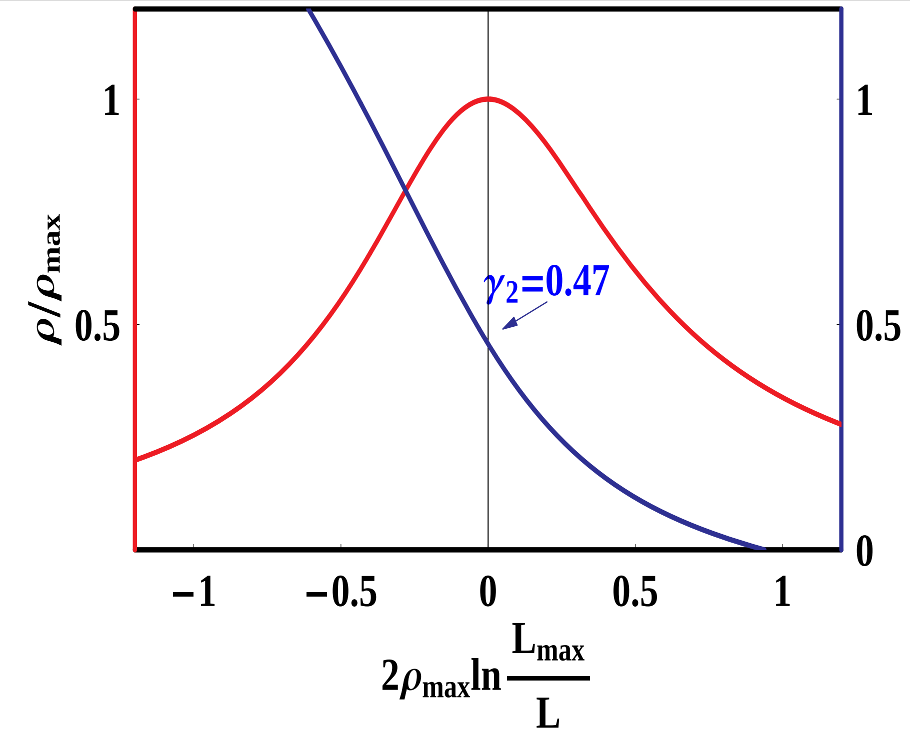
$$\gamma_2 = \frac{F_0^a}{1+F_0^a}$$

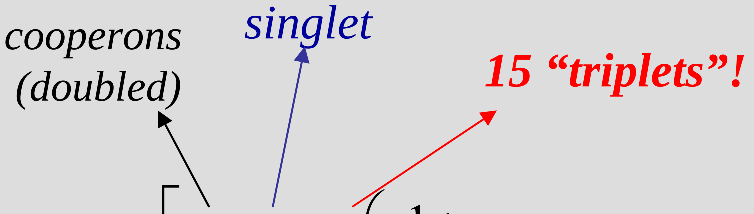
γ_2 **diverges at** $T^* \approx T_{\max}\, e^{-1}$



Magnetic transition is a precursor of MIT ?

One-loop RG analysis in the presence of two valleys





$$\frac{d\rho}{d\xi} = \rho^2\left[2+1-15\left(\left(\frac{1+\gamma_2}{\gamma_2}\right)\ln(1+\gamma_2)-1\right)\right]$$

$$\frac{d\gamma_2}{d\xi} = \rho\frac{(1+\gamma_2)^2}{2}$$

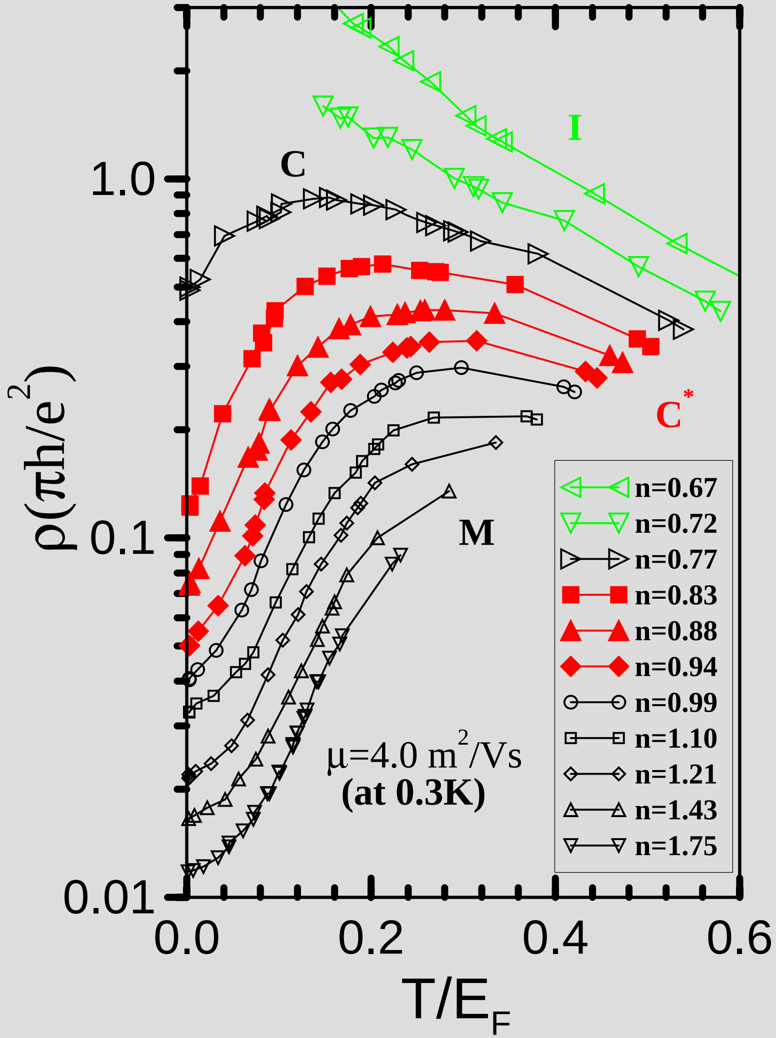
- $\gamma_2^*=0.45$ (<< 2.08 for one valley)

