Нанофизика низких температур, Черноголовка, 20-30 августа 2007 г.

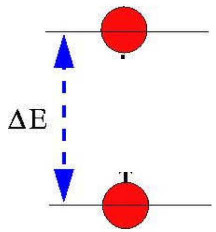
Сверхпроводниковые квантовые биты

(теория)

Юрий Махлин

ИТФ им. Л.Д.Ландау

Продольный шум => *чистая* дефазировка



$$H = -\frac{1}{2}(\Delta E + X)\sigma_z + H_{bath}$$

X – классическое или квантовое

$$\rho_{01} \propto \left\langle \exp\left(-\frac{i}{\hbar}\int_0^t X(\tau)d\tau\right)\right\rangle = \exp\left(-\frac{1}{2\hbar^2}\int_0^t d\tau_1 \int_0^t d\tau_2 \langle X(\tau_1)X(\tau_2)\rangle\right) =$$

$$= \exp\left(-\frac{1}{2h^2}\int \frac{d\omega}{2\pi} S_X(\omega)\frac{\sin^2(\omega t/2)}{(\omega/2)^2}\right)$$

$$\frac{\sin^2(\omega t/2)}{(\omega/2)^2} \approx 2\pi\delta(\omega)t$$

$$\approx \exp\left(-\frac{1}{2h^2}S_X(\omega \approx 0)\cdot t\right) = \exp\left(-\frac{t}{T_2^*}\right)$$

для гладкого спектра

$$S_X(\omega) = \frac{E_{1/f}^2}{|\omega|} \rightarrow \infty \quad \text{for } \omega \rightarrow 0$$

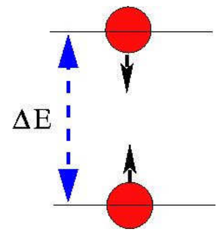
для 1/f шума

$$= \exp\left(-\frac{E_{1/f}^2}{2\pi}t^2 \ln|\omega_{ir} t|\right)$$

e.g., Cottet et al. 01

Поперечный шум => релаксация

$$H = -\frac{1}{2}\Delta E \sigma_z - \frac{1}{2} X \sigma_x + H_{Bath}$$



золотое правило:

$$\Gamma_\uparrow = \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_i \left|\langle i | X | f \rangle\right|^2 \delta\left(E_i + \Delta E - E_f\right)$$

$$= \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_i \langle i | X | f \rangle \langle f | X | i \rangle \frac{1}{2\pi\hbar}\int dt \exp\left[i\left(E_i + \Delta E - E_f\right)t/\hbar\right]$$

$$= \frac{1}{4\hbar^2}\int dt \sum_i \rho_i \langle i | X(t) X(0) | i \rangle \exp\left[i\Delta E t/\hbar\right]$$

$$= \frac{1}{4\hbar^2}\langle X(t) X(0) \rangle_{\omega = \Delta E/\hbar}$$

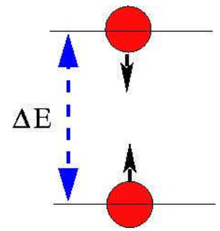
$$\Gamma_\downarrow = \frac{1}{4\hbar^2}\langle X(t) X(0) \rangle_{\omega = -\Delta E/\hbar}$$

$$\frac{1}{T_1} \equiv \Gamma_{\rm rel} = \Gamma_\uparrow + \Gamma_\downarrow = \frac{1}{2\hbar^2} S_X(\omega = \Delta E/\hbar)$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_2^*}$$

Bloch equations, applicability

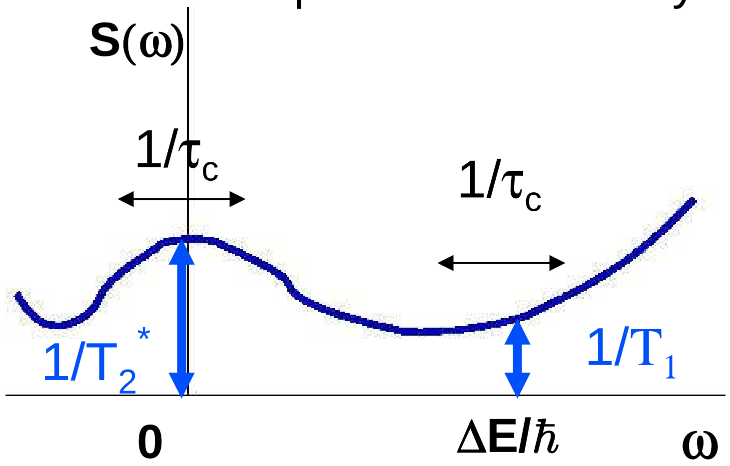
$$H = -\frac{1}{2}\Delta E\,\sigma_z \;-\frac{1}{2}X\cos\eta\,\sigma_z - \frac{1}{2}X\sin\eta\,\sigma_x + H_{bath}$$



$$\frac{d}{dt}\mathbf{S} = \mathbf{B}\times\mathbf{S} - \frac{1}{T_1}(S_z - S_z^{eq})\hat{z} - \frac{1}{T_2}(S_x\hat{x} + S_y\hat{y})$$

Bloch (46,57)
Redfield (57)

perturbation theory



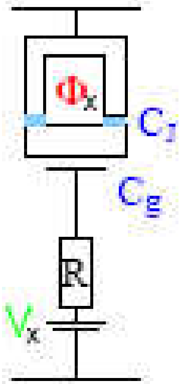
works for $\tau_c \ll T_1, T_2$

weak short-correlated noise

Dissipation by Ohmic control circuit

- gate voltage circuit is dissipative
  characterized by $Z(\omega) = R$
  induces voltage fluctuations

$$S_V(\omega) = \hbar\omega R \coth\left(\frac{\hbar\omega}{2k_\mathrm{B}T}\right) \approx 2k_\mathrm{B}RT\,|_{\hbar\omega \ll k_\mathrm{B}T}$$

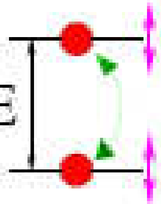


$$\mathcal{H} = \mathcal{H}_0 + \frac{C_g}{C_J}\; e\,\delta V(t)\,\hat\sigma_z + \mathcal{H}_\mathrm{bath}$$

$$= \mathcal{H}_0 + \frac{C_g}{C_J}\; e\,\delta V(t)\,($$

$$\tan\eta = \frac{E_J(\Phi_x)}{\Delta E_{ch}(V_x)}$$

- $R \ll h/e^2 \approx 26\mathrm{k}\Omega$ e.g. $R = 100\Omega$
  and $C_g \ll C_J$ (weak coupling to environment) $\Rightarrow T_1, T_2 \approx 10^{-6} - 10^{-4}\mathrm{s}$
- operation time $\tau_\mathrm{op} \approx \hbar/E_J \approx 10^{-10}\mathrm{s} \Rightarrow$ ratio $\geq 10^4$



