Нанофизика низких температур, Черноголовка, 20-30 августа 2007 г.

Сверхпроводниковые квантовые биты

(теория)

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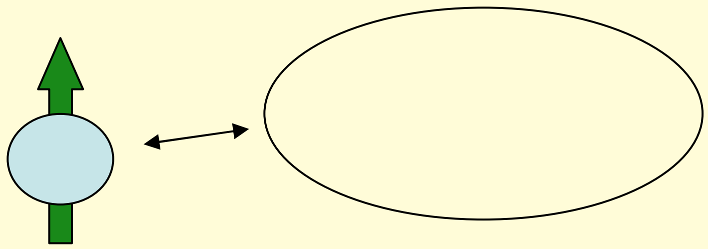


Quantum error correction

- клонирование невозможно
- квантовая информация аналоговая, а не цифровая
- квантовое измерение разрушит состояние

$$a|000\rangle + b|111\rangle$$

пороговая теорема

Матрица плотности кубита

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

$\langle Q\rangle = \langle\psi|\hat{Q}|\psi\rangle = a_j^* Q_{ji} a_i = \mathrm{tr}(\hat{Q}\hat{\rho})$, где $\rho_{ij} = a_i a_j^*$

1 кубит + 1 кубит

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

$\langle Q^{(1)}\rangle = a_{jk}^* Q_{ji} a_{ik} = \mathrm{tr}(\hat{Q}\hat{\rho})$, где $\rho_{ij} = a_{ik} a_{jk}^*$

Для 1 кубита

$$\hat{\rho} = \frac{1}{2}\left(\hat{1} + \mathbf{n}\hat{\vec{\sigma}}\right)$$

|**n**|=1 --- чистое состояние, $\rho = |\psi\rangle\langle\psi|$

|**n**|<1 --- стат. смесь $\rho = \sum w_a |\psi_a\rangle\langle\psi_a|$

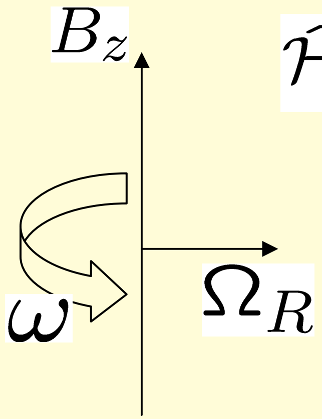
$$\dot{\rho} = -i[\mathcal{H}, \rho]$$

$$\rho_A = \mathrm{tr}_B(\rho_{A+B})$$

2 кубита

$$\rho = \frac{1}{4}\hat{1} + \frac{1}{4}\mathbf{n}^{(1)}\hat{\vec{\sigma}}^{(1)} + \frac{1}{4}\mathbf{n}^{(2)}\hat{\vec{\sigma}}^{(2)} + m_{\alpha\beta}\hat{\sigma}_{\alpha}^{(1)}\hat{\sigma}_{\beta}^{(2)}$$

Осцилляции Раби



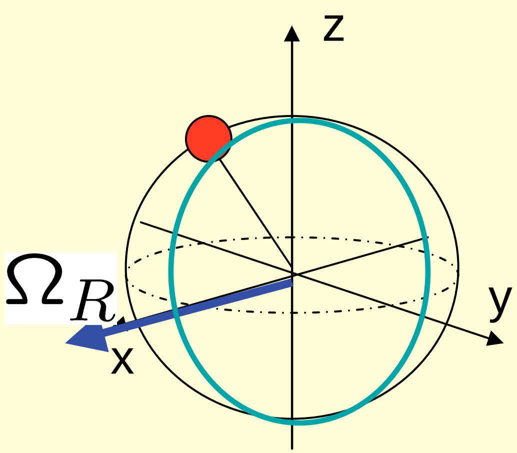
$$\hat{\mathcal{H}} = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}\Omega_R(\cos\omega t\ \hat{\sigma}_x + \sin\omega t\ \hat{\sigma}_y)$$

$$\omega = B_z$$

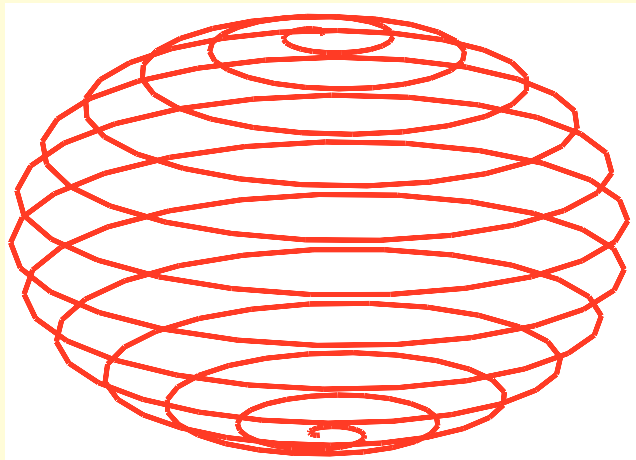
во вращающейся СО

$$\tilde{\mathcal{H}} = \dot{U}U^\dagger + U\mathcal{H}U^\dagger, \qquad U + \exp\left(-i\omega t\sigma_z/2\right)$$

$$\tilde{\mathcal{H}} = -\frac{1}{2}\Omega_R\,\sigma_x$$



в лабораторной системе



Источники шума

внешние:

• управлющая цепь
• детектор – даже в выкл. состоянии

внутренние:

• флуктуации фонового заряда, магнитного потока и крит. тока
• квазичастицы – выморожены при $T \ll \Delta$
• электромагн. излучение
• ядерные спины
• …

Кинетическое уравнение для матрицы плотности

$$\mathcal{H} = \mathcal{H}_0 + V + \mathcal{H}_B \qquad \dot{\rho} = -i[\mathcal{H}, \rho]$$

представление взаимодействия

$$\dot{\rho} = -i[V(t), \rho], \qquad V(t) = e^{i\mathcal{H}_0 t} V e^{-i\mathcal{H}_0 t}$$

$$\rho(t) = \rho(0) - i\int_0^t dt'[V(t'), \rho(t')] =$$
$$\rho(0) - i\int_0^t dt'[V(t'), \rho(0)] -$$
$$-\int_0^t dt' \int_0^{t'} dt''[V(t'), [V(t''), \rho(t'')]]$$

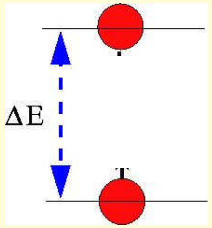
$$t$$

$$\frac{d}{dt}: \dot{\rho} = -i[V(t), \rho(t)] - \int_0^t dt''[V(t), [V(t''), \rho(t'')]]$$

$$\langle V \rangle = 0$$

локальная по времени (марковская) эволюция

Longitudinal coupling ⇒ “pure” dephasing



$$H = -\frac{1}{2}(\Delta E + X)\sigma_z + H_{bath}$$

X – classical or quantum field

$$|\rho_{01}(t)| \propto \left\langle \exp\left(-\frac{i}{\hbar}\int_0^t X(\tau)d\tau\right)\right\rangle = \exp\left(-\frac{1}{2\hbar^2}\int_0^t d\tau_1 \int_0^t d\tau_2 \langle X(\tau_1)X(\tau_2)\rangle\right)$$

$$= \exp\left(-\frac{1}{2\hbar^2}\int \frac{d\omega}{2\pi} S_X(\omega) \frac{\sin^2(\omega t/2)}{(\omega/2)^2}\right)$$

$$\frac{\sin^2(\omega t/2)}{(\omega/2)^2} \approx 2\pi\delta(\omega)t$$

$$\approx \exp\left(-\frac{1}{2\hbar^2} S_X(\omega \approx 0)\cdot t\right) = \exp\left(-\frac{t}{T_2^*}\right)$$

for regular spectrum

$$S_X(\omega) = \frac{E_{1/f}^2}{|\omega|} \to \infty \quad \text{for } \omega \to 0$$

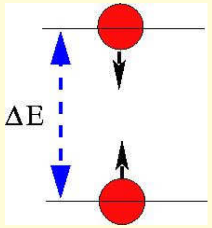
for 1/f noise

$$= \exp\left(-\frac{E_{1/f}^2}{2\pi} t^2 \ln|\omega_{ir} t|\right)$$

e.g., Cottet et al. 01

Transverse coupling ⇒ relaxation

$$H = -\frac{1}{2}\Delta E \sigma_z - \frac{1}{2} X \sigma_x + H_{Bath}$$



Golden Rule:

$$\Gamma_{\uparrow} = \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_i \left|\langle i \,|\, X \,|\, f\rangle\right|^2 \delta\left(E_i + \Delta E - E_f\right)$$

$$= \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_i \langle i \,|\, X \,|\, f\rangle\langle f \,|\, X \,|\, i\rangle \frac{1}{2\pi\hbar}\int dt \exp\left[i\left(E_i + \Delta E - E_f\right)t/\hbar\right]$$

$$= \frac{1}{4\hbar^2}\int dt \sum_i \rho_i \langle i \,|\, X(t) X(0) \,|\, i\rangle \exp\left[i\Delta E t/\hbar\right]$$

$$= \frac{1}{4\hbar^2}\left\langle X(t) X(0)\right\rangle_{\omega=\Delta E/\hbar}$$