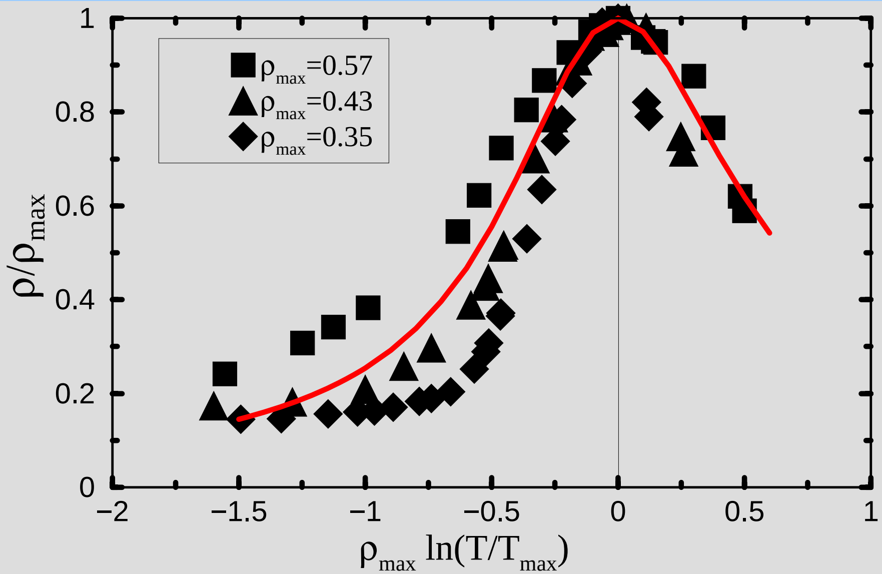
the strong antilocalization effect makes it much easier for two valleys to reach the stage where the resistance begins to decrease.

Notations: $\gamma_2\,(1+\gamma_2) = -F_0^a$

Punnoose, Finkelstein, PRL (2002)

Comparison with experimental data for a high-mobility sample





• **no adjustable parameters** are used

VP et al., (1998)

A. Punnoose and A. Finkelstein, *PRL (2002)*

Character of the e-e interaction

The experiments provide an evidence that electrons do interact in the wide energy interval $\delta E >> kT$

Bosonic character of the e-e interactions !

Pis'ma v ZhETF, vol. 76, iss. 9, pp. 660 – 664 © 2002 November 10

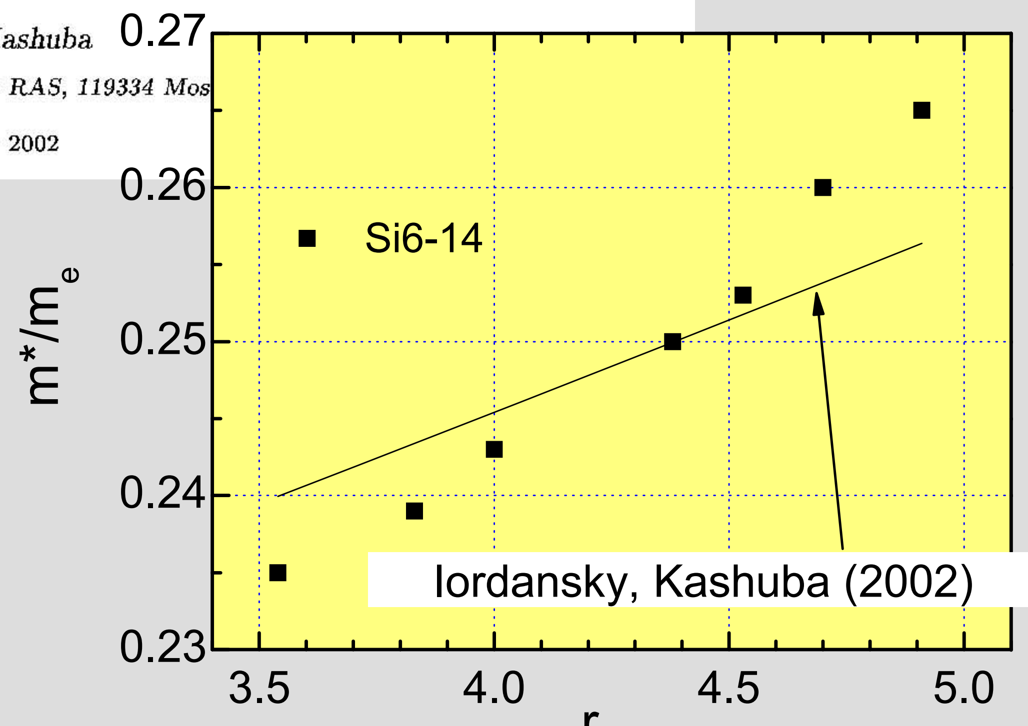
Multicomponent dense electron gas as a model of Si MOSFET

S. V. Iordanski, A. Kashuba

L. D. Landau Institute for Theoretical Physics RAS, 119334 Mos

Submitted 2 October 2002

$N_v = \infty$ model



Interplay of the disorder and interaction

RG-result in 2-loop approximation

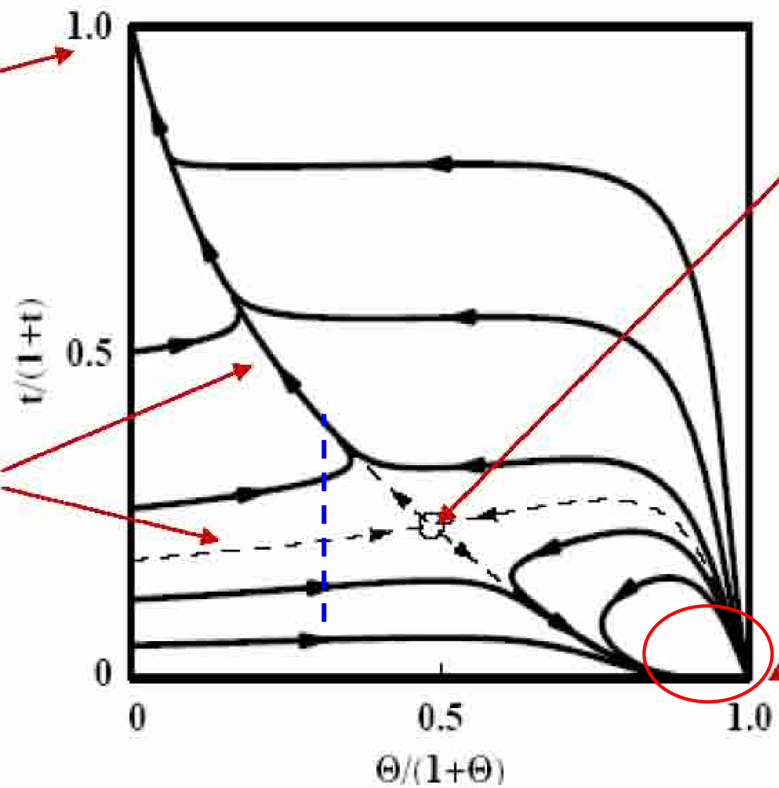
ρ increases γ_2

γ_2 decreases ρ

Finkelstein, Punnoose, *Science* (2005)

Insulator

$t \to \infty,\ \Theta \to 0$



unstable MIT fixed point

$t \to 0.3, \Theta \to 0.9 \quad (\alpha = 1)$

when the cooperon channel is included.