$\delta V \;\cos\eta$

fluctuatio

eigenenergies

between eigenstates

$$\Gamma_\mathrm{rel} = 4\pi\frac{R}{h/e^2}\left(\frac{C_g}{C_J}\right)^2 \sin^2\eta\,\frac{\Delta E}{\hbar}\,\coth\left(\frac{\Delta E}{2k_\mathrm{B}T}\right)$$

$$\Gamma_\varphi = \frac{1}{2}\Gamma_\mathrm{rel} + 4\pi\frac{R}{h/e^2}\left(\frac{C_g}{C_J}\right)^2 \cos^2\eta\,\frac{k_\mathrm{B}T}{\hbar}$$

Распад рабиевских колебаний

$$H = -\frac{1}{2}B_z\,\sigma_z - \frac{1}{2}\Omega_R\,(\cos\omega t\;\sigma_x + \sin\omega t\;\sigma_y) - \frac{1}{2}X\,\sigma_z + H_{\mathrm{bath}}$$

No driving ($\Omega_R = 0$)

$$H = -\frac{1}{2}B_z\,\sigma_z - \frac{1}{2}X\;\sigma_z + H_{\mathrm{bath}} \quad ; \qquad \text{purely longitudinal}$$

$$\Rightarrow \quad 1/T_1 = 0\;,\; 1/T_2 = \frac{1}{2\hbar^2}\,S_X(\omega \approx 0)$$

With driving $\Omega_R \neq 0$, in the rotating frame

$$\tilde{H} = \dot{U}U + U\,HU^{\dagger} \quad ; \quad U = \exp\left(-i\omega\frac{\sigma_z}{2}t\right)$$

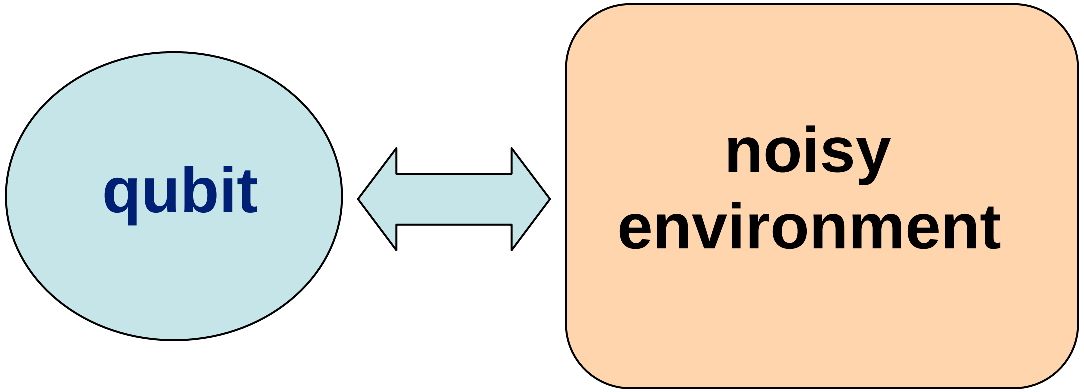
$$H = -\frac{1}{2}\Omega_R\,\sigma_x - \frac{1}{2}X\;\sigma_z + H_{\mathrm{bath}} \quad ; \qquad \text{purely transverse}$$

$$1/T_1 = \tfrac{1}{2\hbar^2}\,S_X(\omega = \Omega_R) \qquad\qquad 1/T_2 = 1/(2T_1)$$

In general noise at $\omega = 0, B_z, \pm\Omega_R, B_z \pm \Omega_R$ may be involved

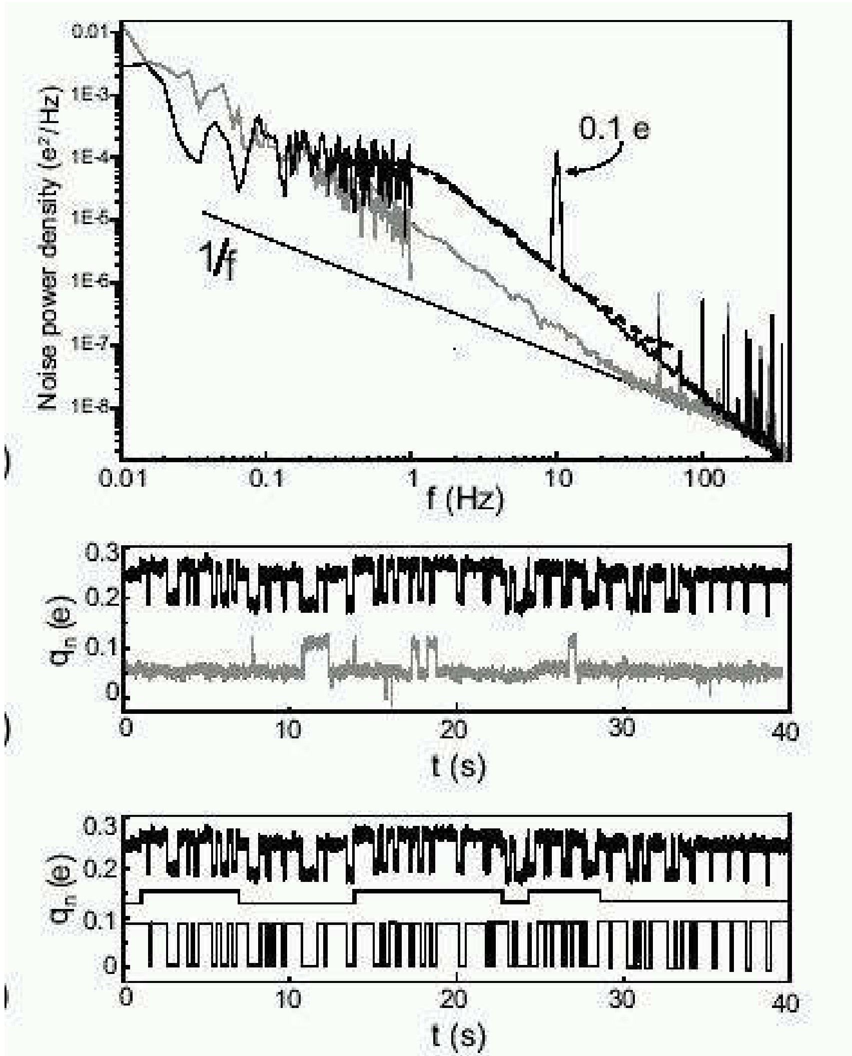
Note: T_1 and T_2 are modified by driving