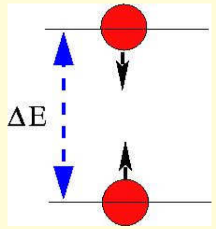
$$\Gamma_{\downarrow} = \frac{1}{4\hbar^2}\left\langle X(t) X(0)\right\rangle_{\omega=-\Delta E/\hbar}$$

$$\Rightarrow \quad \frac{1}{T_1} \equiv \Gamma_{\text{rel}} = \Gamma_{\uparrow} + \Gamma_{\downarrow} = \frac{1}{2\hbar^2} S_X(\omega = \Delta E/\hbar)$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_2^*}$$

Bloch equations, applicability

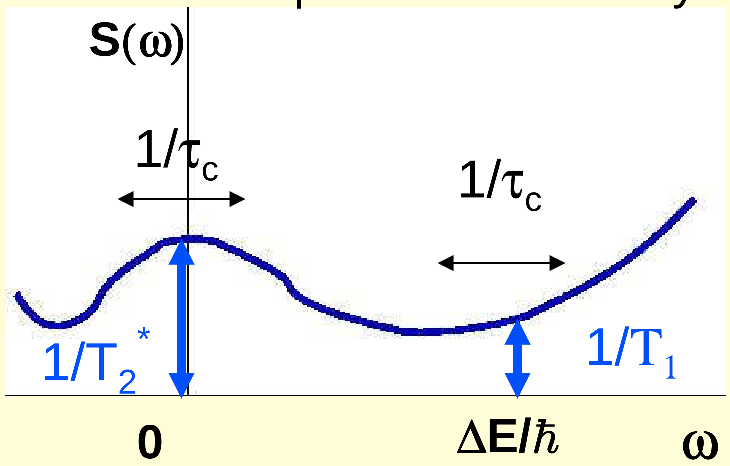
$$H = -\frac{1}{2}\Delta E\,\sigma_z \; -\frac{1}{2} X\cos\eta\,\sigma_z - \frac{1}{2} X\sin\eta\,\sigma_x + H_{bath}$$



$$\frac{d}{dt}\mathbf{S} = \mathbf{B}\times\mathbf{S} - \frac{1}{T_1}(S_z - S_z^{eq})\hat{z} - \frac{1}{T_2}(S_x\hat{x} + S_y\hat{y})$$

Bloch (46,57)
Redfield (57)

perturbation theory



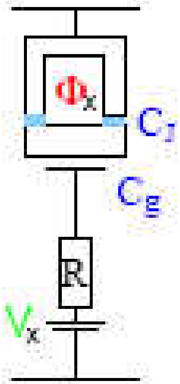
works for $\tau_c \ll T_1, T_2$

weak short-correlated noise

Dissipation by Ohmic control circuit

- gate voltage circuit is dissipative
  characterized by $Z(\omega) = R$
  induces voltage fluctuations

$$S_V(\omega) = \hbar\omega R \coth\left(\frac{\hbar\omega}{2k_B T}\right) \approx 2k_B R T \,|_{\hbar\omega \ll k_B T}$$



$$\mathcal{H} = \mathcal{H}_0 + \frac{C_g}{C_J}\, e\, \delta V(t)\, \hat{\sigma}_z + \mathcal{H}_{\rm bath}$$

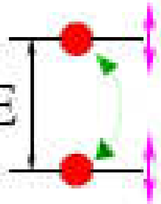
$$= \mathcal{H}_0 + \frac{C_g}{C_J}\, e\, \delta V(t)\, ($$

$$\tan\eta = \frac{E_J(\Phi_x)}{\Delta E_J(V_x)}$$

- $R \ll h/e^2 \approx 26\text{k}\Omega$ e.g. $R = 100\Omega$

and $C_g \ll C_J$ (weak coupling to environment) $\Rightarrow T_1, T_2 \approx 10^{-6} - 10^{-4}$s

- operation time $\tau_{op} \approx \hbar/E_J \approx 10^{-10}$s $\Rightarrow$ ratio $\geq 10^4$



$\delta V \cos\eta$

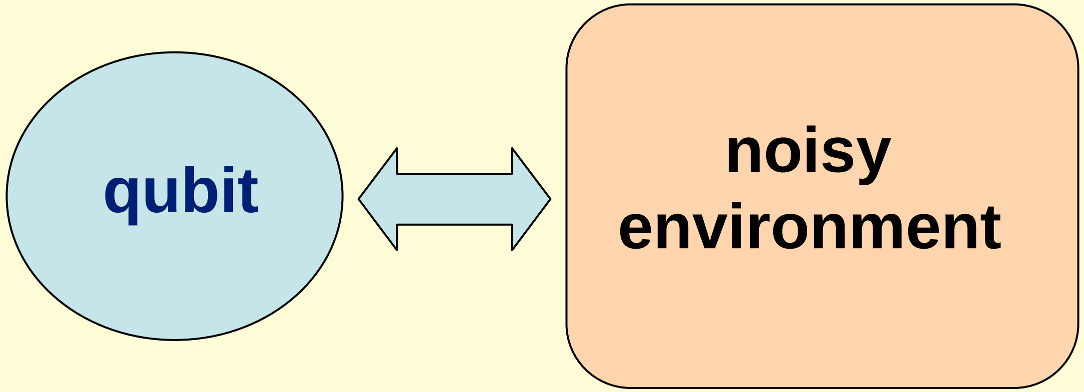
fluctuation

eigenenergies

$$\Gamma_{\rm rel} = 4\pi \frac{R}{h/e^2}\left(\frac{C_g}{C_J}\right)^2 \sin^2\eta\, \frac{\Delta E}{\hbar} \coth\left(\frac{\Delta E}{2k_B T}\right)$$

