Electrons in Quantum Wires

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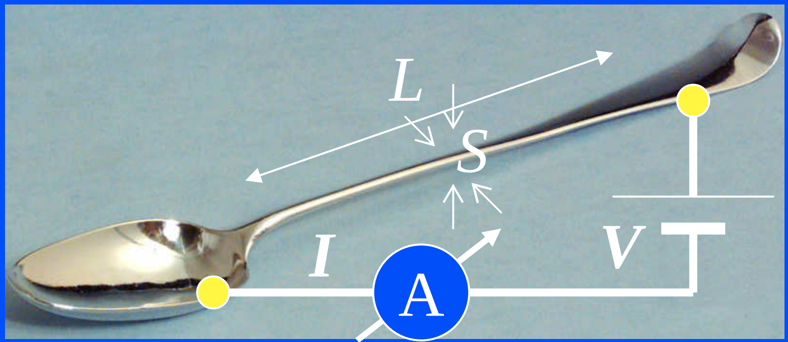
Outline

- Electrical resistance – Sharvin resistance and Landauer formula
- Interaction effects: scattering of electron waves off a Friedel oscillation
- Dynamics of electron fluid in 1D, intro to bosonization
- Multi-mode wires, spin-charge separation
- Effects of non-linear dispersion [$\varepsilon = p^2/2m$]

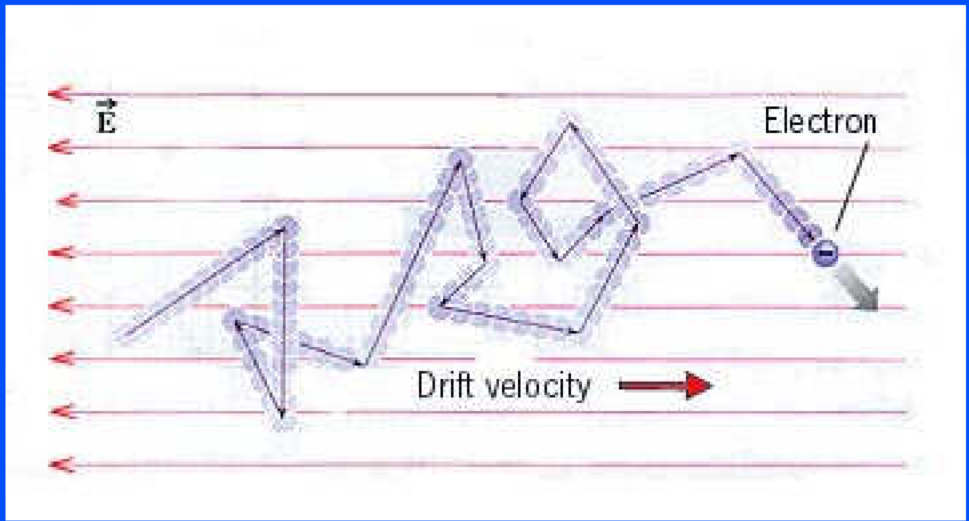
Resistance, Conductance, Conductivity



Ohm's law: $V=IR$

$$\text{Conductance: } G = 1/R = \sigma \cdot S/L$$

$$\text{Metals–high conductivity } [\text{Cu: } \sigma \sim 10^8\,(\Omega \cdot m)^{-1}\,]$$



Drude conductivity:

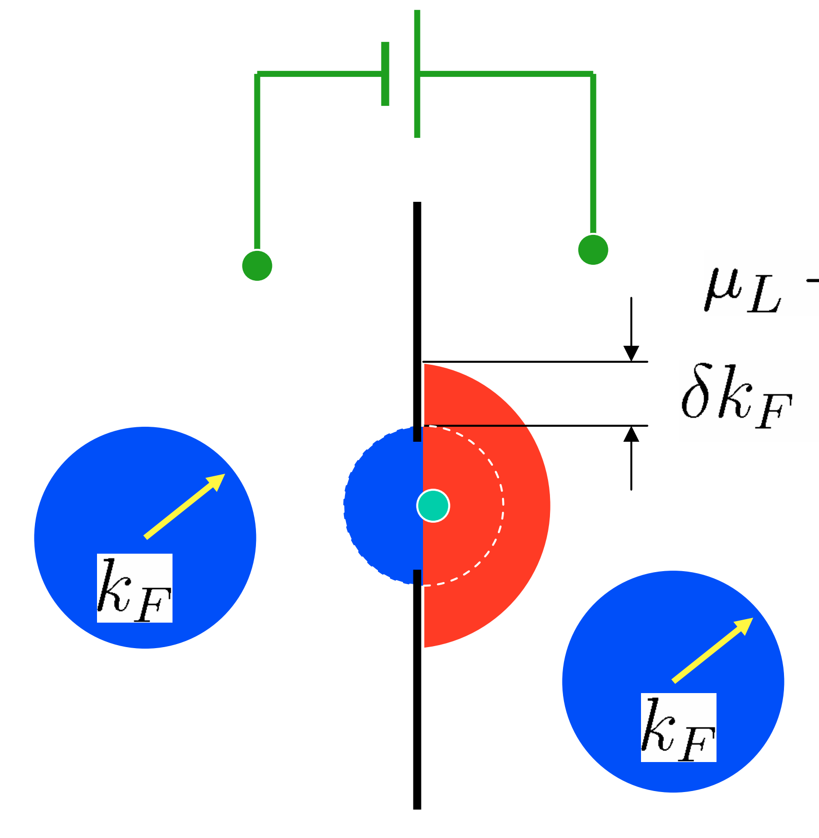
$$\sigma = \frac{n_e e^2 \ell}{m_e v_F}$$

Ballistic Electron Conductance

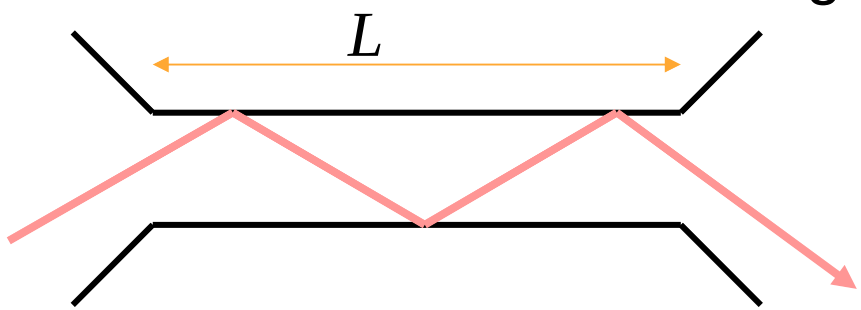
Conductance: $G \propto S/L$

Does it always hold?

Point contact (Sharvin,1965)



Ballistic channel - same thing



$G = \frac{e^2}{\hbar}\frac{Sk_F^2}{4\pi^2}$, independent of L

$\mu_L - \mu_R = eV$

$\delta k_F = eV/\hbar v_F$

$\delta n \sim k_F^2 \cdot \delta k_F$

$I \sim e v_F \delta n S$

$$G = \frac{e^2}{\hbar}\frac{Sk_F^2}{4\pi^2}$$

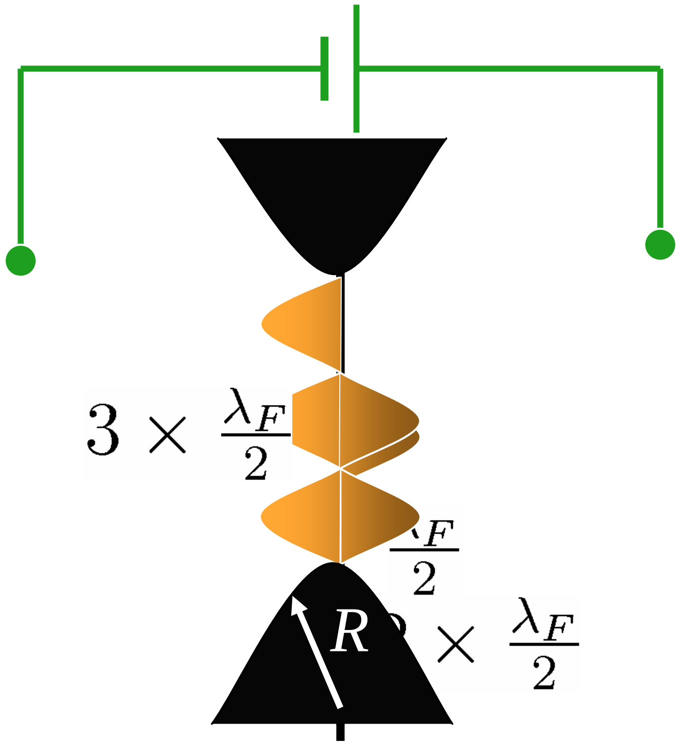
HW: work out the coefficients

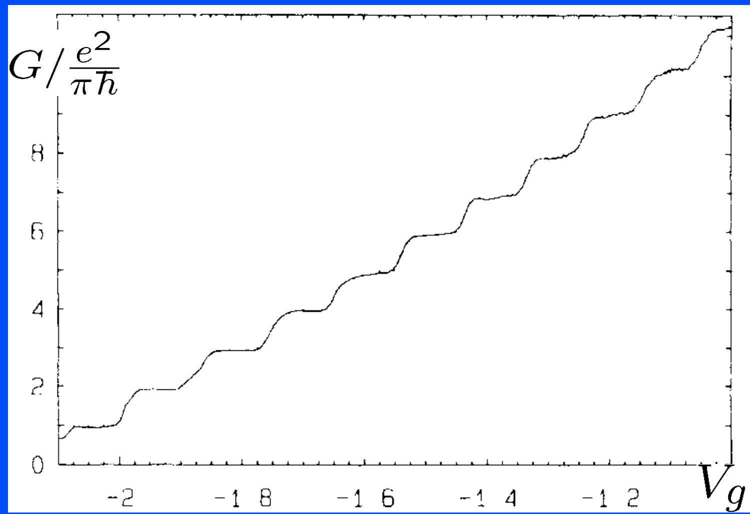
Quantum Ballistic Electron Conductance

Conductance: $G \propto S$

Does it always hold?