Qubits and environment



- Decoherence induced by noise
- Qubits as spectrometers

Nanoelectronic circuits and 1/f noise

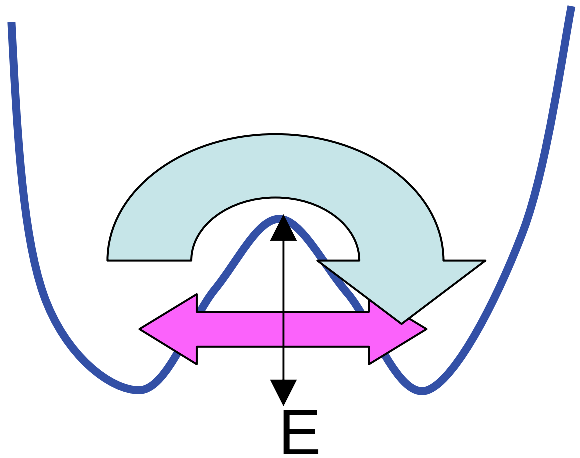


Bouchiat et al. ´97

Charge noise:

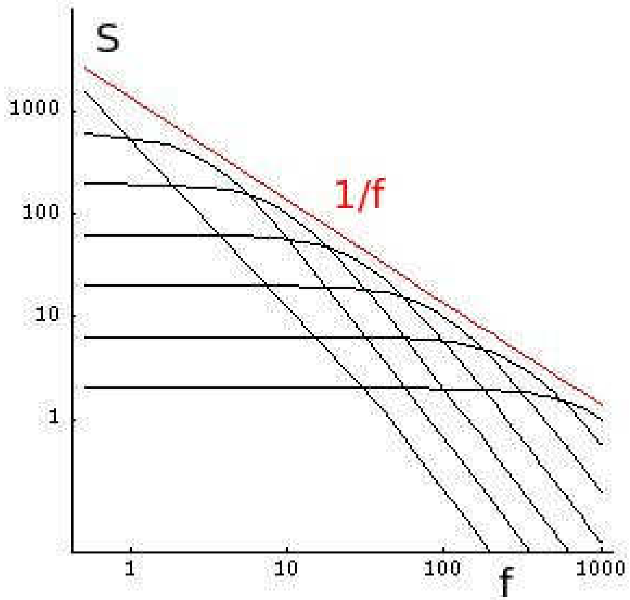
- 1/f noise
- individual Lorentzians – bistable fluctuators
- T-dependence saturated at low T$\lesssim$ 300 mK

1/f noise from bistable fluctuators



$\Gamma \propto e^{-E/kT}$ or $\Gamma \propto e^{-E/h\omega_0}$

$$dw = g(E)dE \propto \begin{Bmatrix} h\omega_0 \\ kT \end{Bmatrix} \frac{d\Gamma}{\Gamma}$$



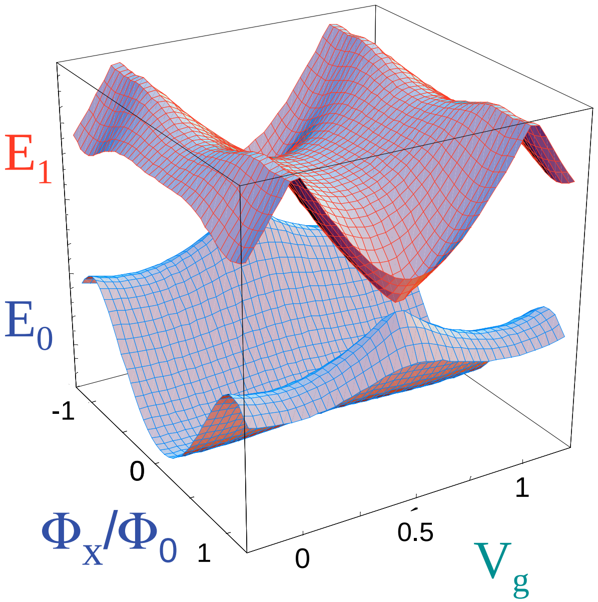
$$S(\omega) \propto \int \frac{d\Gamma}{\Gamma} \frac{\Gamma}{\omega^2 + \Gamma^2} \propto \frac{1}{\omega}$$