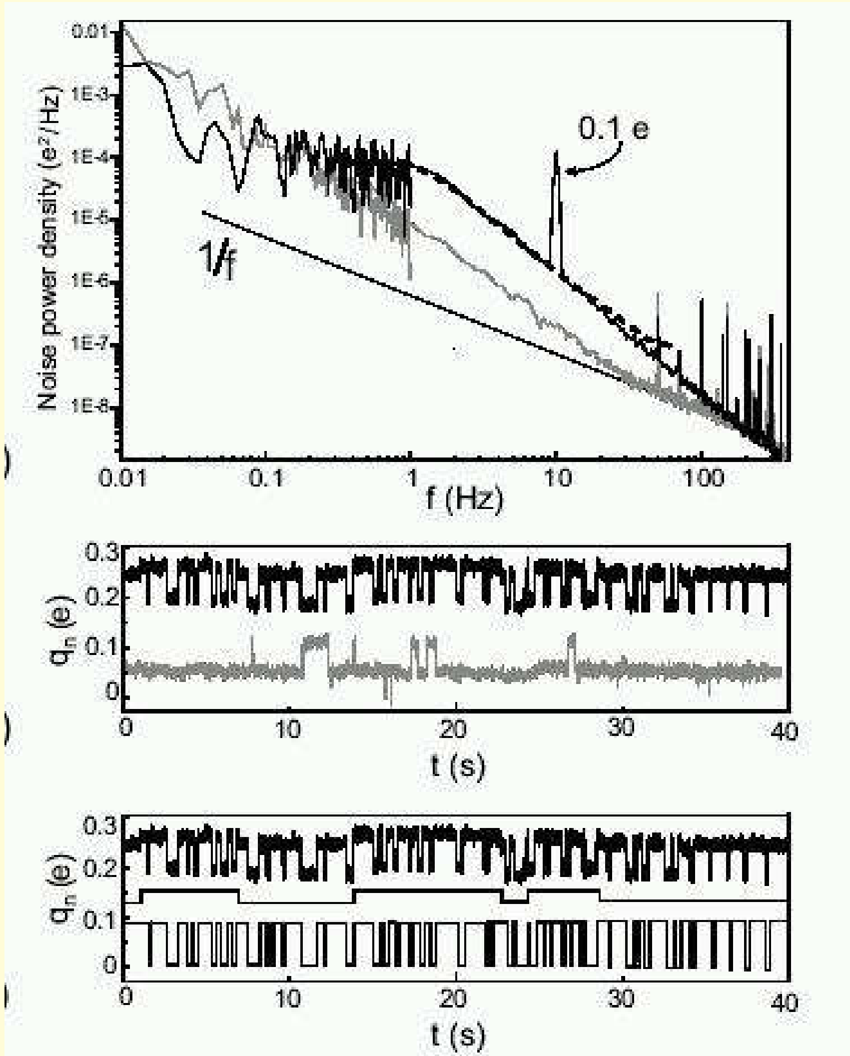
$$\Gamma_\varphi = \frac{1}{2}\Gamma_{\rm rel} + 4\pi \frac{R}{h/e^2}\left(\frac{C_g}{C_J}\right)^2 \cos^2\eta\, \frac{k_B T}{\hbar}$$

Qubits and environment



- Decoherence induced by noise
- Qubits as spectrometers

Nanoelectronic circuits and 1/f noise

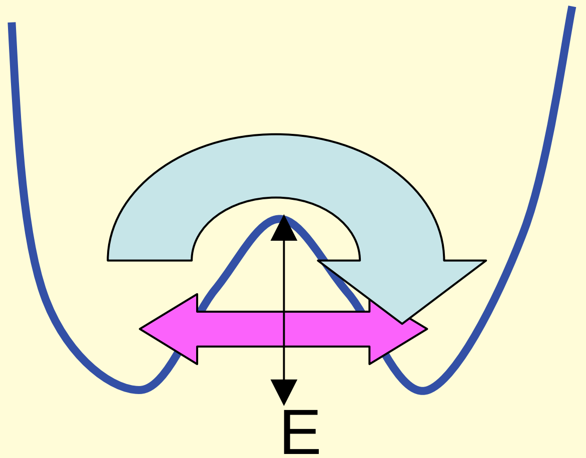


Bouchiat et al. ´97

Charge noise:

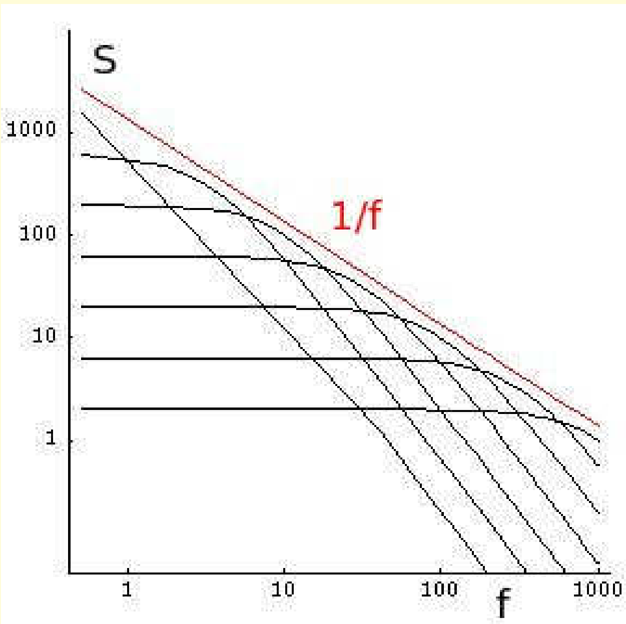
- 1/f noise
- individual Lorentzians – bistable fluctuators
- T-dependence saturated at at low T $\lesssim$ 300 mK

1/f noise from bistable fluctuators



$\Gamma \propto e^{-E/kT}$ or $\Gamma \propto e^{-E/h\omega_0}$

$$dw = g(E)dE \propto \begin{Bmatrix} h\omega_0 \\ kT \end{Bmatrix} \frac{d\Gamma}{\Gamma}$$

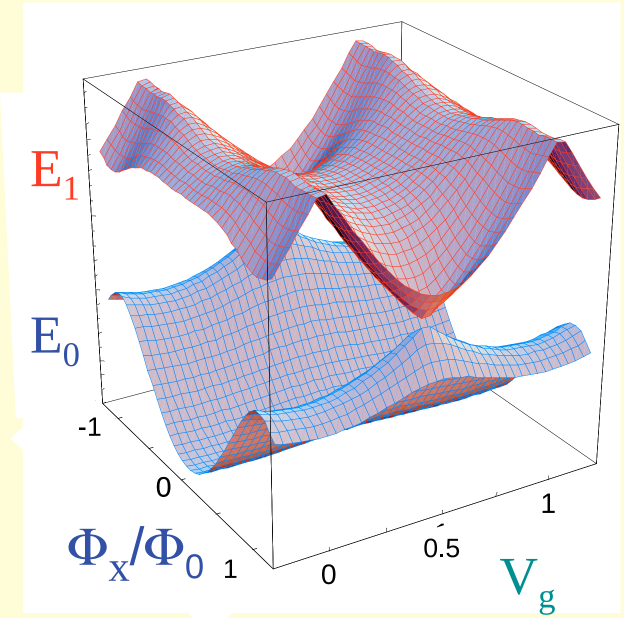


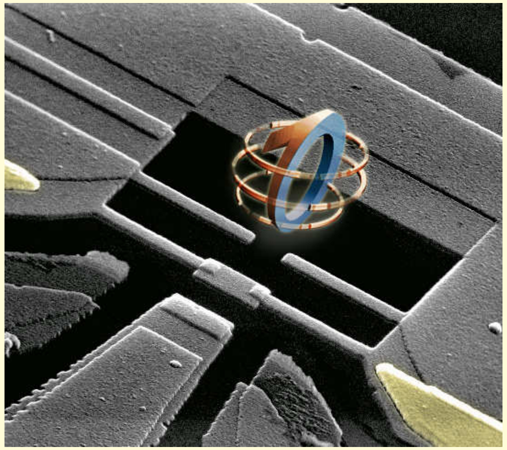
$$S(\omega) \propto \int \frac{d\Gamma}{\Gamma} \frac{\Gamma}{\omega^2 + \Gamma^2} \propto \frac{1}{\omega}$$