Quantum point contact
(van Wees et al; 1988
M. Pepper et al; 1988)

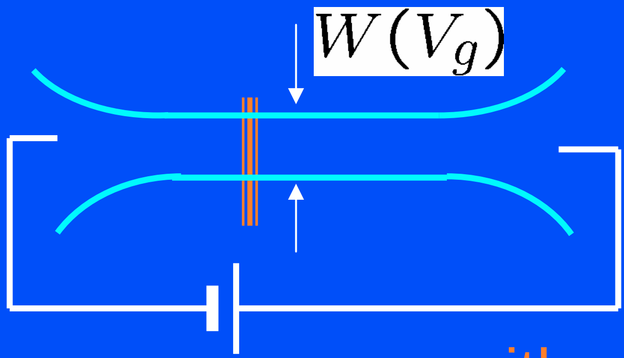




$R \gtrsim \lambda_F$ is crucial for the conductance quantization

$$G = \frac{e^2}{\pi\hbar}\frac{W k_F}{\pi} \longrightarrow G = \frac{e^2}{\pi\hbar}N$$

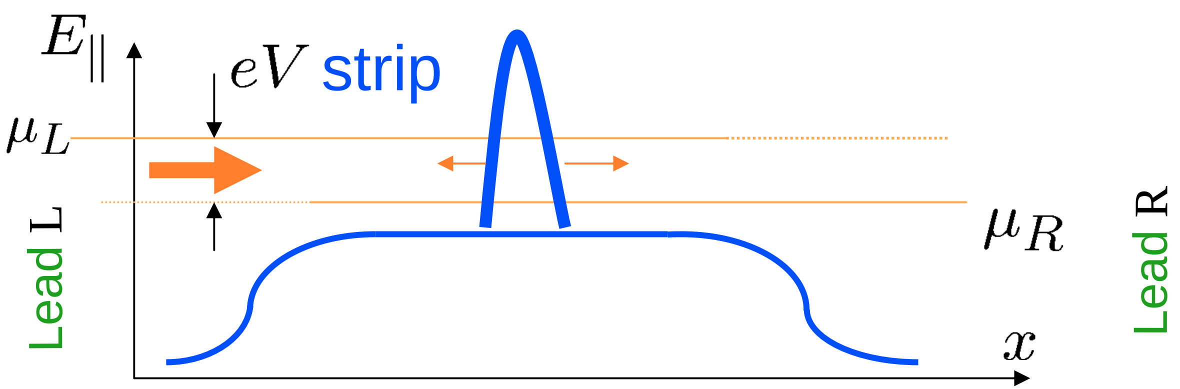
Conductance of a 1D channel, free electrons



Ideal, adiabatic channel:
quantized conductance

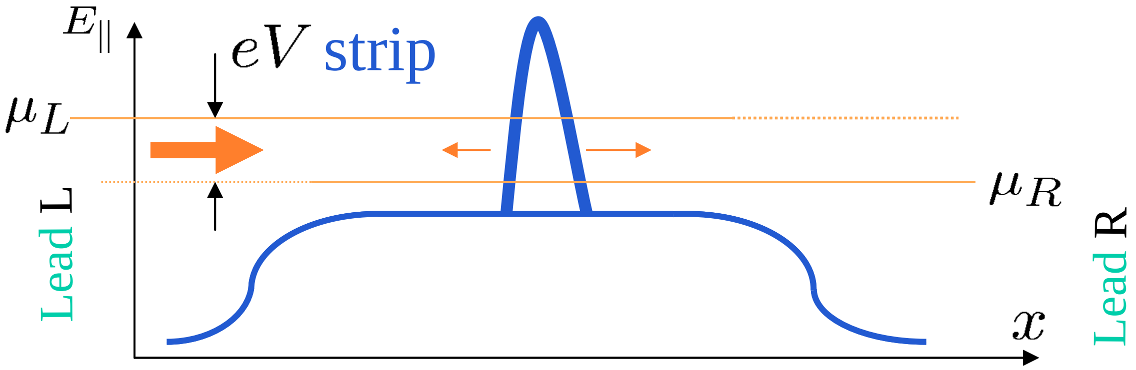
~~Ballistic~~ conductance (with ~~no~~ scatterers) is less than

$\frac{e^2}{2\pi\hbar}$ per mode per spin



$$I = e\int \frac{dp}{2\pi\hbar} v(p) = \frac{e}{2\pi\hbar}\underbrace{\int dE}_{\text{strip } eV} = \frac{e}{2\pi\hbar}(\mu_L - \mu_R) = \frac{e^2}{2\pi\hbar}V$$

Conductance of a 1D channel, free electrons



$$I = \frac{e}{2\pi\hbar}\int_{\text{strip } eV} T_0(E)\,dE$$

$T_0(E)$ - transmission coefficient of the barrier

Current = sum of partial currents at different energy "slices"

For each "slice" $[E, E+dE]$, the partial current depends on T_0 at the same energy E only.

The linear conductance

$$G = \frac{e^2}{2\pi\hbar}T_0(E_F)$$

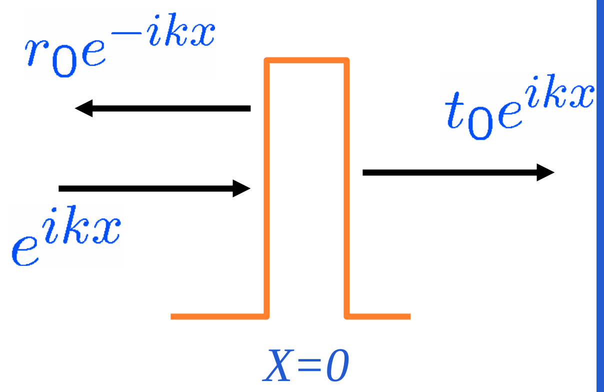
(Landauer formula).

Friedel oscillation (Friedel, 1952)

Reflection at the barrier changes all electron states, including those with energy $E<E_F$.

r_0 - reflection amplitude

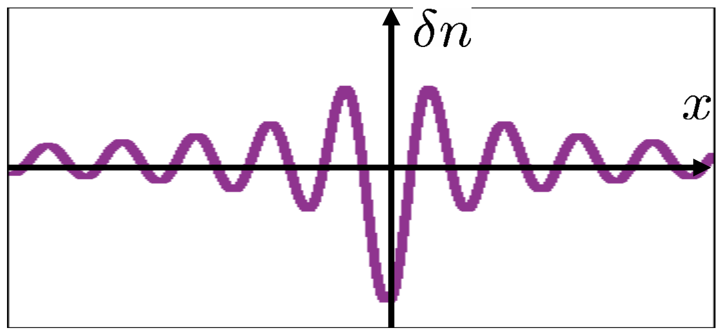
t_0 - transmission amplitude



$x < 0$

$$\psi_k(x) \sim \left(e^{ikx} + r_0e^{-ikx}\right)$$

$$\delta n(x) \sim -\frac{|r_0|}{|x|}\sin[2k_F|x| - \delta], \quad |x| \gg \lambda_F$$



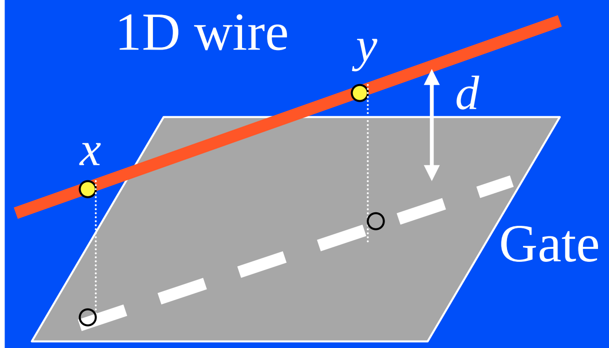
Friedel oscillation: Hartree potential

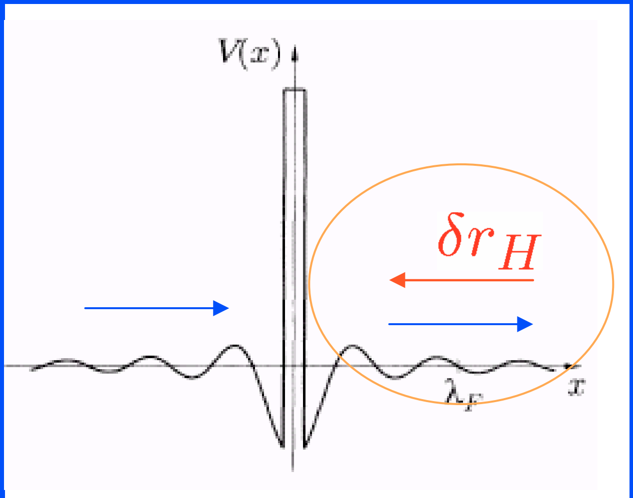
Hartree potential

$$V_H(x) = \int dx_1 V(x - x_1)\delta n(x_1)$$

oscillates with the period $\lambda_F/2$; at $|x| \gg d$

$$V_H(x) \sim |r_0| V(2k_F) \frac{\sin[2k_F|x| - \delta]}{|x|}$$





Scattering off the Friedel oscillation:

$$\delta r_H \sim r_0 \frac{V(2k_F)}{\hbar v_F} \int_d^\infty dx e^{2ikx} \frac{\sin(2k_F x)}{x}$$

$$\sim r_0 \frac{V(2k_F)}{\hbar v_F} \ln \frac{1}{|k_F - k| d}$$

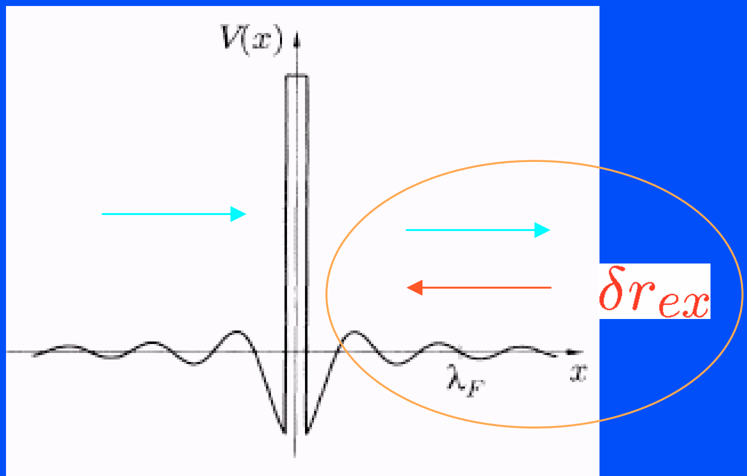
Exchange contribution — similar

Friedel oscillation: Exchange contribution

Exchange potential

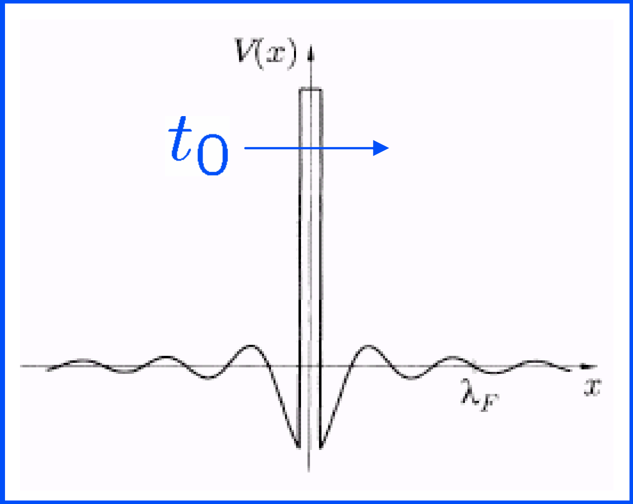
$$V_{ex}(x,y) = V(x-y)\int_0^{2k_F}\frac{dk}{2\pi}\psi_k^*(x)\psi_k(y)$$