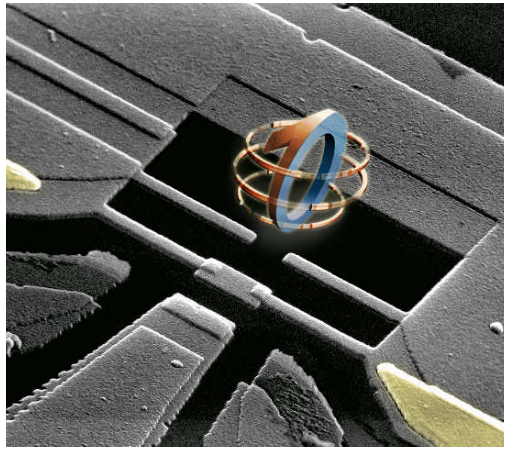
McWhorter, 1958; Dutta, Horn, RMP'81

Charge-phase qubit

$E_C \approx E_J$

$$H = -\frac{1}{2}E_{ch}(V_g)\,\sigma_z - \frac{1}{2}E_J(\Phi_x)\,\sigma_x$$





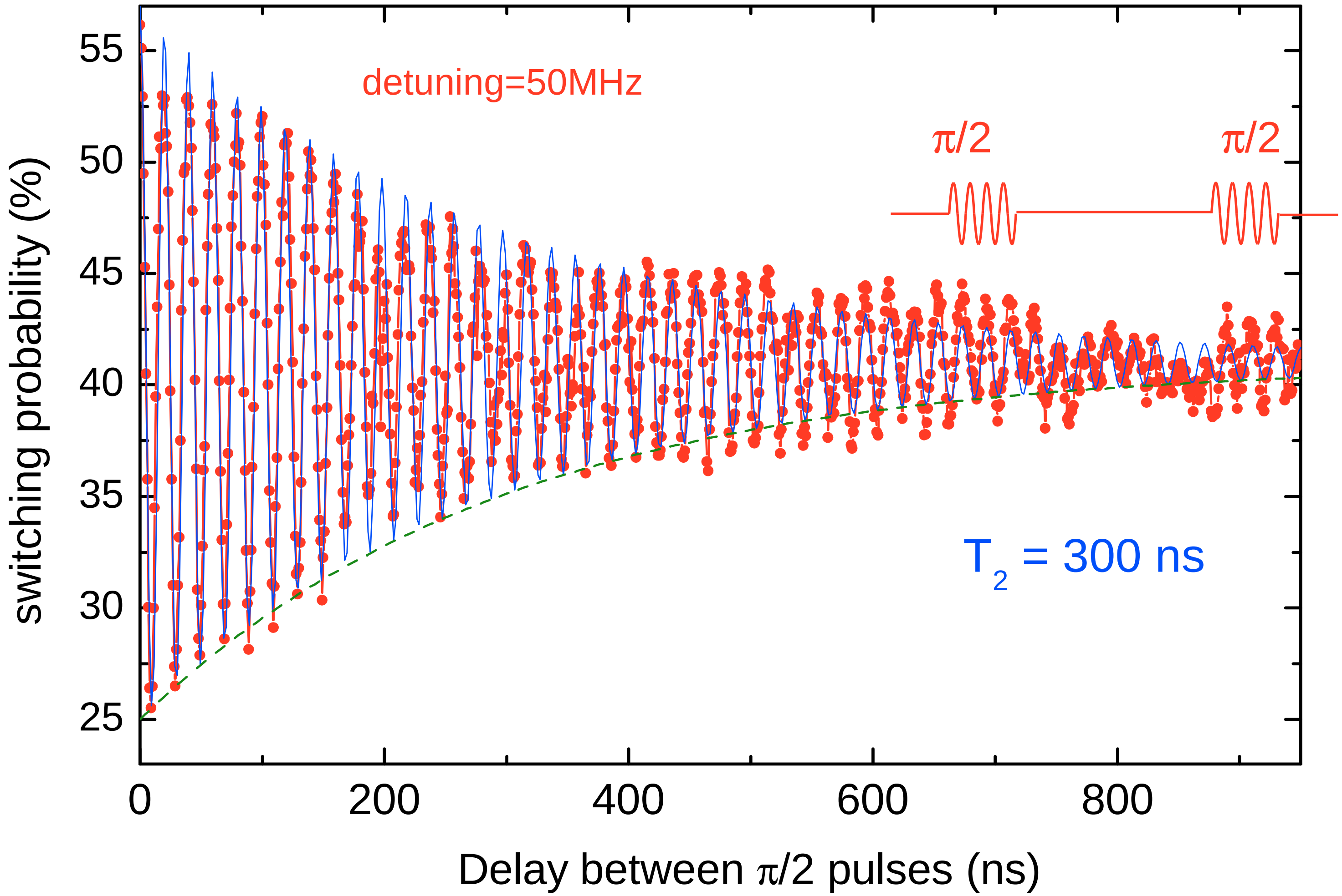
Quantronium (Saclay)

Operation at optimal point (saddle)

- minimizes noise effects
- voltage fluctuations couple transversely
- flux fluctuations couple quadratically

$$H = -\frac{1}{2}E_J(\Phi_{x0})\sigma_x - \frac{1}{2}\frac{\partial E_{ch}}{\partial V_g}\Big|_{V_{go}}\delta V_g\,\sigma_z - \frac{1}{4}\frac{\partial^2 E_J}{\partial \Phi_x^2}\Big|_{\Phi_{xo}}\delta\Phi_x^2\,\sigma_x$$

Decay of Ramsey fringes at optimal point



Vion et al., Science 02, …

Flux qubit: Bertet et al. ´04

Quadratic longitudinal coupling:

$$H = -\frac{1}{2}\left(\Delta E + \lambda X^2\right)\sigma_z$$

- Spectrum of fluctuations of $X^2(t)$?
- Distribution of fluctuations of $X^2(t)$?

Even if $X(t)$ is distributed Gaussian (central limit theorem), $X^2(t)$ is not!

- **1/f noise**

$$S_X = \frac{E_{1/f}^2}{|\omega|} \quad \Rightarrow \quad S_{X^2} \approx \frac{E_{1/f}^4}{|\omega|}\ln\frac{\omega}{\omega_{ir}}$$

again 1/f noise with different scale

if X^2 is Gaussian $\Rightarrow$

$$|\rho_{01}(t)| = \exp\left(-\frac{1}{\pi}\Gamma_f^2 t^2 \ln^2|\omega_{ir} t|\right)$$

$$\Gamma_f \equiv \lambda E_{1/f}^2$$

$$P(t) = P(t)^{static} \cdot P(t)^{hf}$$

$$static \equiv \omega < 1/t$$

$$hf \equiv \omega > 1/t$$

$$P(t)^{static} = \frac{1}{\sqrt{1 + i\frac{2}{\pi}\Gamma_f t \ln(\omega_{ir} t)}}$$

in general

$p(X) \propto \exp(-X^2/2\sigma_X^2)$

$$P(t)^{static} = \int dX\, p(X)\, e^{i\lambda X^2 t} = \frac{1}{\sqrt{1 - 2i\lambda\sigma_X^2 t}}$$