McWhorter, 1958; Dutta, Horn, RMP'81

Charge-phase qubit $E_C \approx E_J$

$$H = -\frac{1}{2}E_{ch}(V_g)\sigma_z - \frac{1}{2}E_J(\Phi_x)\sigma_x$$





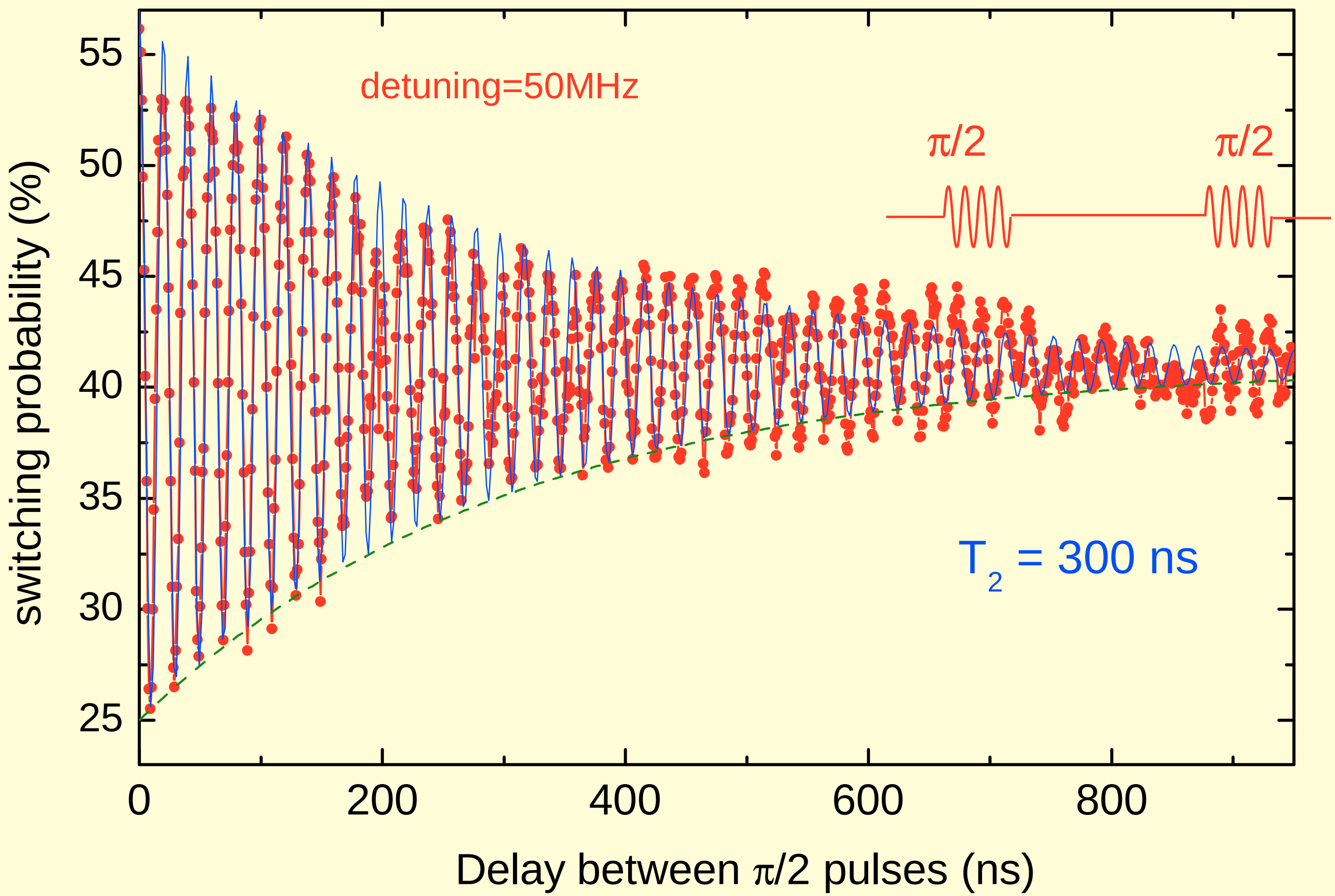
Quantronium (Saclay)

Operation at optimal point (saddle)

- minimizes noise effects
- voltage fluctuations couple transversely
- flux fluctuations couple quadratically

$$H = -\frac{1}{2}E_J(\Phi_{x0})\sigma_x - \frac{1}{2}\frac{\partial E_{ch}}{\partial V_g}\Big|_{V_{go}}\delta V_g\,\sigma_z - \frac{1}{4}\frac{\partial^2 E_J}{\partial \Phi_x^2}\Big|_{\Phi_{xo}}\delta\Phi_x^2\,\sigma_x$$

Decay of Ramsey fringes at optimal point



Vion et al., Science 02, …

Flux qubit: Bertet et al. ´04

Quadratic longitudinal coupling:

$$H = -\frac{1}{2}\left(\Delta E + \lambda X^2\right)\sigma_z$$

- Spectrum of fluctuations of $X^2(t)$?
- Distribution of fluctuations of $X^2(t)$?

Even if $X(t)$ is distributed Gaussian (central limit theorem), $X^2(t)$ is not!