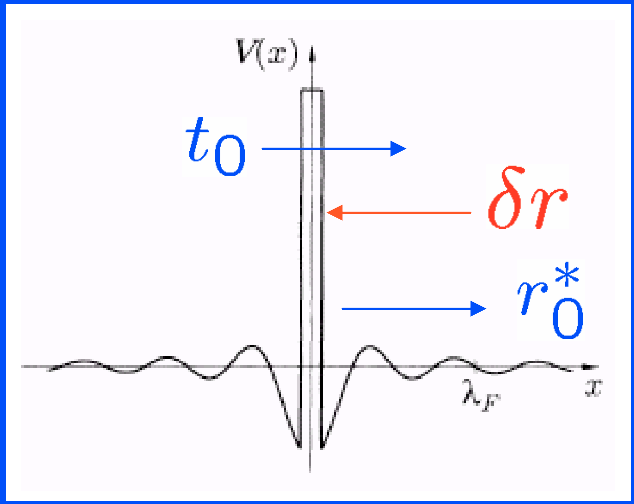
$$\sim \frac{V(x-y)}{x+y} r_0 \exp[-ik_F(x+y)]$$

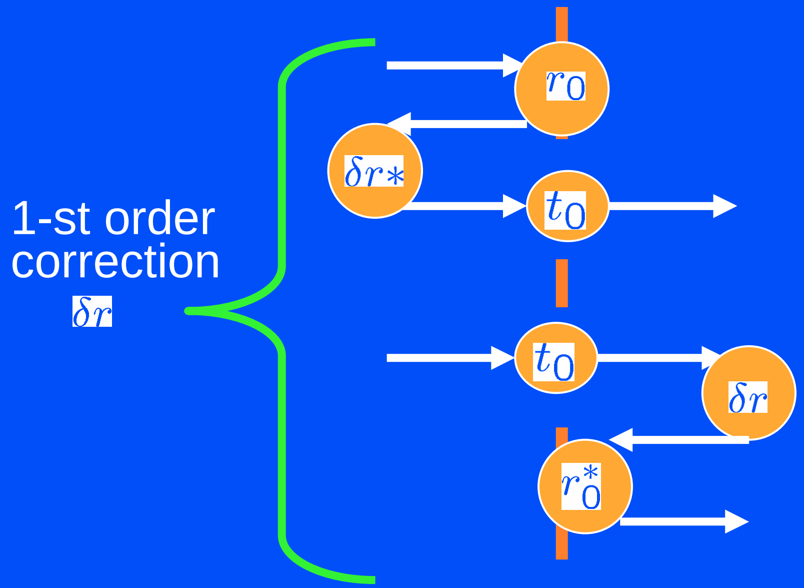


$$\delta r_{ex} \sim r_0 \frac{V(k=0)}{4\pi\hbar v_F} \ln\frac{1}{|k_F-k|d}$$

Transmission modified by the Friedel oscillation



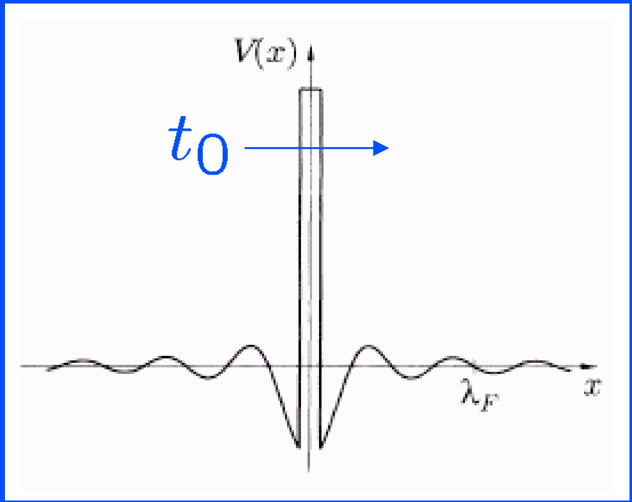


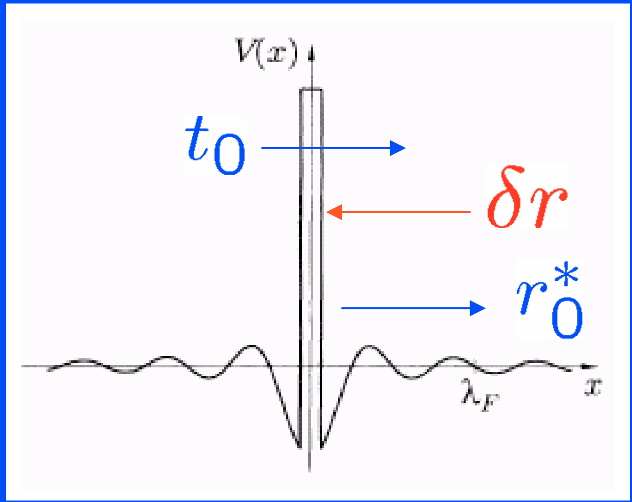


Transmission modified by the Friedel oscillation

Transmission coefficient of a "composite" barrier:

$$T = T_0 + 2T_0 \mathrm{Re}(r_0^* \delta r)$$





correction to T_0:

Hartree

exchange

$$\delta T = T_0(1 - T_0)\frac{V(2k_F) - V(0)}{\pi\hbar v_F} \ln \frac{1}{|k_F - k|d}$$

First-order interaction correction to the transmission coefficient

Transmission coefficient becomes **energy-dependent** :

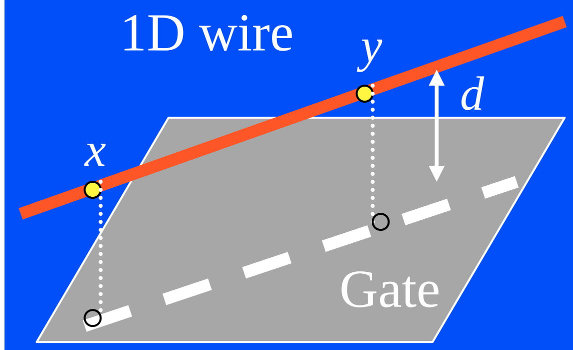
$$\boxed{\delta T(\varepsilon) = -2\alpha T_0(1-T_0)\ln\left|\frac{D_0}{\varepsilon}\right|}$$

$$\varepsilon = \hbar v_F(k-k_F)$$

$$D_0 = \hbar v_F/d$$

$$\boxed{\alpha = \frac{1}{2\pi\hbar v_F}\left[V(0) - V(2k_F)\right]}$$

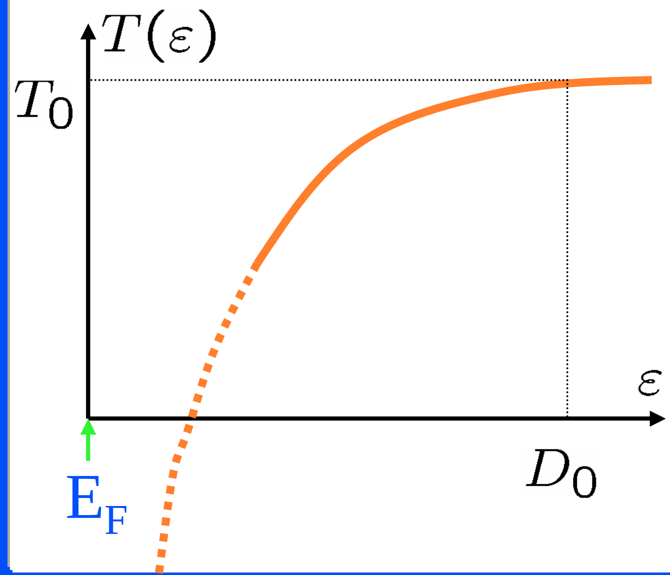
suppression of the transmission — enhancement of the transmission



$$\alpha \approx \frac{e^2}{\hbar v_F}\ln(k_F d)$$

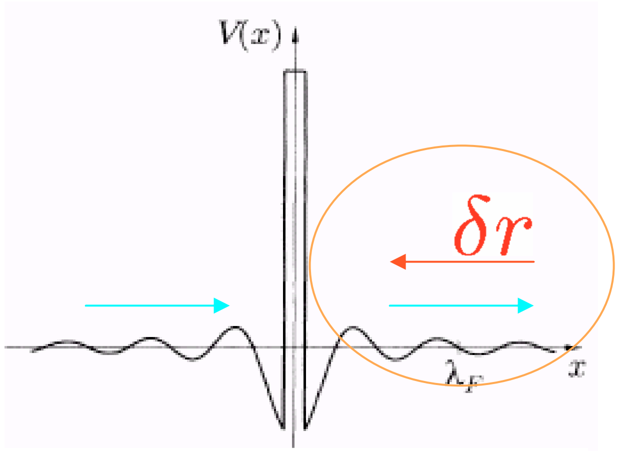
at $k_F d \gg 1$

The first-order correction diverges at low energies:



Cure: the leading—logarithm approximation

$$\delta r \sim r_0 \alpha \int_d^\infty \frac{dx}{x} \sin(2k_F x) e^{2ikx}$$



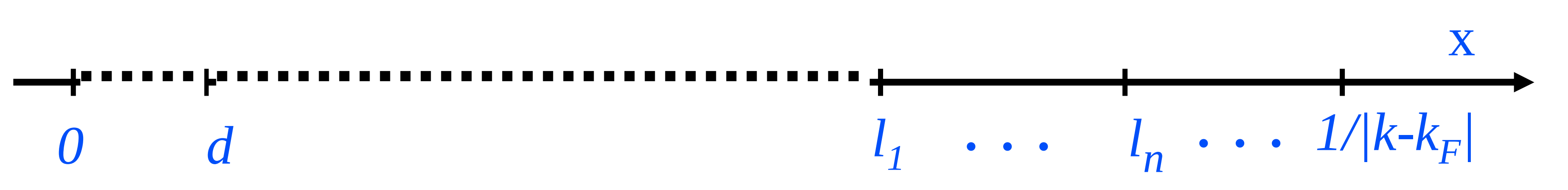
$$= r_0 \alpha \underbrace{\int_d^{\frac{1}{|k-k_F|}} \frac{dx}{x} \sin(2k_F x) e^{2ikx}}_{} + r_0 \alpha \int_{\frac{1}{|k-k_F|}}^\infty \frac{dx}{x} \sin(2k_F x) e^{2ikx}$$

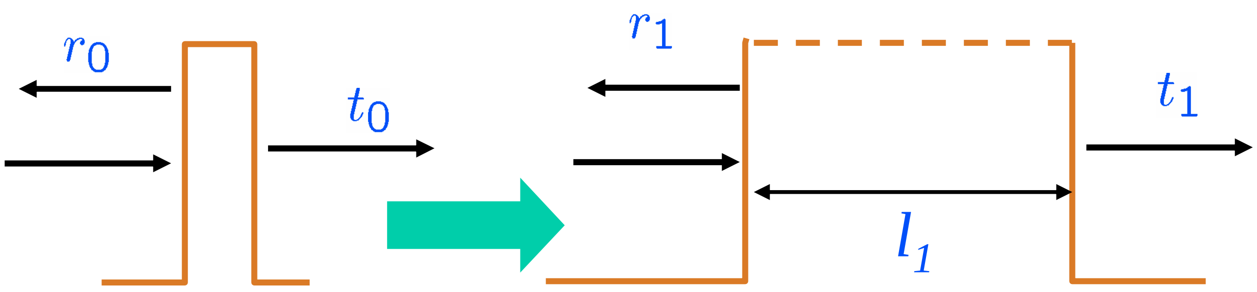
$$\boxed{\propto \alpha \ln \frac{1}{|k-k_F| d} = \alpha \ln \frac{D_0}{\varepsilon}}$$