YM, Shnirman 04
D. Averin et al. 04
E. Paladino et al. 04

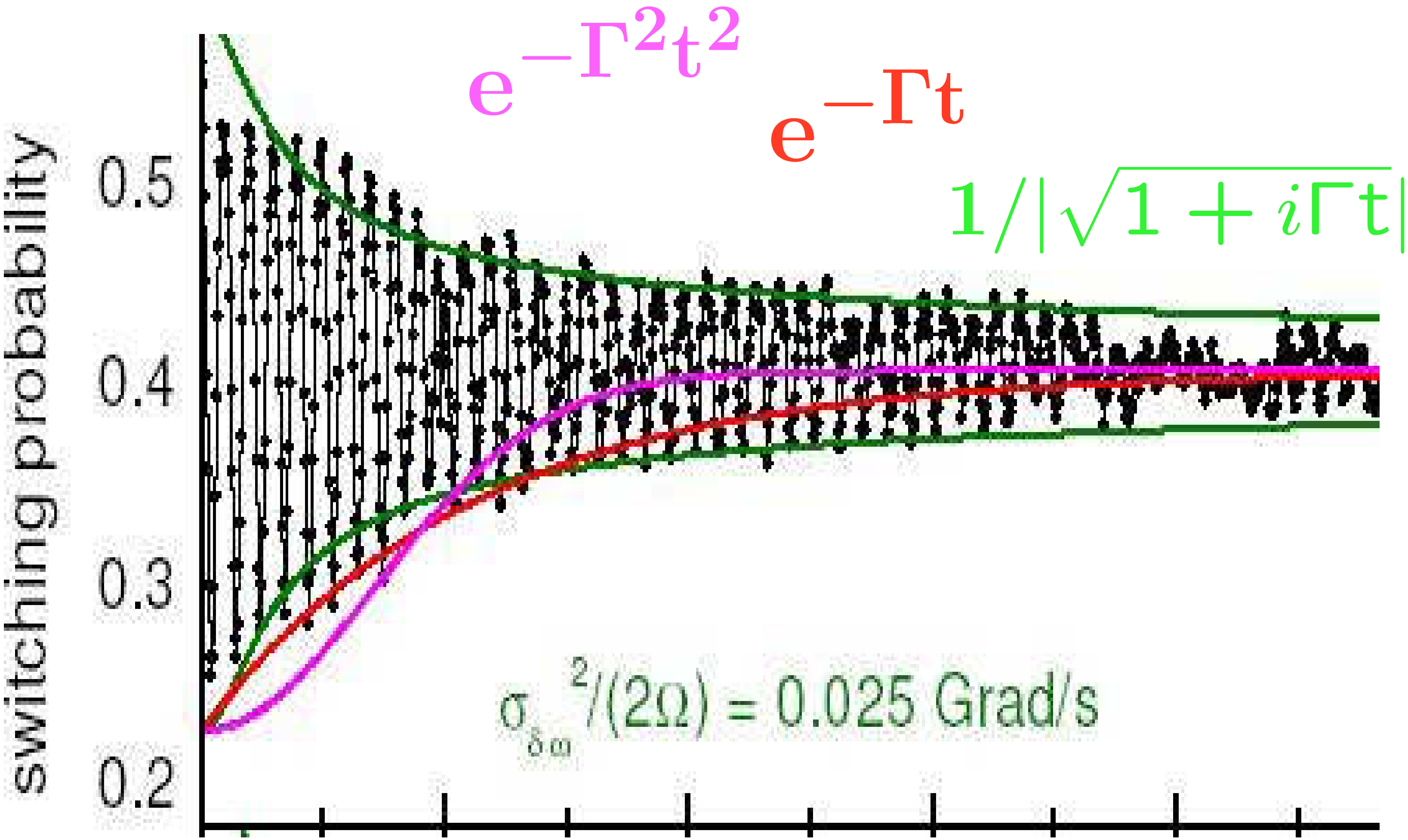
1/f spectrum „quasistatic“

$$\sigma_X^2(t) \propto \int_{\omega_{ir}}^{1/t} d\omega \frac{1}{\omega} \propto |\ln(\omega_{ir} t)|$$

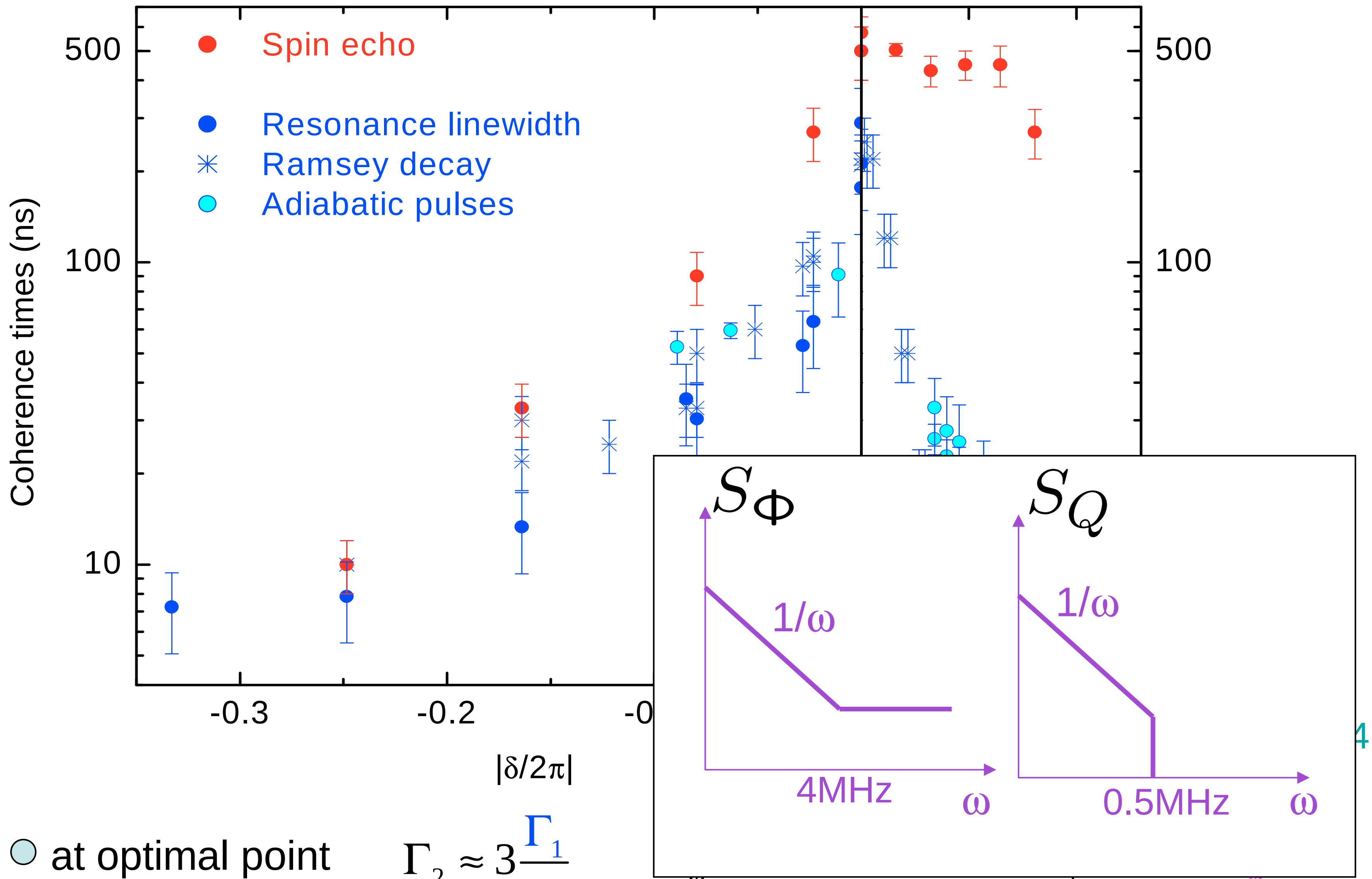
even more general

$$p(X) \text{ smooth at } X \approx 0 \rightarrow P(t)^{static} \propto 1/\sqrt{t}$$