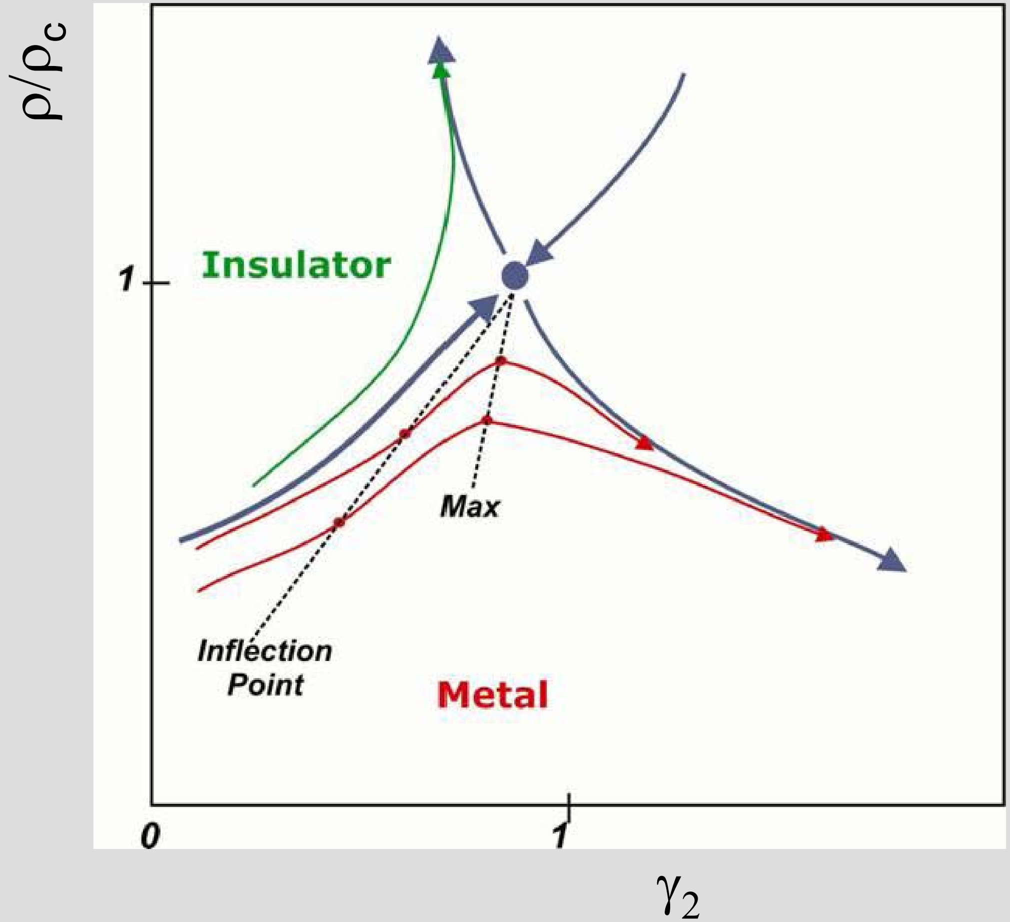
separatrices

metal

Good: Magnetic transition is missing in the *e*-liuqid,

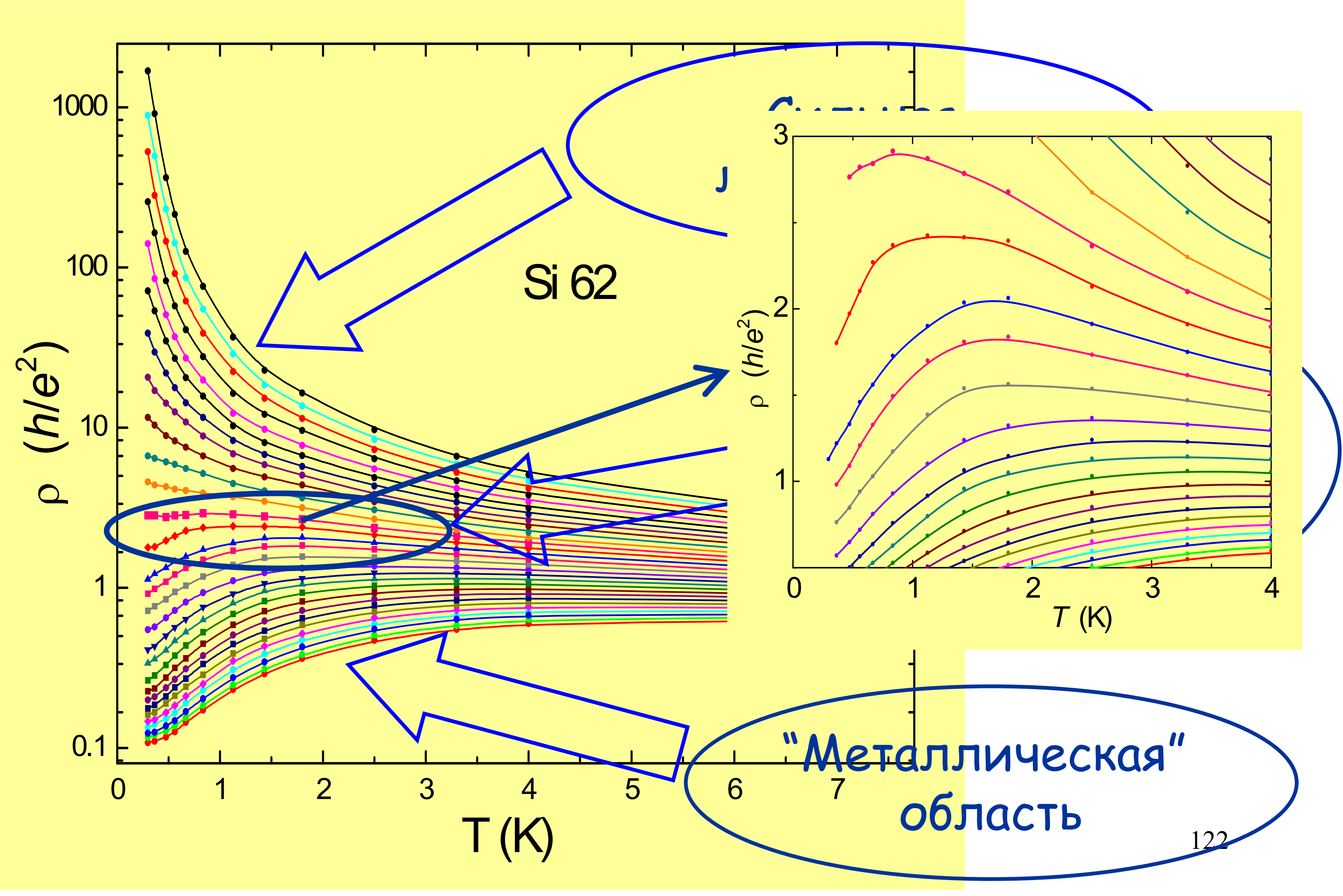
RG results

Two-loop, infinite n_v



ρ_{xx} maxima is important feature; $T_{max} \longrightarrow 0$ as $n \longrightarrow n_c$

Metal-Insulator transition: **studies in the critical regime**

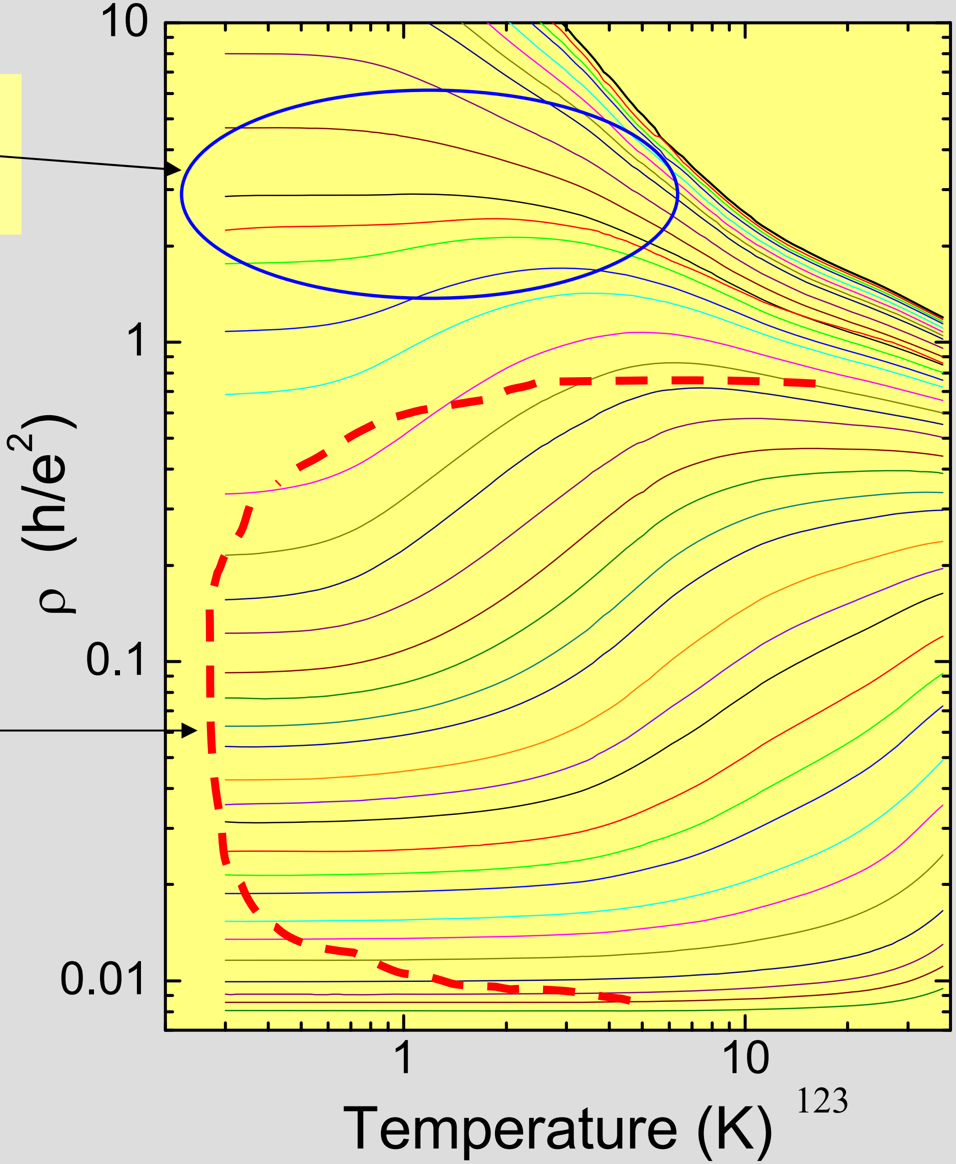


A big surprize: diffusive interaction borderline rises to 10K !

Critical regime falls into the diffusive interaction regime

In metallic regime:

$$T^* = \frac{\hbar}{\tau}\frac{(1+F_0^\sigma)}{2\pi}$$



Точное решение при $B = 0$

В первом порядке по ρ для n_v=2 функции $\beta_\sigma(\sigma,\gamma_2)$ и $\beta_{\gamma_2}(\sigma,\gamma_2)$ известны, и система RG уравнений решается численно:

$$\rho(T) = \frac{1}{\sigma(T)}$$

$$\gamma_2(T)$$

$$\gamma_2 = -\frac{F_0^\sigma}{1+F_0^\sigma}$$

Удобные граничные условия:

$$\rho(T_{\max}) = \rho_{\max}$$

где (T_{max}, ρ_{max}) - точка максимума на кривой $\rho(T)$

Условия применимости:

1. $\rho < \pi \frac{h}{e^2}$

2. Достаточно высокие T

1998-2002 –
Сравнение с теорией только $\rho(T)$.

А что с $\gamma_2(T)$?