Leading—log: sums up the most divergent terms, $\left[\alpha \ln \left|\frac{D_0}{\varepsilon}\right|\right]^n$, of the perturbation theory

Real-space RG

Split the important interval $[d,\ 1/|k-k_F|]$ on smaller pieces, so that $l_n - l_{n-1} \gg d$, but $\alpha \int_{l_{n-1}}^{l_n} dx/x \ll 1$





$$T_0 \quad \Longrightarrow \quad T_1 = T_0 + \delta T_1$$

Transmission in the leading-log approximation

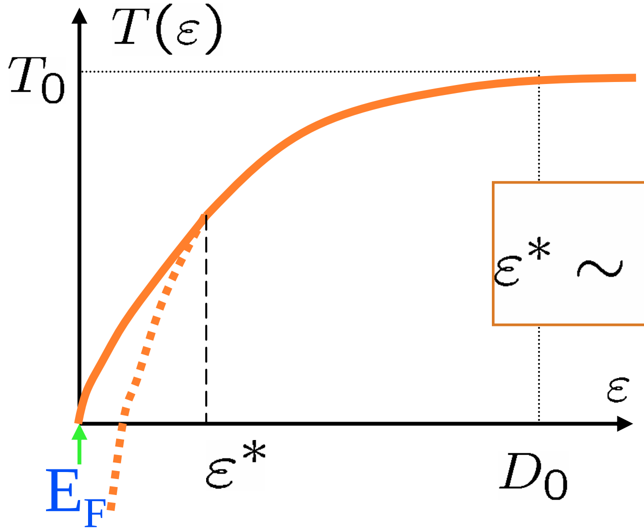
$$\boxed{T_n - T_{n-1} = -2\alpha T_{n-1}(1 - T_{n-1}) \ln \frac{l_n}{l_{n-1}}}$$

$$1 \leq n \leq D_0/|\varepsilon|$$

$$\boxed{T(\varepsilon) = \frac{T_0 \left|\frac{\varepsilon}{D_0}\right|^{2\alpha}}{R_0 + T_0 \left|\frac{\varepsilon}{D_0}\right|^{2\alpha}}}$$

$$R_0 \equiv 1 - T_0$$

The leading-log result does not diverge



$$\varepsilon^* \sim \left(\frac{R_0}{T_0}\right)^{\frac{1}{2\alpha}} D_0$$

Conductance in the leading-log. approximation

scattering remains elastic → Landauer formula works

$$G(k_B T) = \frac{e^2}{2\pi\hbar}\int d\varepsilon \left(-\frac{df_F}{d\varepsilon}\right) T(\varepsilon)$$

Within log-accuracy:

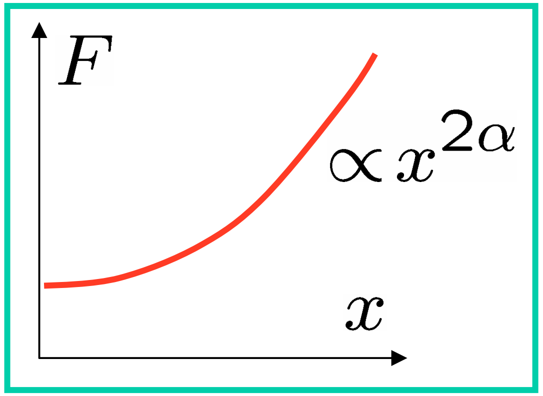
$$G(k_B T) = \frac{e^2}{2\pi\hbar}\frac{T_0\left|\frac{k_B T}{D_0}\right|^{2\alpha}}{R_0 + T_0\left|\frac{k_B T}{D_0}\right|^{2\alpha}}$$

At low energies

$$\left[k_B T, eV \ll \varepsilon^* \sim \left(\frac{R_0}{T_0}\right)^{\frac{1}{2\alpha}} D_0\right]$$

$$\boxed{\frac{dI}{dV} = V^{2\alpha} F\left(\frac{k_B T}{eV}\right)}$$

scaling
$F(x)$



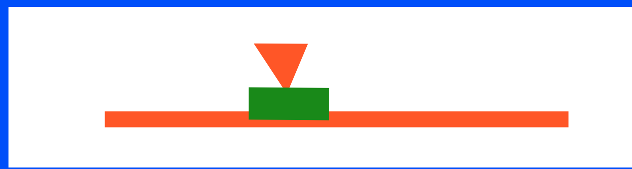
Effects of interaction – Friedel oscillation picture

1. Tunneling across a barrier is modified, $\left.\frac{dI}{dV}\right|_{T=0} \propto (eV)^{2\alpha}$

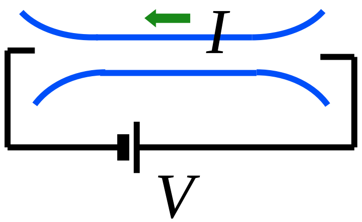
2. No barrier ⇨ no Friedel oscillation; properties of an ideal 1D channel are not modified **?**

Tunneling into the "bulk":

$\left.\frac{dI}{dV}\right|_{\text{bulk}} \propto \text{const}$



$I = GV$



Two-terminal conductance remains quantized $G = \frac{e^2}{2\pi\hbar}$

1D electron fluid: phenomenology

Dynamical variable:
displacement of a unit 1D volume $u(x,t)$

Lagrangian:

$$\delta L = \delta K - \delta U = \frac{1}{2} n_0 m \left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\frac{\partial \mu}{\partial n} n_0^2 \left(\frac{\partial u}{\partial x}\right)^2$$

Kin. energy Potential energy

External field: $\delta U_{\text{ext}} = e n_0 E(x,t) u$

Wave equation:

$$\frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} = -\frac{e}{m} E(x,t)$$

Conductivity & charge waves

$$\frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} = -\frac{e}{m} E(x,t)$$

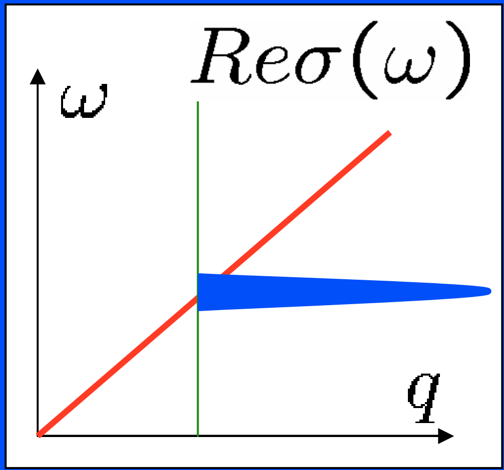
$$v = \left[v_F^2 + \frac{2}{\pi} V(0) v_F \right]^{\frac{1}{2}} > v_F$$

Velocity of 1D plasmon wave

Fermi gas rigidity + Coulomb repulsion

current: $$I(x,t) = e n_0 \frac{\partial u(x,t)}{\partial t}$$

$$\sigma(q,\omega) = \frac{e^2 v_F}{\pi \hbar} \frac{-i\omega}{(qv)^2 - \omega^2 - i0 \cdot \omega}$$



Quantum wires - continuation

From conductivity to the conductance

Ideal (homogeneous) wire

$$I(x,\omega) \qquad V = \int_0^L E(x,\omega)$$

$$\int dx_1 \cancel{e^{-iqx_1}} E(x_1,\omega)$$

$$\omega \to 0 \Rightarrow q = \omega/v \to 0 \ [\text{pole in } \sigma(q,\omega)]$$

$$\lim_{\omega\to 0} I(x,\omega) = I$$ independent of x

$$\frac{I}{V} = G = \frac{e^2}{2\pi\hbar} g; \ \ g = \frac{v_F}{v}$$

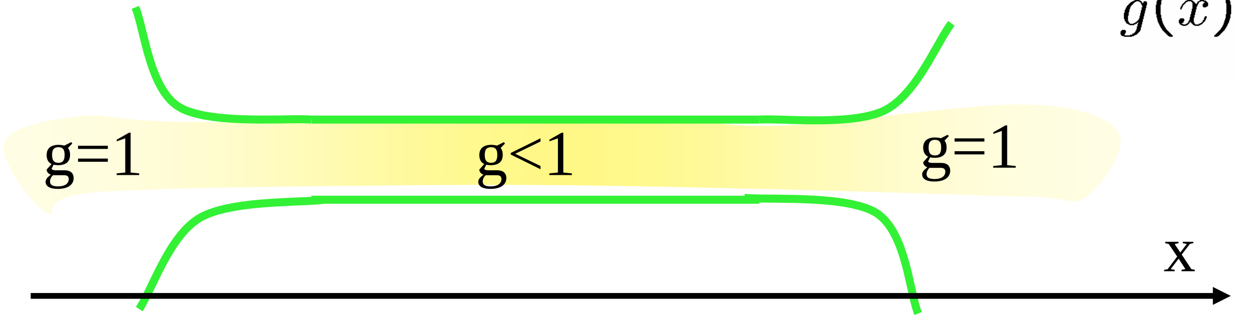
Conductance G(ω)=emission of plasmon waves of wavelength ~v/ω.

Conductance of a finite channel

Dissipative conductance=emission of plasmons of wavelength ~v/ω.

Finite-length channel: *g(x)* slowly varies in space

$$g(x) = \frac{v_F}{v(x)}$$



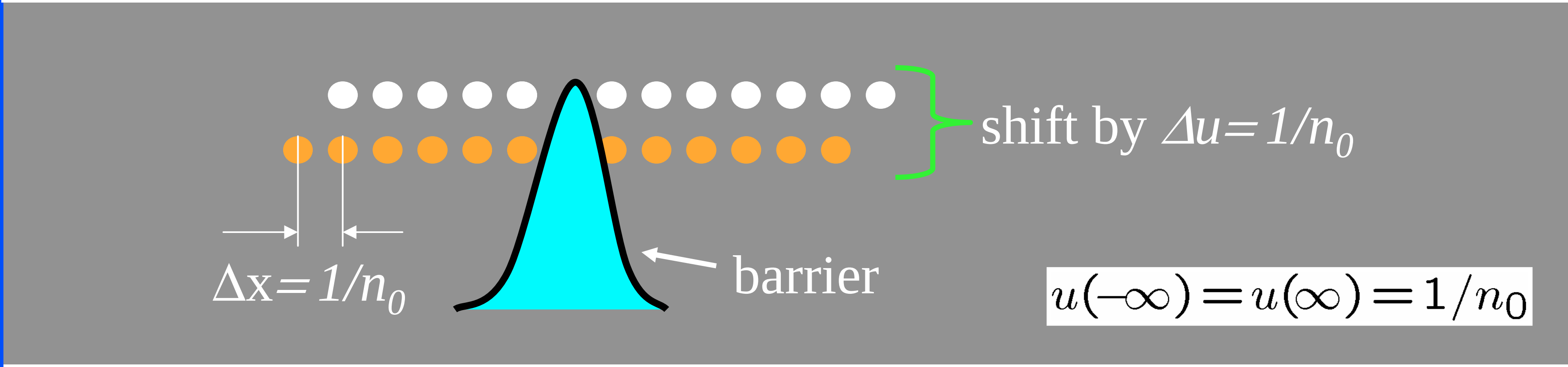
$$\boxed{G(\omega) = \frac{e^2}{2\pi\hbar} g(x \sim v/\omega)}$$