Fitting the experiment



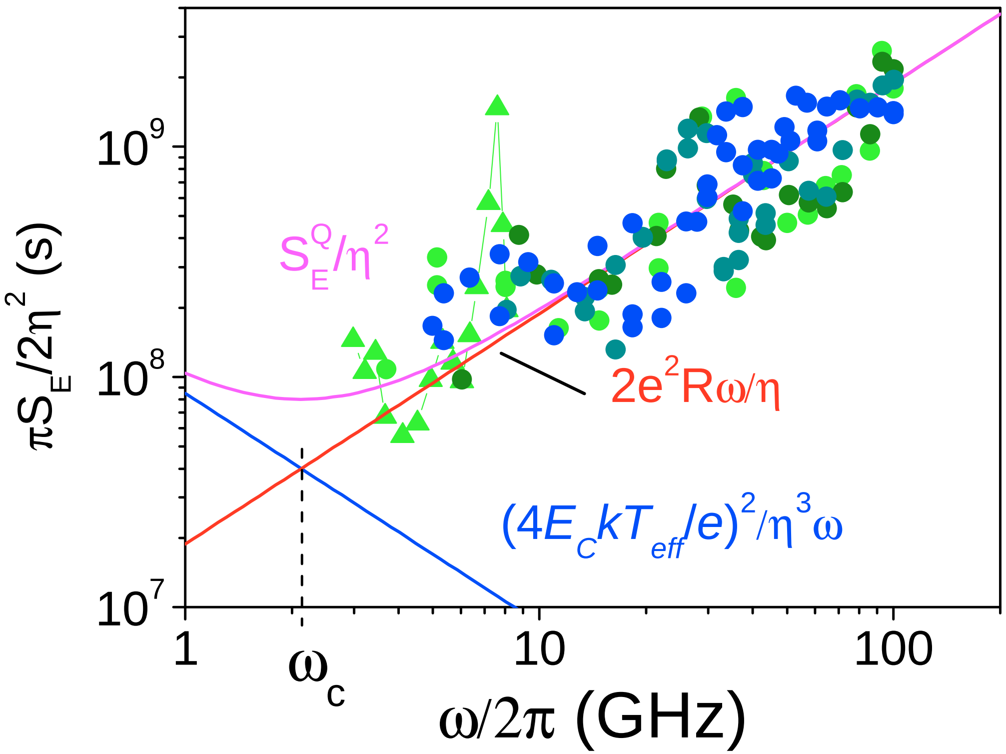
Decoherence during free evolution: summary



○ at optimal point $\Gamma_2 \approx 3\frac{\Gamma_1}{2}$

pure dephasing still strong

Recent experiments on decoherence in JJ qubits



Astafiev et al. ´04

- Ohmic high-frequency noise - very strong, not $Z(\omega)$

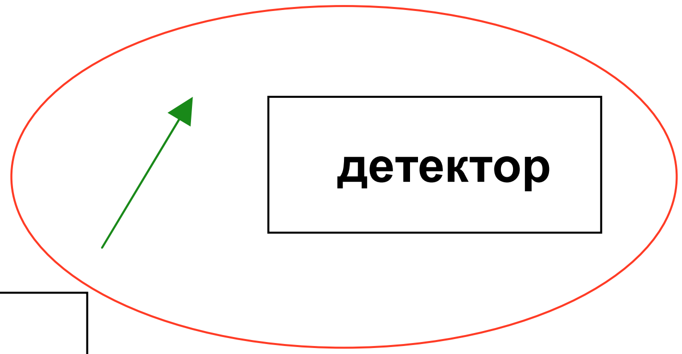
What is it?

- relation between high- and low-frequency noise
- $S(\omega) \propto \frac{T^2}{\omega} + \omega$

also: Wellstood, Urbina, Clarke 2004

Квантовое измерение

$$a|0\rangle + b|1\rangle$$



результат	0	1
состояние	$\vert 0\rangle$	$\vert 1\rangle$
вероятность	$\vert a\vert^2$	$\vert b\vert^2$

измерение как перепутывание

унитарная эволюция системы кубит + детектор

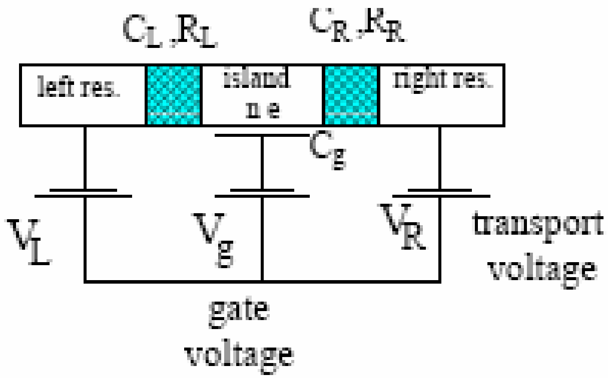
$$(a|0\rangle + b|1\rangle) \otimes |M\rangle$$