Теория: случай $B_{\|} \neq 0$

$$\begin{cases} \dfrac{d\sigma}{d\eta} = \beta_\sigma(\sigma,\gamma_2) + f_\sigma(\sigma,\gamma_2,B_{\|}^2) \\ \dfrac{d\gamma_2}{d\eta} = \beta_{\gamma_2}(\sigma,\gamma_2) + f_{\gamma_2}(\sigma,\gamma_2,B_{\|}^2) \end{cases}$$

Burmistrov, Chtchelkachev, JETP Lett. (2006)

Решение в первом порядке по $B_{\|}^2$

$$\frac{\rho(T,B_{\|})}{\rho(B_{\|})_{\max}} = \frac{\rho(T,0)}{\rho_{\max}} + \nu(\gamma_2)\left(\frac{g\mu_B B_{\|}}{kT}\right)^2 \qquad \frac{g\mu B_{\|}}{kT} < 1$$

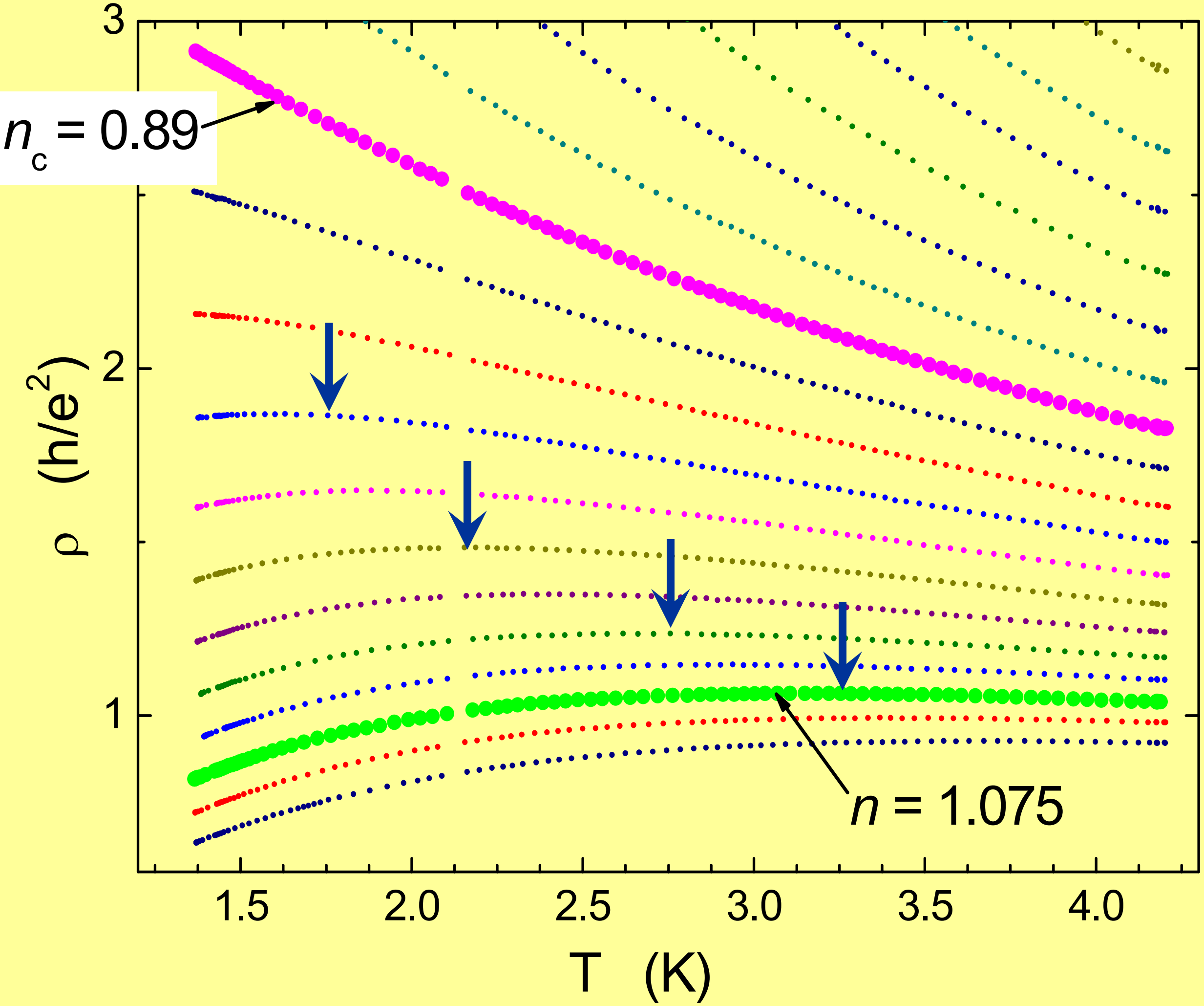
Измерение $\rho(T,B_{\|})$ в слабых магнитных полях ⇨ $\gamma_2(T)$

Проверка теоретической $\gamma_2(T)$!

Anissimova, Kravchenko, Punnoose, Finkel'stein, Klapwijk, cond-mat/0609181

Knyazev, Omel'yanovsky, VP, Burmistrov, JETP Lett. (2006).