Outside the channel: $g(x \to \pm\infty) \to 1 \Rightarrow \quad G_{\mathrm{dc}} = \frac{e^2}{2\pi\hbar}$

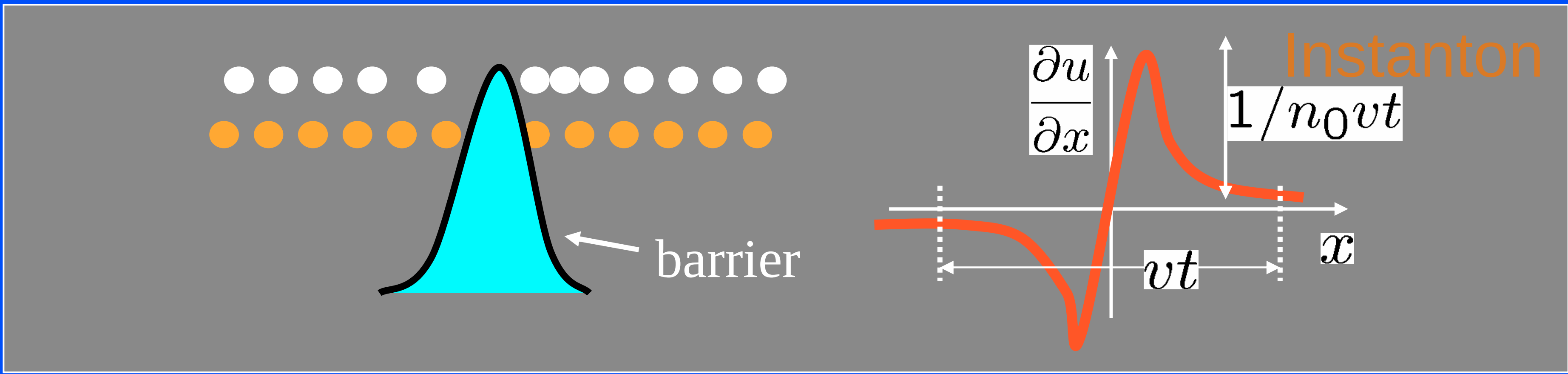
regardless the interaction strength within the channel

Tunneling across a barrier

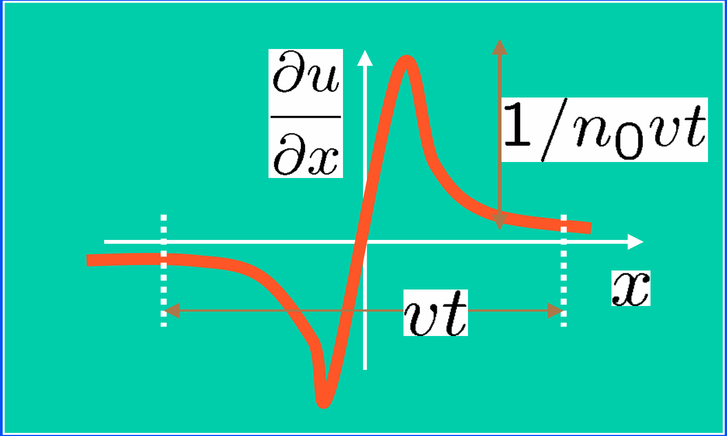
A barrier reveals the discreteness of the fermions; the Hamiltonian must be invariant only under the discrete shifts



Each particle changed state – zero overlap of the old and new ground states (Anderson Orthogonality Catastrophe)



Tunneling amplitude



Instanton action (WKB tunneling):

$$S(t) \sim \int_{t_0}^{t} d\tau E(\tau) \sim i\frac{1}{g} \ln \frac{|t|}{t_0}$$

Tunneling amplitude:

$$A(\varepsilon) \propto \int dt e^{-i\varepsilon t} \exp\left[iS(t)\right] \propto |\varepsilon|^{\frac{1}{g}-1}$$

Tunneling rate:

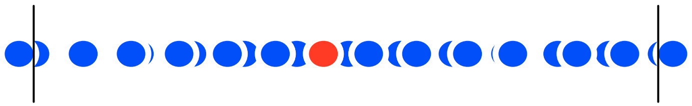
$$T(\varepsilon) \propto |\varepsilon|^{2\left(\frac{1}{g}-1\right)}$$

$$g = v_F/v$$

Weak interaction: $\frac{1}{g} - 1 \rightarrow \alpha$ fits perturbation theory

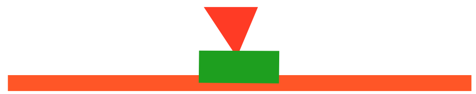
Tunneling density of states

New particle ⇨ a finite shift of the liquid



$$-u(-\infty) = u(\infty) = 1/2n_0$$

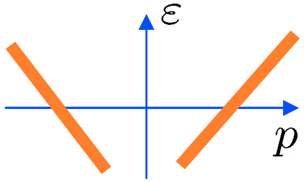
$$\left.\frac{dI}{dV}\right|_{\text{end}} \propto |V|^{2\left(\frac{1}{g}-1\right)} \qquad \left.\frac{dI}{dV}\right|_{\text{bulk}} \propto |V|^{\frac{1}{2}\left(g+\frac{1}{g}-2\right)}$$



$$g = v_F/v < 1$$

C.L. Kane, M.P.A. Fisher (1992)

$$\boxed{\nu_{\text{bulk}}(\varepsilon) = -\frac{1}{\pi}\mathrm{Im}\,G_R(x,x,\varepsilon)}$$



$$\varepsilon(p) = v_F(|p| - k_F)$$

Tomonaga (1950);
Luttinger (1963)

Dzyaloshinkii &
Larkin(1973)

Related work:
Luther & Peschel;
Luther & Emery
(1974)

Many modes – many fluids

Example: spin&charge $u_{\uparrow,\downarrow} = u_\rho \pm u_s$

rigidity: Fermi+Coulomb

u_ρ + - + - + - + - + - + - + - + - + - + - -

rigidity: Fermi only

u_s ↑↓↑↓↑↓↑↓↑↓↑↓↑↓↑↓

$v_\rho > v_F$

$$v = \left[v_F^2 + \cancel{\frac{2}{\pi} V(0) v_F} \right]^{\frac{1}{2}} > v_F$$

Velocity of 1D plasmon wave

Fermi gas rigidity

Coulomb repulsion

$v_s = v_F$

Tunneling density of states

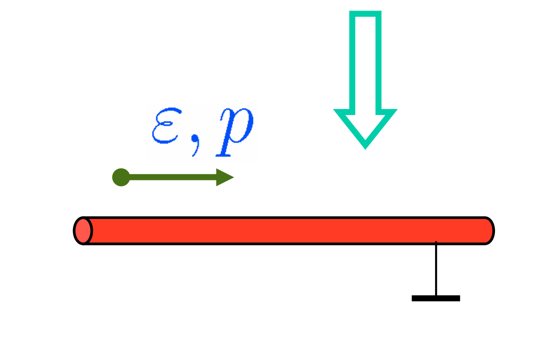
Inserting electron: finite shift $\Delta u = 1/n_0$

Many (N>>1) modes: $\Delta u = \sum \Delta u_i$

$\boxed{\nu(\varepsilon) \propto |\varepsilon|^\alpha}$ $\alpha \propto \sum (\Delta u_i)^2 \propto 1/\sqrt{N}$

Spectral Function

$$A(\varepsilon,p) = -\frac{1}{\pi}\mathrm{Im}G_R(p,\varepsilon)$$



$$G_R(\varepsilon,k) = \frac{1}{\varepsilon - \xi(k) - \Sigma(\varepsilon,k)} = \frac{1}{\varepsilon - \tilde{\xi}_k - i/2\tau_\varepsilon}$$

Free electrons: $1/2\tau_\varepsilon = 0$ $A(\varepsilon,p) \propto \delta(\varepsilon - \xi(p))$

Interacting fermions: 1D vs. 3D

$$G_R(\varepsilon,k) = \frac{1}{\varepsilon - \xi(k) - \Sigma(\varepsilon,k)} = \frac{1}{\varepsilon - \tilde{\xi}_k - i/2\tau_\varepsilon}$$

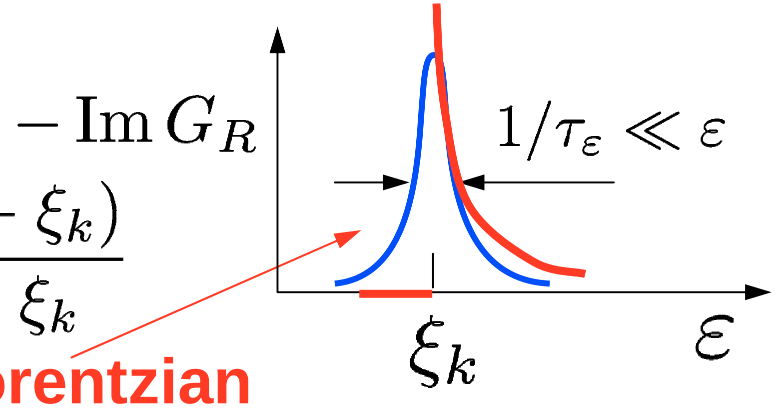
3D Fermi liquid:

$$-\mathrm{Im}\Sigma(\varepsilon=\xi_k) = \frac{1}{2\tau_\varepsilon} \propto r_s^2 \frac{\varepsilon^2}{\epsilon_F}$$

$$r_s = e^2/\hbar v_F$$

$$A(\varepsilon,k) = -\mathrm{Im}\, G_R(\varepsilon,k) \propto V_{LR}^2 \frac{\theta(\varepsilon-\xi_k)}{\varepsilon-\xi_k}$$