$$\Downarrow$$

$$a|0\rangle \otimes |M_0\rangle + b|1\rangle \otimes |M_1\rangle$$

$|M_i\rangle$ --- макроскопически различимые состояния

Одноэлектронный транзистор

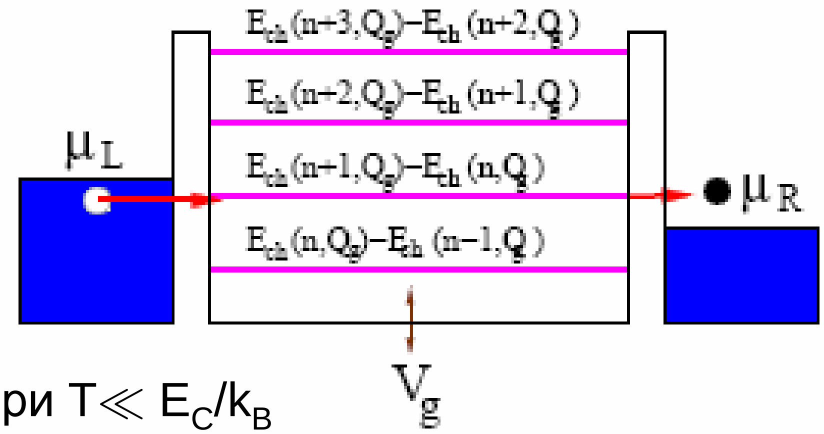


$$E_{ch}(n, Q_g) = \frac{(ne - Q_g)^2}{2C}$$

$$Q_g = C_g V_g + C_L V_L + C_R V_R$$

$$C = C_g + C_L + C_R$$

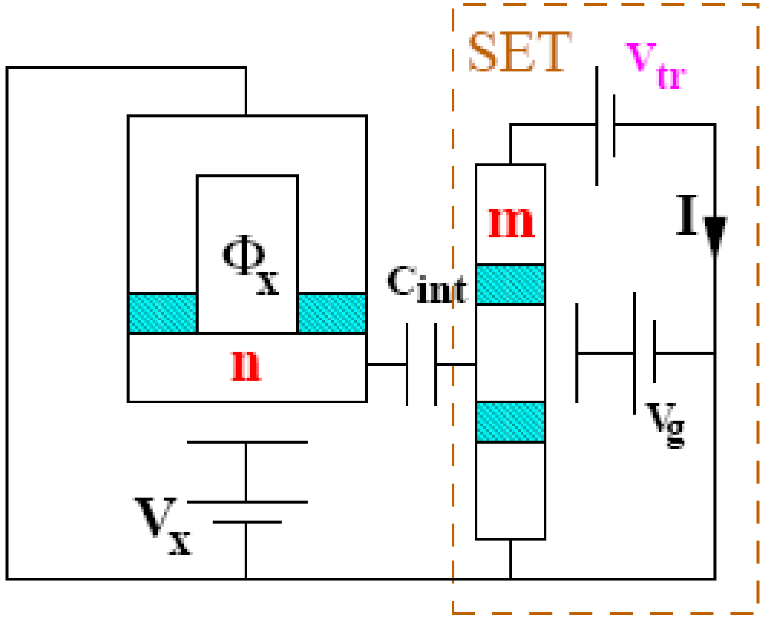
$$E_{ch}(n+1, Q_g) - E_{ch}(n, Q_g) = \left(n + \frac{1}{2}\right)\frac{e^2}{C} - \frac{Q_g}{C}$$



кулоновская блокада при $T \ll E_C/k_B$

последовательное туннелирование при $\mu_L > E_{ch}(n+1, Q_g) - E_{ch}(n, Q_g) > \mu_R$

Квантовое измерение одноэлектронным транзистором



- $V_{tr} = 0 \implies$ no dissipative currents
  no additional decoherence
  SET just renormalizes capacitances

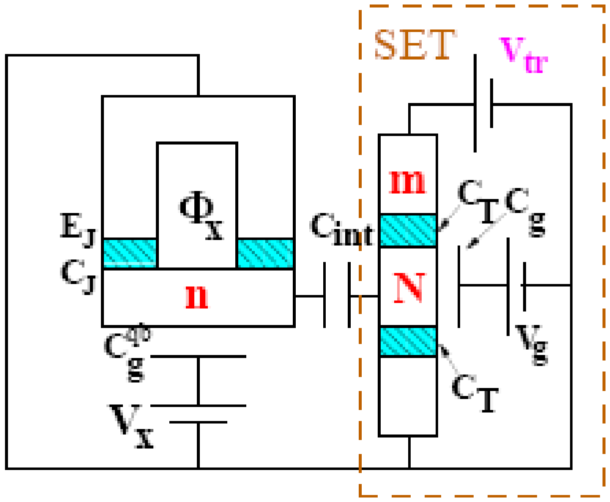
- $V_{tr} \neq 0 \implies$ dissipative current $I(n, V_{tr})$

  $n = 0$: $\quad I = I_0 \quad m = I_0 t$

  $n = 1$: $\quad I = I_1 \quad m = I_1 t$

а если $\quad |\psi\rangle = a|0\rangle = b|1\rangle$?

квантовая система: кубит и транзистор



$$C, C_{int}, C_g^{SET} \ll C_T \ll C_J$$

$$\begin{aligned} \mathcal{H} = & -\frac{1}{2}\Delta E_{ch}\,\hat{\sigma}_z - \frac{1}{2}E_J\,\hat{\sigma}_x && \text{qubit} \\ & + E_{set}\frac{(Ne - Q_{set})^2}{e^2} && \text{SET} \\ & + 2E_{int}\, n\, N && \text{interaction } (2n = 1 + \hat{\sigma}_z) \\ & + \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_I && \text{microscopic terms} \\ & + \mathcal{H}_T && \text{tunneling} \end{aligned}$$

$$E_{int} \approx e^2 C_{int}/2C_J C_T \qquad E_{set} \approx e^2/C_T$$

$$\mathcal{H}_r = \sum_{k\sigma} \epsilon^r_{k\sigma} c^{r\dagger}_{k\sigma} c^r_{k\sigma} \qquad (r = \mathrm{R,L,I})$$

$$\begin{aligned} \mathcal{H}_T = & \sum_{kk'\sigma} T^L_{kk'} c^{I\dagger}_{k\sigma} c^L_{k'\sigma} \{N \to N+1\} + \text{h.c.} \\ + & \sum_{kk'\sigma} T^R_{kk'} c^{R\dagger}_{k\sigma} c^I_{k'\sigma} \{N+1 \to N\}\{m \to m+1\} + \text{h.c.} \end{aligned}$$

Эволюция матрицы плотности

разложим $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{tun}}$ $\alpha = \frac{R_K}{4\pi^2 R_T} \ll 1$

проинтегрируем по микроскопическим степеням свободы

$$\langle iNm|\rho|i'Nm\rangle$$

$$\frac{d}{dt}\hat{\rho} = \frac{i}{\hbar}[\hat{\rho}, \mathcal{H}_0] + \int_{t_0}^{t} dt' \Sigma(t-t')\hat{\rho}(t')$$

$$\boxed{\frac{d}{dt}\begin{pmatrix} \hat{\rho}^N \\ \hat{\rho}^{N+1} \end{pmatrix} = \begin{pmatrix} i[\mathcal{H}^0_{\text{qb}}, \hat{\rho}^N] \\ i[\mathcal{H}^0_{\text{qb}} + E_{\text{int}}\sigma_z, \hat{\rho}^{N+1}] \end{pmatrix} + \begin{pmatrix} -\check{\Gamma}_L & e^{ik}\check{\Gamma}_R \\ \check{\Gamma}_L & -\check{\Gamma}_R \end{pmatrix} \begin{pmatrix} \hat{\rho}^N \\ \hat{\rho}^{N+1} \end{pmatrix}}$$