Критическая область: $n > n_c$

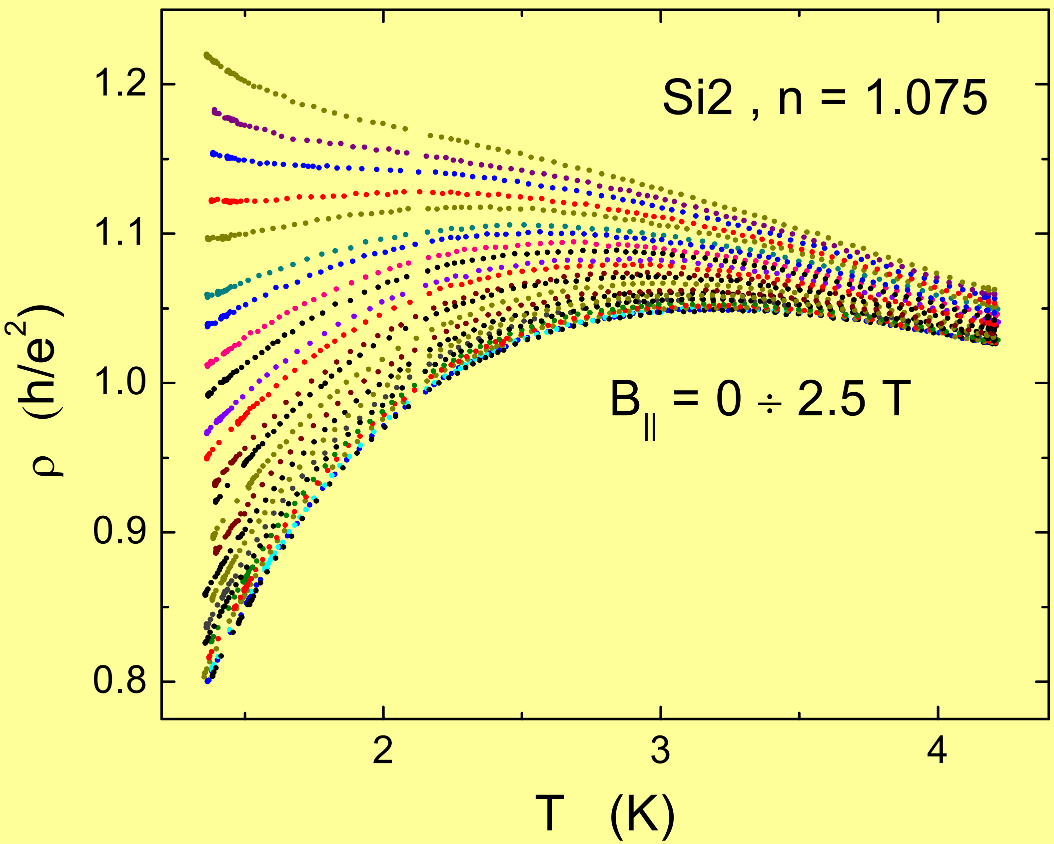


$T = (1.4 - 4.2)$K, подвижность μ = 30,000cm²/Vs при $T = 1.4$K

стрелки - положения $\rho_{max}(T_{max})$ на кривых

При T = 4.2K и n = 1.075 $T\tau = 0.3$ диффузный режим !!!!

Магнитосопротивление в крит. области



Приложение магнитного поля движет 2D систему в изолятор

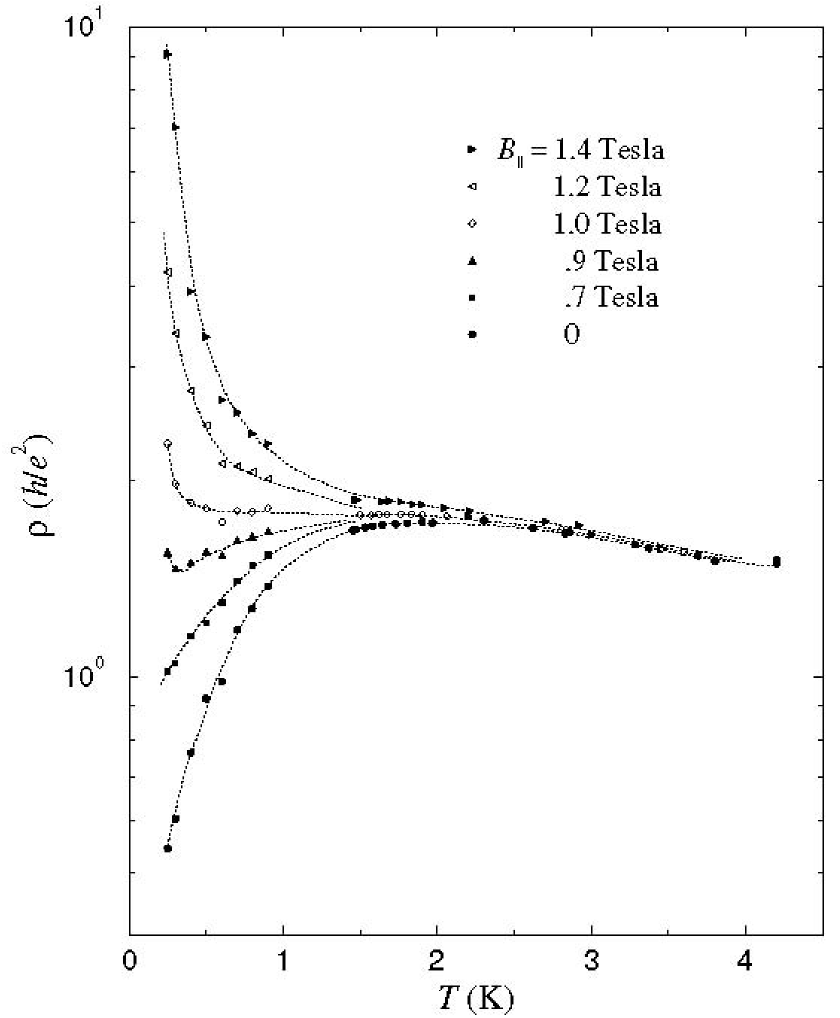
Вся область (ρ, T) - диффузный режим взаимодействия $T\tau < 1$

T_{max} смещается влево с ростом магнитного поля.

Максимум исчезает при:

$$g\mu_B B_{||} \sim kT_{max}$$

Puzzling reentrant ρ(B||) dependence



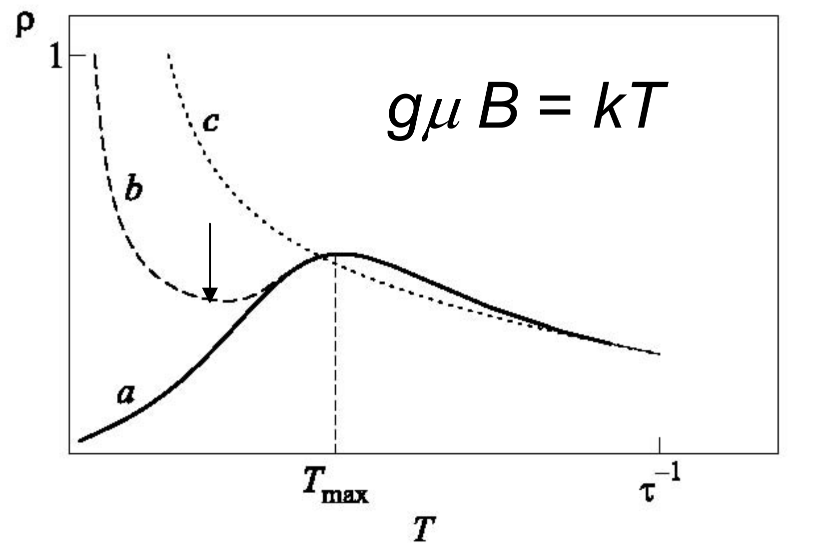
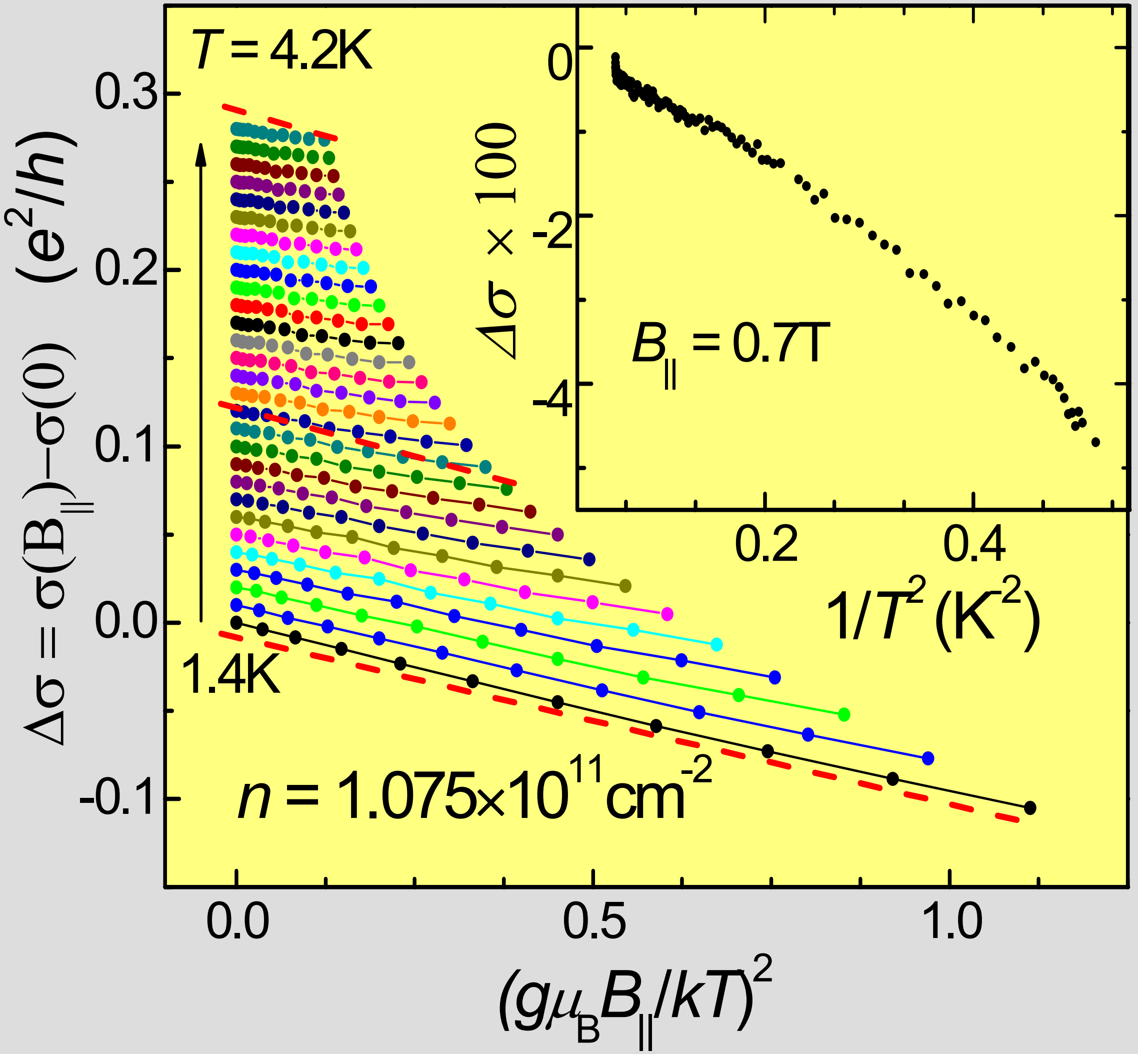


Fig.1. The sketch of the temperature dependence of the resistance in the presence of $B_{||}$. The curve *a* corresponds to the case $B_{||} = 0$, the curve *b* to the case $g_L\mu_B B_{||} \ll T_{\max}$ and the curve *c* to the case $g_L\mu_B B_{||} \gg T_{\max}$

Burmistrov, Chtchelkachev, JETP Lett. (2006)

Температурная зависимость $\gamma_2(T)$



Для баллистического режима $T\tau > 1$:

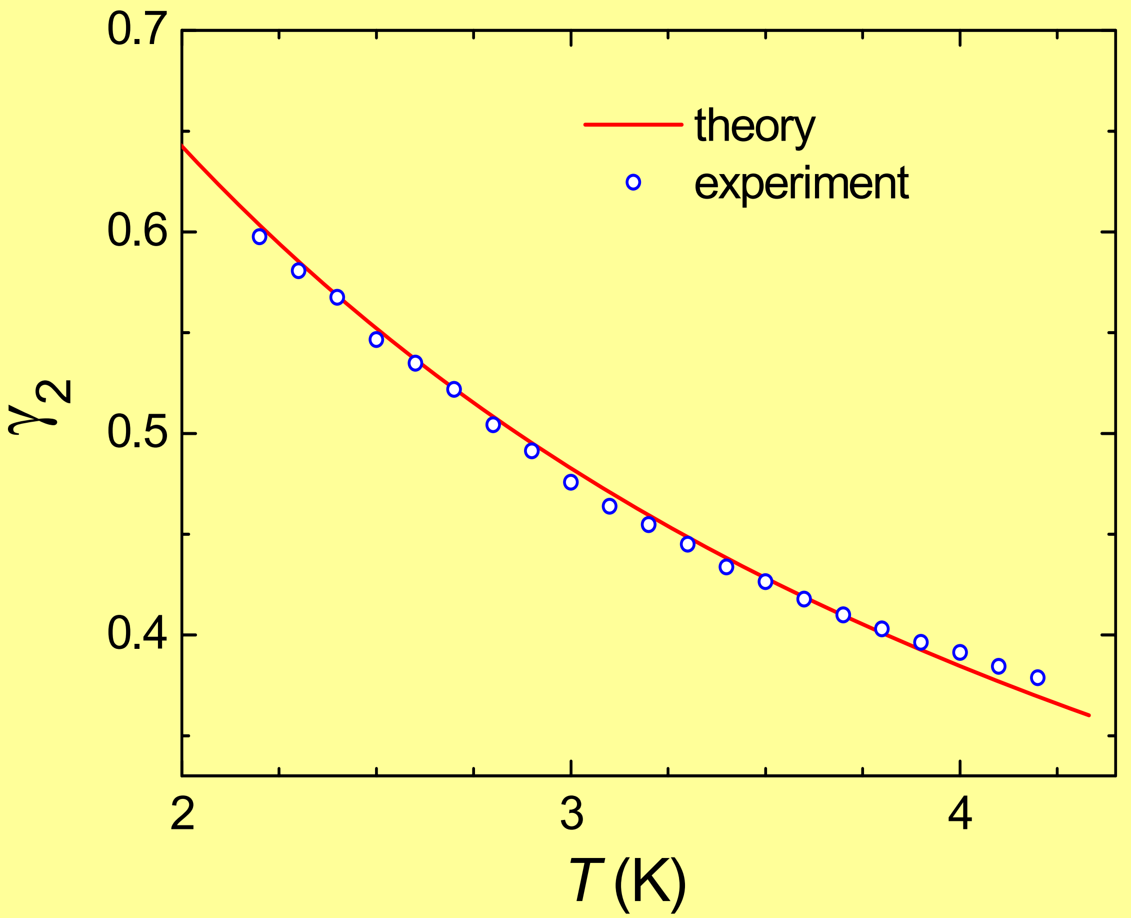
$$\Delta\sigma \propto \frac{\tau}{\hbar}\frac{(g\,\mu_B B_{\|})^2}{kT}$$