spectral function

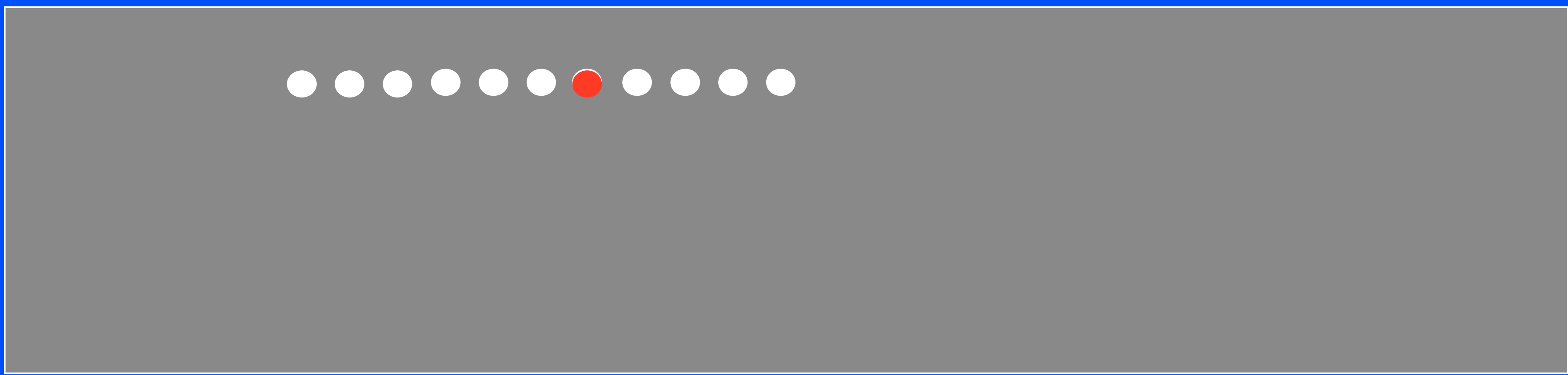


Construction of the Fermion creation operator

$$\hat{\varphi} = n_0 \hat{u}$$

$$[\theta(x), \nabla\varphi(y)] = \pi\delta(x-y)$$

$$\psi^\dagger(x) \propto e^{i\theta(x)}$$

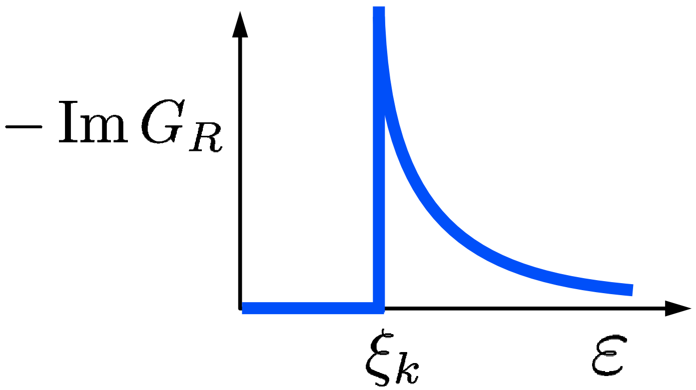


Spectral Function in a Luttinger Liquid

threshold

$$-\operatorname{Im} G_R(\varepsilon,k) \propto \alpha^2 \frac{\theta(\varepsilon-\xi_k)}{\varepsilon-\xi_k}\left\{1-\alpha^2\ln\left(\frac{\varepsilon}{\varepsilon-\xi_k}\right)+\ldots\right\}$$

$$\approx \alpha^2 \frac{\theta(\varepsilon-\xi_k)}{\varepsilon-\xi_k}\left[\frac{\varepsilon}{\varepsilon-\xi_k}\right]^{-\alpha^2}$$



$G_R(\varepsilon,k)$: **branch-cut**
rather than quasiparticle pole

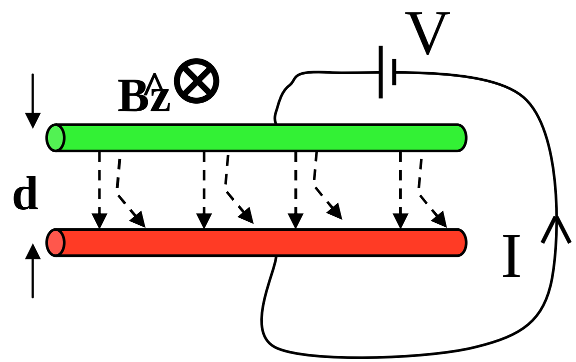
arbitrary interaction strength: $\alpha^2 \rightarrow \frac{1}{2}\left(g+\frac{1}{g}-2\right)$

- **exact** for **linear** spectrum $\xi_k = \pm v k$

Spectral function: Experiment

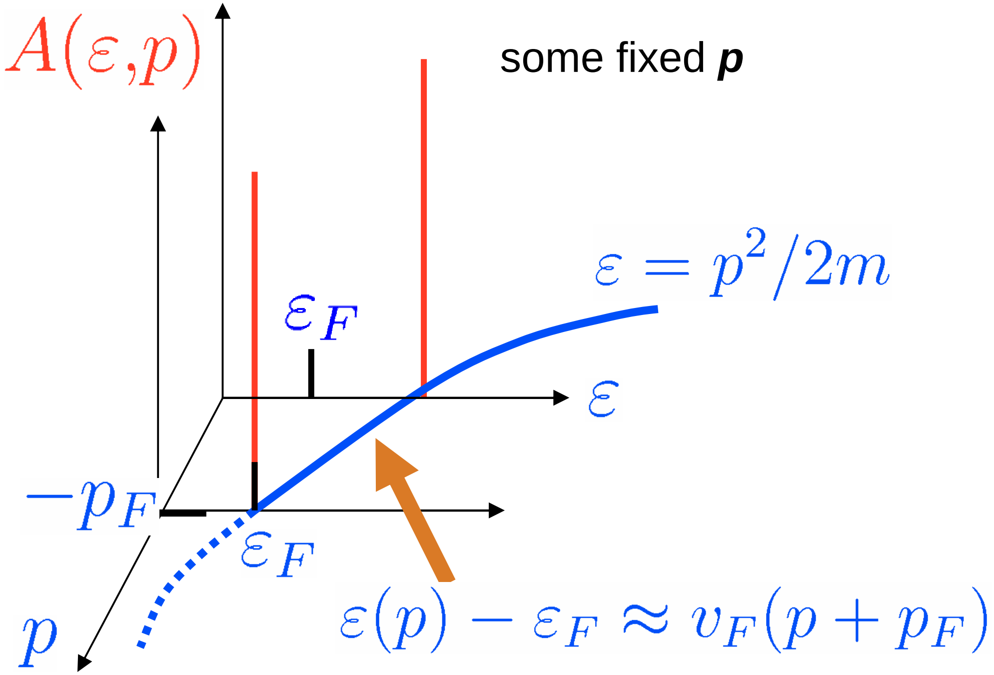
A. Yacoby group, Weizmann Inst., 2003-…

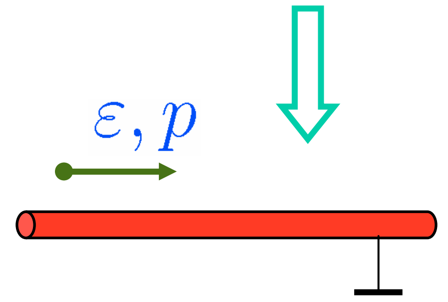
Tunneling between parallel wires in a magnetic field