- **1/f noise**

$$S_X = \frac{E_{1/f}^2}{|\omega|} \quad \Rightarrow \quad S_{X^2} \approx \frac{E_{1/f}^4}{|\omega|}\ln\frac{\omega}{\omega_{ir}}$$

again 1/f noise with different scale

if X^2 is Gaussian $\Rightarrow$

$$\left|\rho_{01}(t)\right| = \exp\left(-\frac{1}{\pi}\Gamma_f^2 t^2 \ln^2|\omega_{ir} t|\right)$$

$$\Gamma_f \equiv \lambda E_{1/f}^2$$

$$P(t) = P(t)^{static} \cdot P(t)^{hf}$$

$$static \equiv \omega < 1/t$$
$$hf \equiv \omega > 1/t$$

$$P(t)^{static} = \frac{1}{\sqrt{1 + i\frac{2}{\pi}\Gamma_f t \ln(\omega_{ir} t)}}$$

in general

$\mathbf{p(X) \propto \exp(-X^2/2\sigma_X^2)}$

$$P(t)^{static} = \int dX\, p(X)\, e^{i\lambda X^2 t} = \frac{1}{\sqrt{1 - 2i\lambda\sigma_X^2 t}}$$

YM, Shnirman 04
D. Averin et al. 04
E. Paladino et al. 04

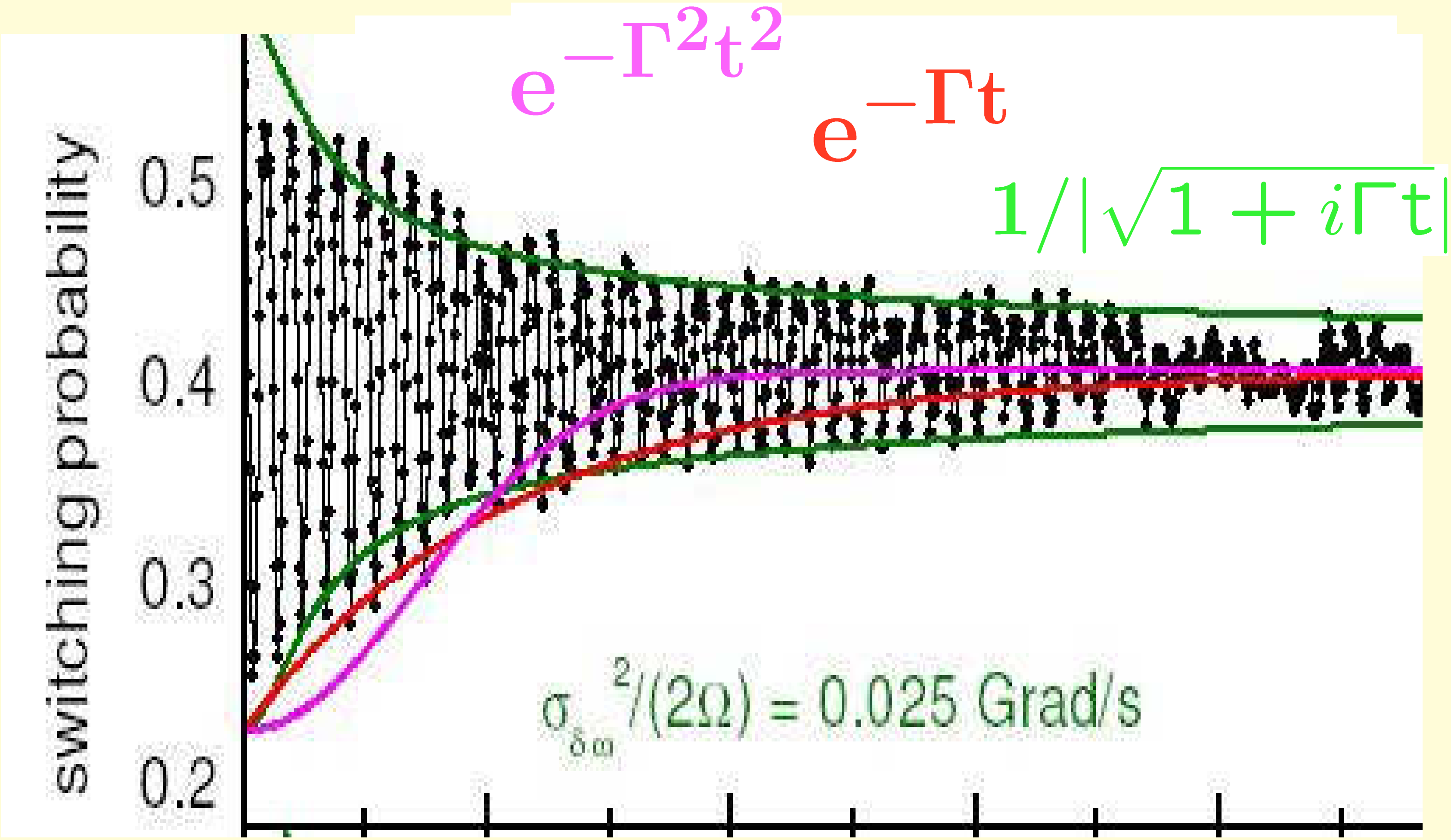
1/f spectrum „quasistatic“

$$\sigma_X^2(t) \propto \int_{\omega_{ir}}^{1/t} d\omega \, \frac{1}{\omega} \propto |\ln(\omega_{ir} t)|$$