Для диффузного режима $T\tau < 1$:

$$\Delta\sigma \propto z^2 = \left(\frac{g\,\mu_B B_{\|}}{kT}\right)^2$$

Уменьшение наклонов $d\sigma / dz$ с ростом $T \Rightarrow$ наличие **температурной зависимости** $\gamma_2(T)$!

γ_2(T) – сравнение с теорией

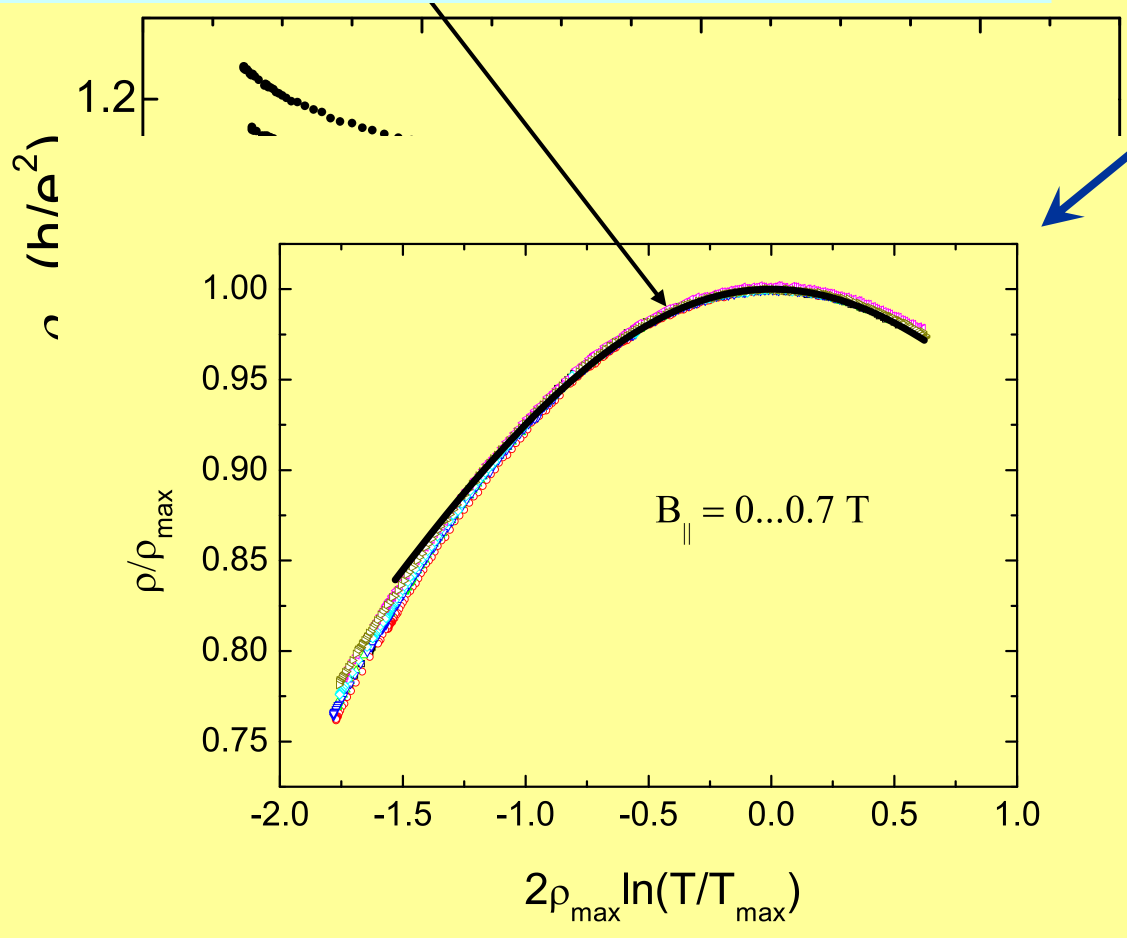


Количественное согласие теории (Finkelstein, B=0) и эксперимента

Но:
при $T < 2$K
необходимо решать RG уравнения во 2-м порядке по ρ

Магнитосопротивление в критической области: эксперимент и теория

Finkelstein equations solution for B=0



The main effect in low fields – shift of (T_{max}, ρ_{max}) with field.

1. Для $B_{\parallel} < 0.7\text{T}$ – скейлинг кривых $\rho(T,B=0)$ в координатах ρ/ρ_{max}, $\rho_{max}\ln(T/T_{max})$

2. Для $B_{\parallel} < 1.5\text{T}$ – количественное согласие

3. При $B_{\parallel} > 1.5\text{T}$ – Согласие только качественное: расхождение теории и эксперимента ~ 30%

Conclusions

✓Analysis of the $\rho(T, B_{||})$ data shows that $\gamma_2(T)$ grows strongly as T decreases, in accord with the 2-parameter RG theory

✓Both, $\rho(T)$ and $\gamma_2(T)$ agree with the 2-parameter RG theory

✓$\rho(T)$ data obey the 2-parameter scaling in the vicinity of n_c

✓Generic $\rho_c(T)$ "separatrix" **is a power-law function,** the result that resolves much of the earlier discrepancies

The agreement with RG theory and the two-parameter scaling support the interpretation of the 2D MIT as a QPT

Both, interaction and disorder are equally relevant at the MIT

Current understanding of the 2D system

- "Metallic" conduction observed in 2D systems at $\sigma >> e^2/h$ - is a result of the e-e interaction

- Interplay of the disorder and e-e interaction fundamentally changes a common belief : "2DE system must always become insulating at sufficiently low T "

- 2D electron liquid remains paramagnetic down to the M-I transition.

- In theory, the 2D metal exists always for n_v=2 (for n_v=1 – at sufficiently large γ_2), and M-I-transition is a quantum phase transition

- Theory needs a detailed experimental test.
- Theory needs 2-loop result for a finite number of valleys

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Mitya Maslov
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