momentum “boost” $\Delta p = edB/c$

Spectral density – free electrons

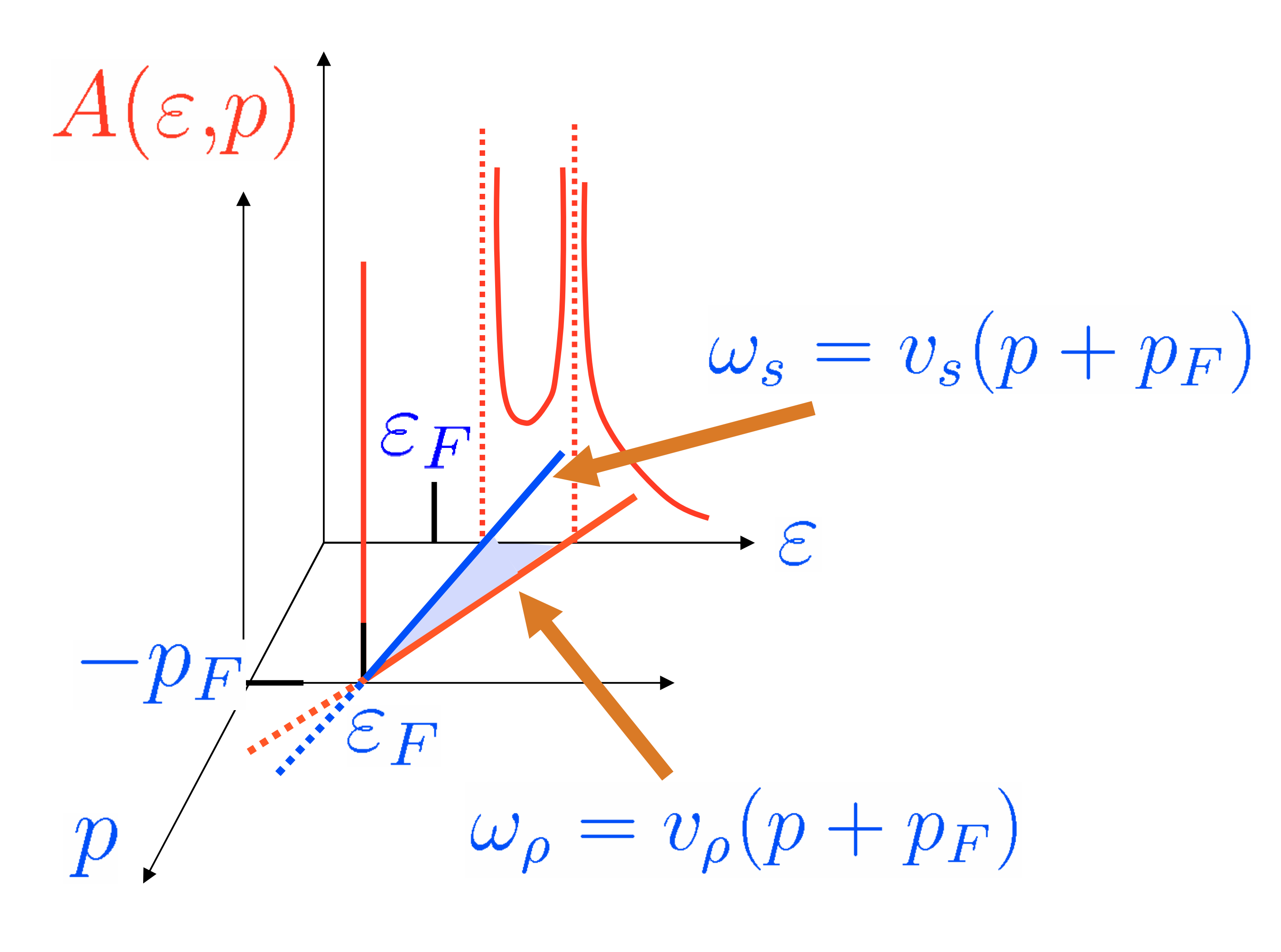




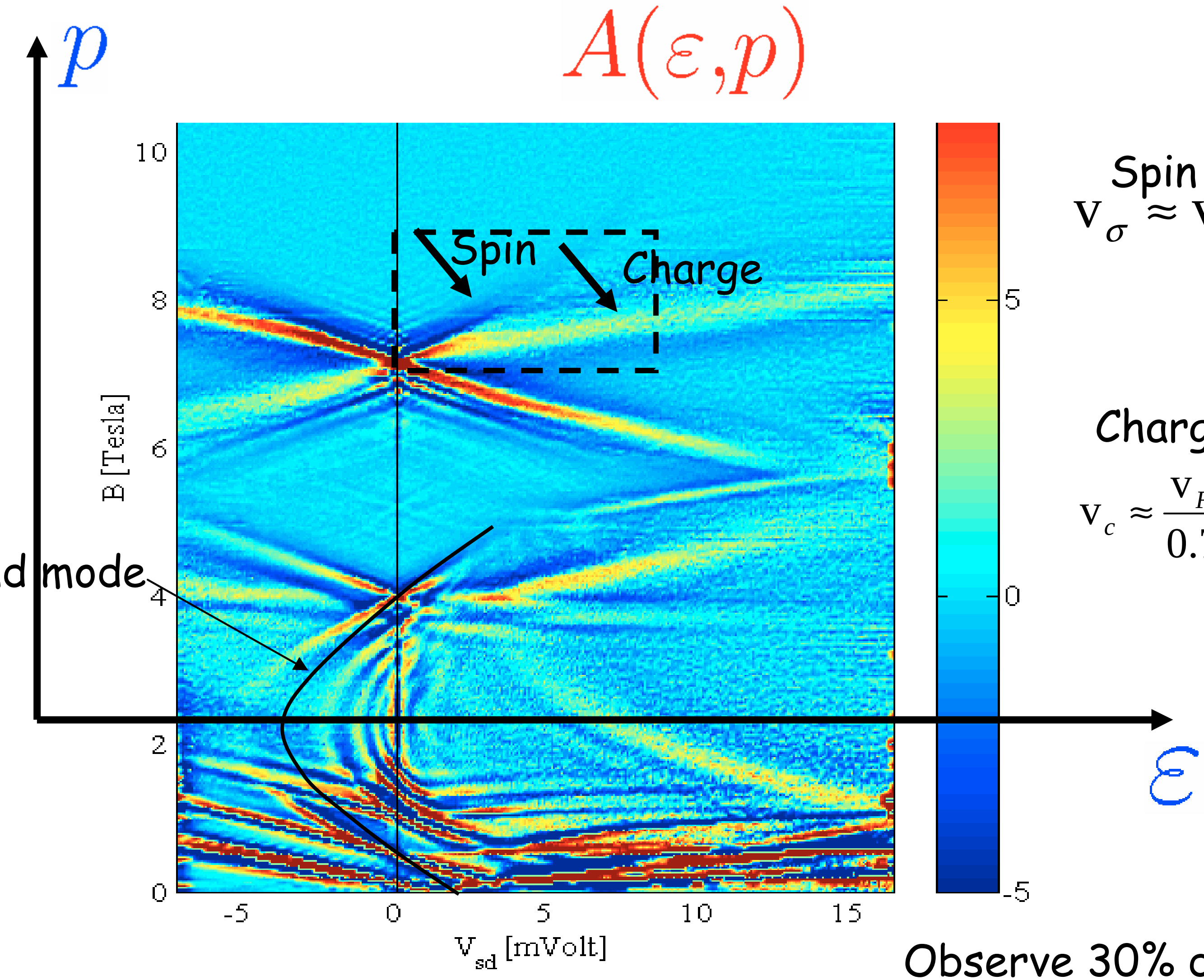
$$A(\varepsilon,p) = -\frac{1}{\pi}\mathrm{Im}G_R(p,\varepsilon)$$

Experiment: Charge-Spin Separation

Spectral density: spin and charge modes



Experiment: Charge-Spin Separation



Spin
$$v_\sigma \approx v_F$$

Charge
$$v_c \approx \frac{v_F}{0.7}$$

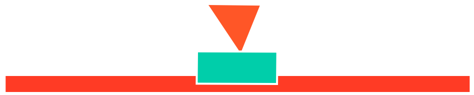
Observe 30% deviations

Tunneling Experiments: Carbon Nanotubes

Single-wall nanotubes – 4-mode (incl. spin) Luttinger liquids

tunneling density of states:

$$\left.\frac{dI}{dV}\right|_{\text{end}} \propto |V|^{\alpha_{\text{end}}}$$

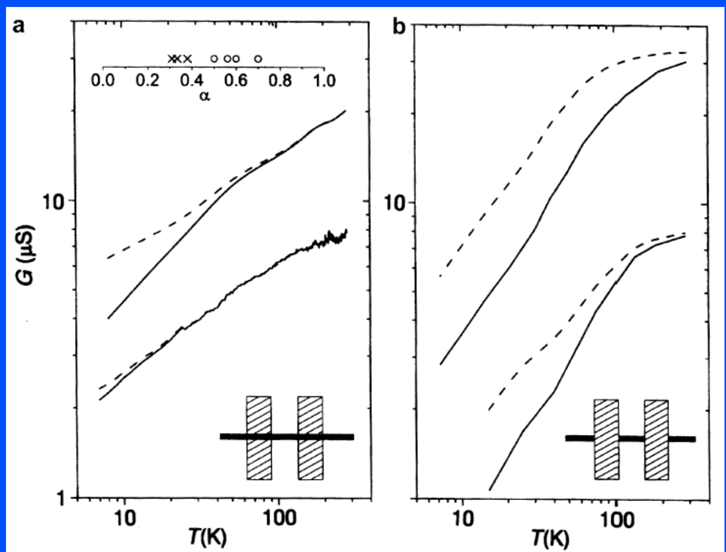


$$\left.\frac{dI}{dV}\right|_{\text{bulk}} \propto |V|^{\alpha_{\text{bulk}}}$$

$$\alpha_{\text{end}} = \frac{1}{4}\left[\frac{1}{g} - 1\right] \qquad \alpha_{\text{bulk}} = \frac{1}{8}\left[g + \frac{1}{g} - 2\right]$$

+ data scaling

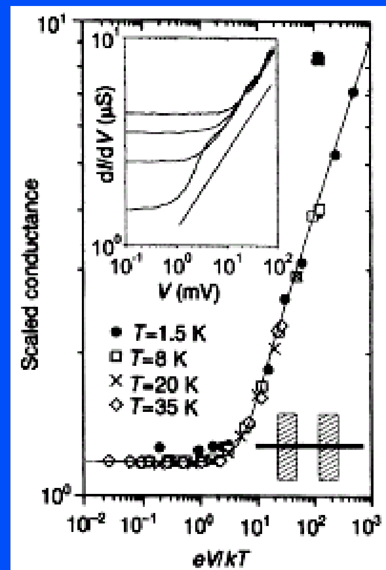
$$\frac{dI}{dV} = V^{\alpha} f\left(\frac{V}{T}\right)$$



$$\alpha_{\text{bulk}} \approx 0.3$$

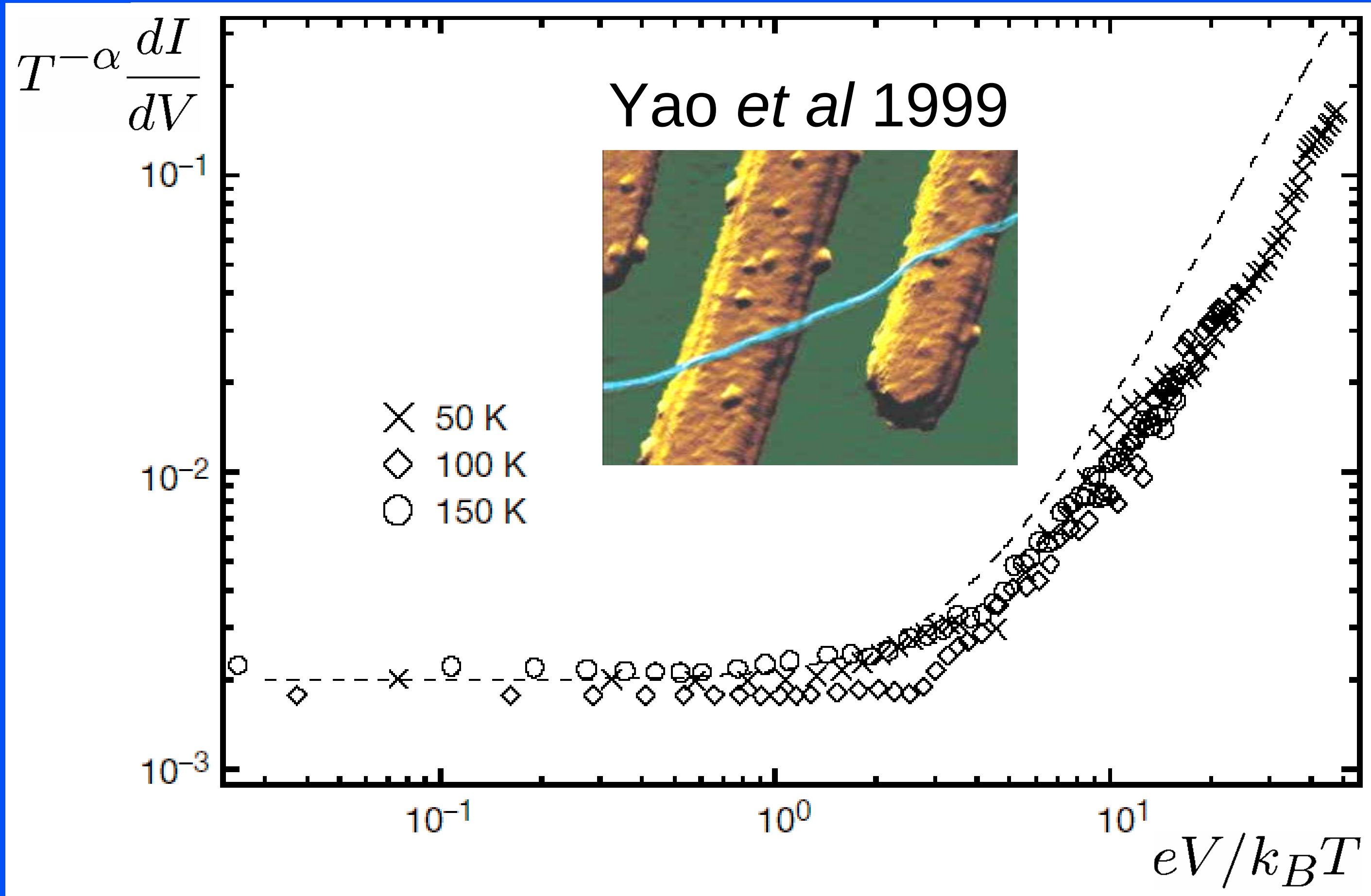
$$\alpha_{\text{end}} \approx 0.6$$

Bockrath *et al* 1999



Carbon nanotubes – tunneling

corroborating experiment



$$\alpha_{\text{end}} \approx 2\alpha_{\text{bulk}}$$

corresponds to $g \approx 0.22$

But…

Carbon nanotubes – tunneling

In a multi-wall nanotube *dI/dV* is also a power-law…

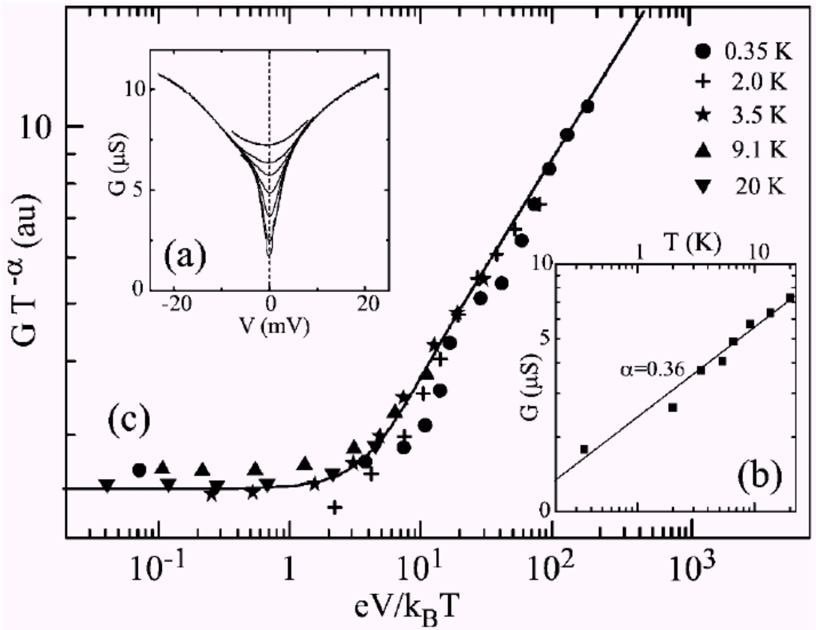


FIG. 2. (a) $G(V,T = \text{const}) = dI/dV$ of a second MWNT for $T = 0.35, \ldots, 20$ K. (b) The linear conductance $G(0,T)$ in a double logarithmic plot demonstrating power-law scaling. (c) $G(V,T)T^{-\alpha}$ versus eV/k_BT. Similar to the T dependence, $G \propto V^\alpha$ for $eV \gg k_BT$ with power $\alpha = 0.36$.