N-degree of freedom

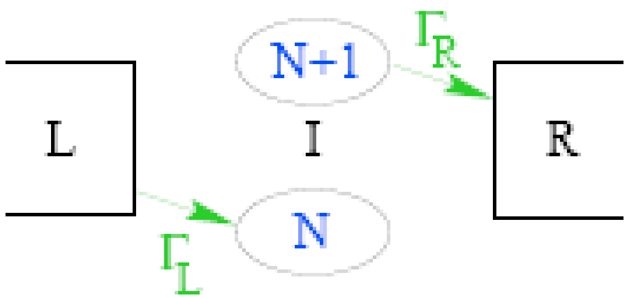
Balance of N and $N+1$

Time scale: $\frac{1}{\Gamma}$

$$p_N = \sum_{i,m} \sigma_{ii}(N,m)$$

Decay leads to:

$$\frac{p_{N+1}}{p_N} = \frac{\Gamma_L}{\Gamma_R}$$



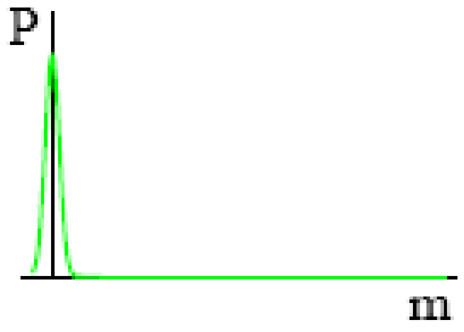
$$\Gamma_L \, p_N - \Gamma_R \, p_{N+1} = 0$$

Динамика тока в транзисторе

$$P(m,t) = \sum_{i,N} \sigma_{ii}(N,m)$$

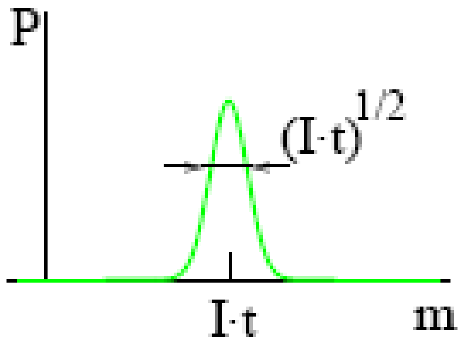
вероятность, что *m* электронов протуннелировало

- $t = 0$: $\quad P(m,t) = \delta_{m,0}$



- t small: $\quad m \approx I \cdot t$

  $+$ shot noise

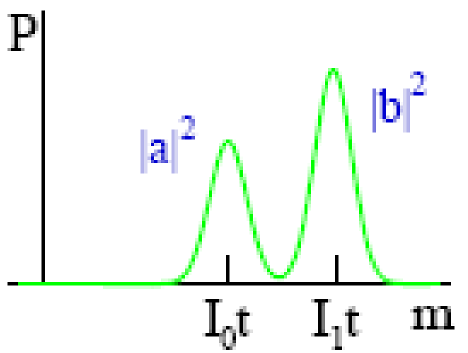


- different current for $|0\rangle$ and $|1\rangle$

$$\sqrt{It} \approx I_1 t - I_0 t$$

$$\Downarrow$$

$$t_{\text{meas}} \approx \frac{\hbar}{2\pi\alpha} \frac{E_{\text{set}}}{E_{\text{int}}^2}$$



Динамика кубита

Reduced DM of qubit:

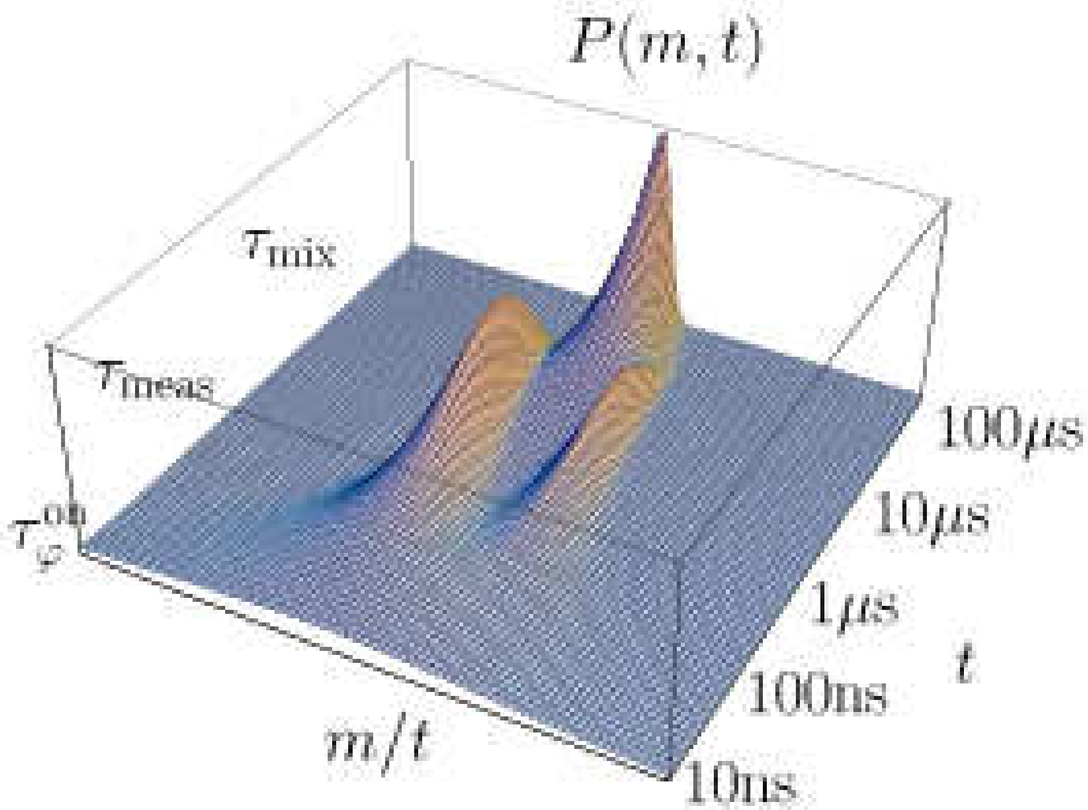
$$\rho_{ij} = \sum_{N,m} \sigma_{ij}(N,m)$$

$$\rho(t=0) = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix} \xrightarrow{t \gg \tau_\varphi^*} \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}$$

Time scale:

$$\tau_\varphi^* \approx \hbar \frac{\Gamma}{E_{\mathrm{int}}^2}$$

Перемешивание (релаксация)



$$\begin{pmatrix} |a|^2 \\ |b|^2 \end{pmatrix} \xrightarrow{t \gg \Gamma_{\rm mix}^{-1}} \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

Эффективность квантового детектора: параметры

М.Нильсен, И.Чанг «Квантовые вычисления и квантовая информация»

M.H.Devoret, A.Wallraff, J.Martinis, cond-mat/0411174

D.Esteve, D.Vion, cond-mat/0505676

YM, G.Schoen, A.Shnirman, cond-mat/0011269

Ч.Слихтер, «Основы теории магнитного резонанса», Мир, 1981

G.Ithier et al., cond-mat/0508588