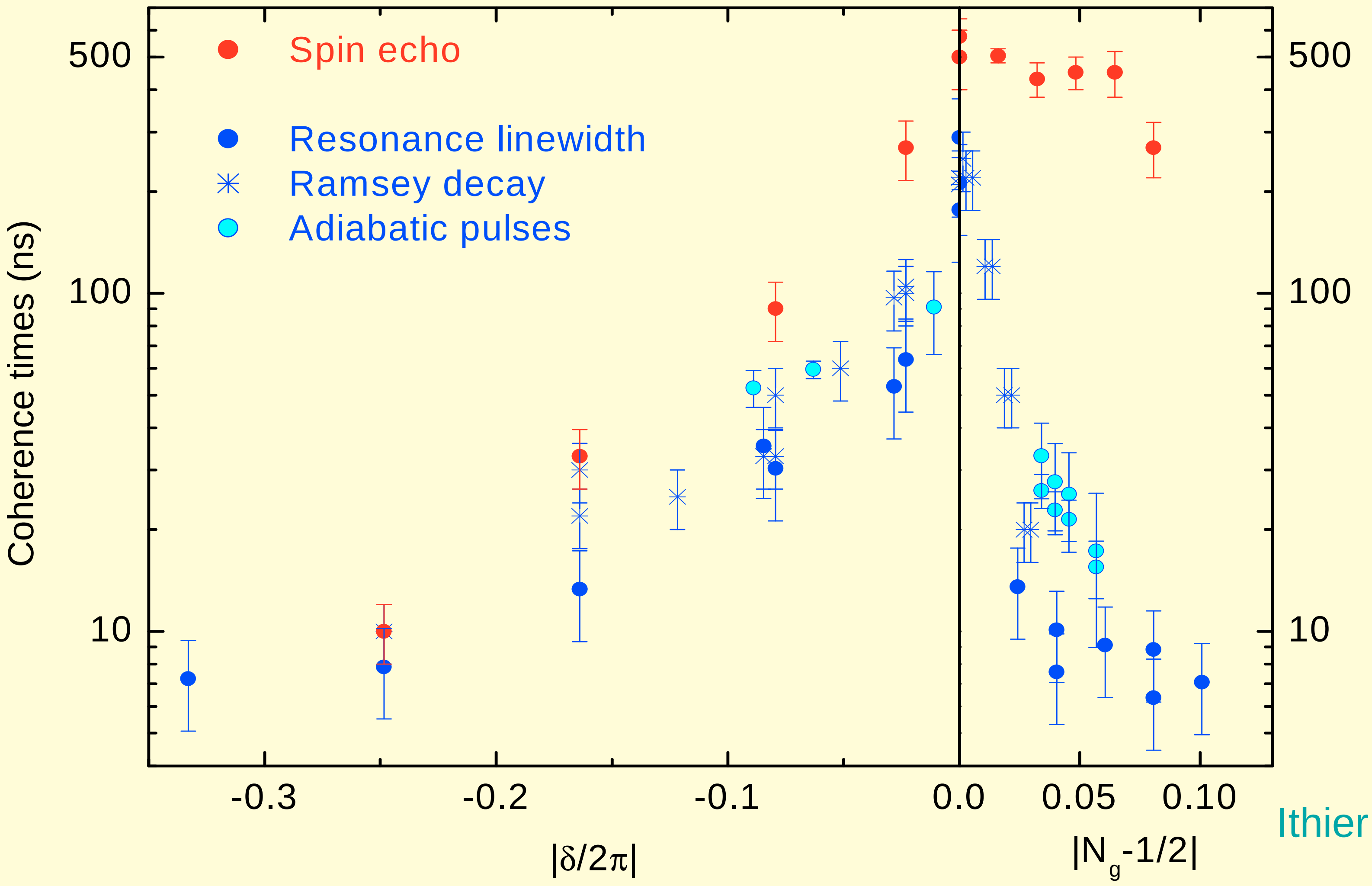
even more general

$$p(X) \text{ smooth at } X \approx 0 \rightarrow P(t)^{static} \propto 1/\sqrt{t}$$

Fitting the experiment



Decoherence during free evolution: summary



Ithier et al. 04

● at optimal point $\Gamma_2 \approx 3\frac{\Gamma_1}{2}$ $\Gamma_\varphi^* \approx 2\frac{\Gamma_1}{2}$ $\Gamma_2 = \frac{\Gamma_1}{2} + \Gamma_\varphi^*$

pure dephasing still strong