…instead of a different function (incl. disorder):

$$\nu(\epsilon, T) \propto \exp\left\{-\sqrt{\frac{\epsilon^*}{T}}\, F\left(\frac{\epsilon}{\sqrt{\epsilon^* T}}\right)\right\}$$

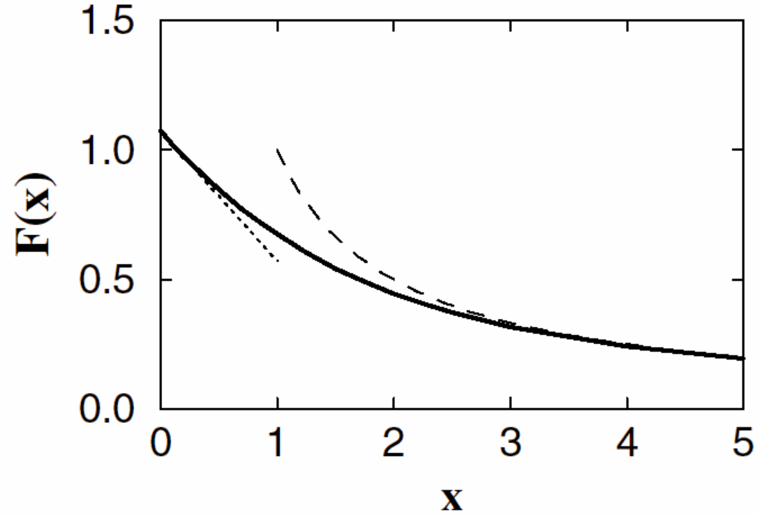


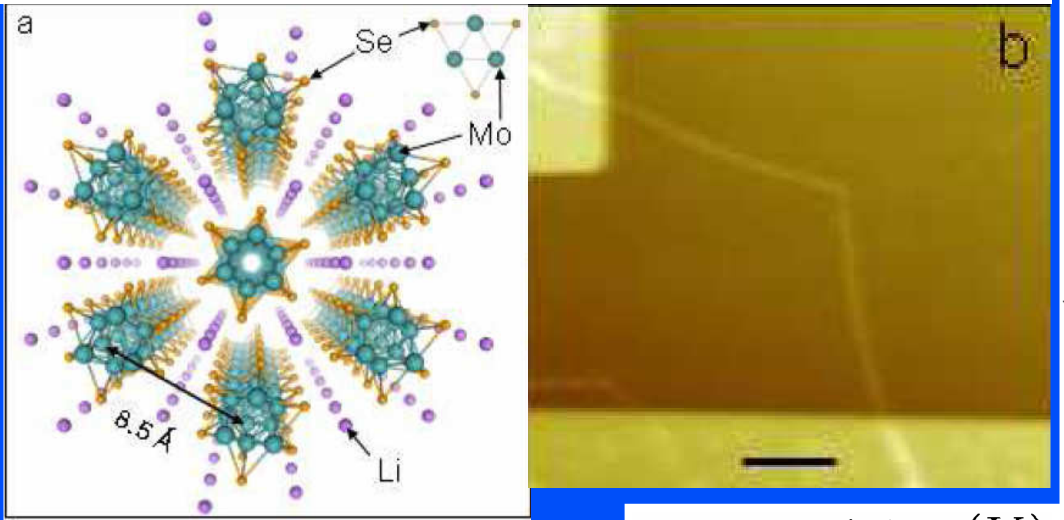
FIG. 1. The scaling function $F(x)$ and its asymptotics: $F(x) = 1.07 - x/2$ for $x \ll 1$ (dotted line), and $F(x) \sim 1/x$ for $x \gg 1$ (dashed line).

[Bachtold et al (2001)]

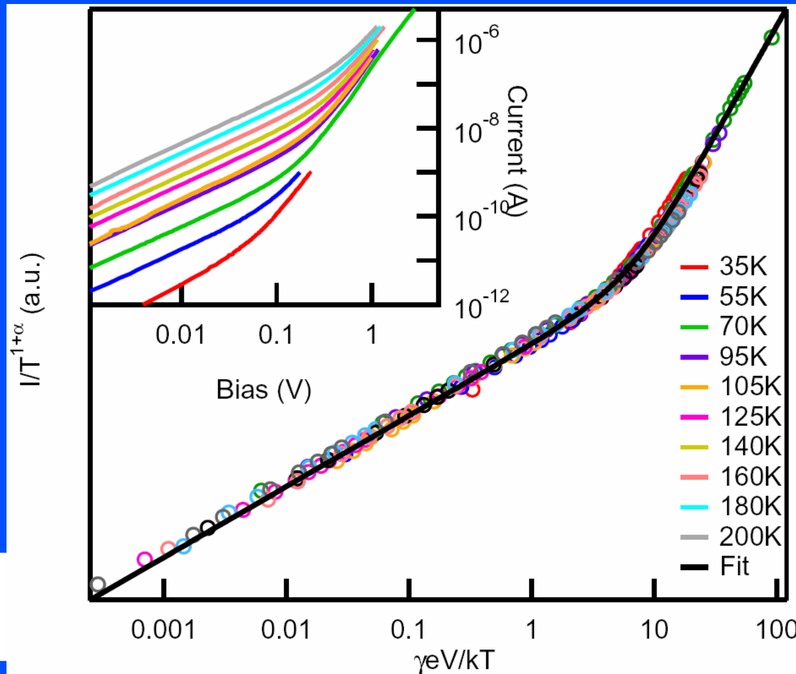
Variable number of modes: MoSe Nanowires

FIG. 1: (color online) (a) Structural model of a 7-chain MoSe nanowire along with the triangular Mo_3Se_3 unit cell,

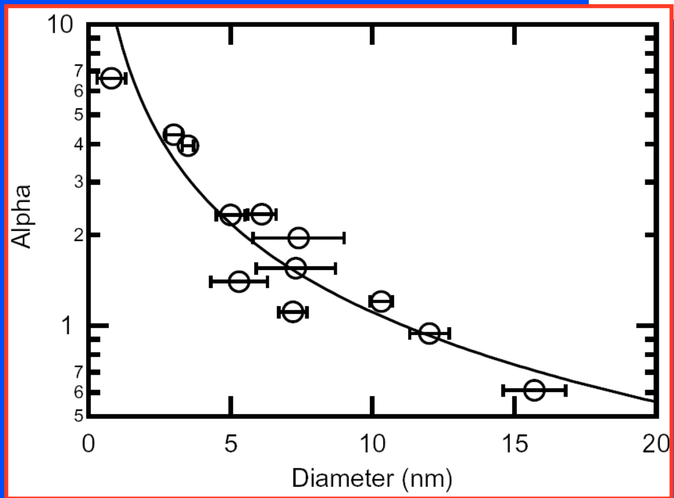
(b) AFM height image between two Au electrodes ~1μm apart



Philip Kim group
(cond-mat/Jan 2006)



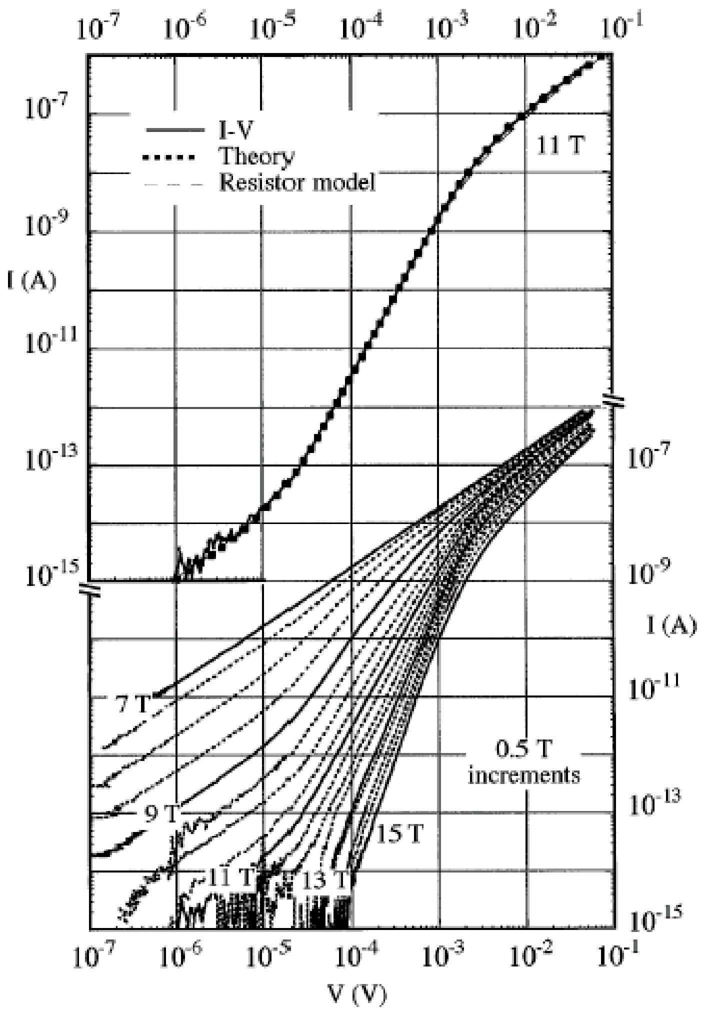
$$I = T^{\alpha+1} f\left(\frac{V}{T}\right)$$



$$\alpha = \frac{2}{N}\left[(1+NU)^{1/2} - 1\right]$$

$$N \propto (\text{Diameter})^2$$

Tunneling: Edge States

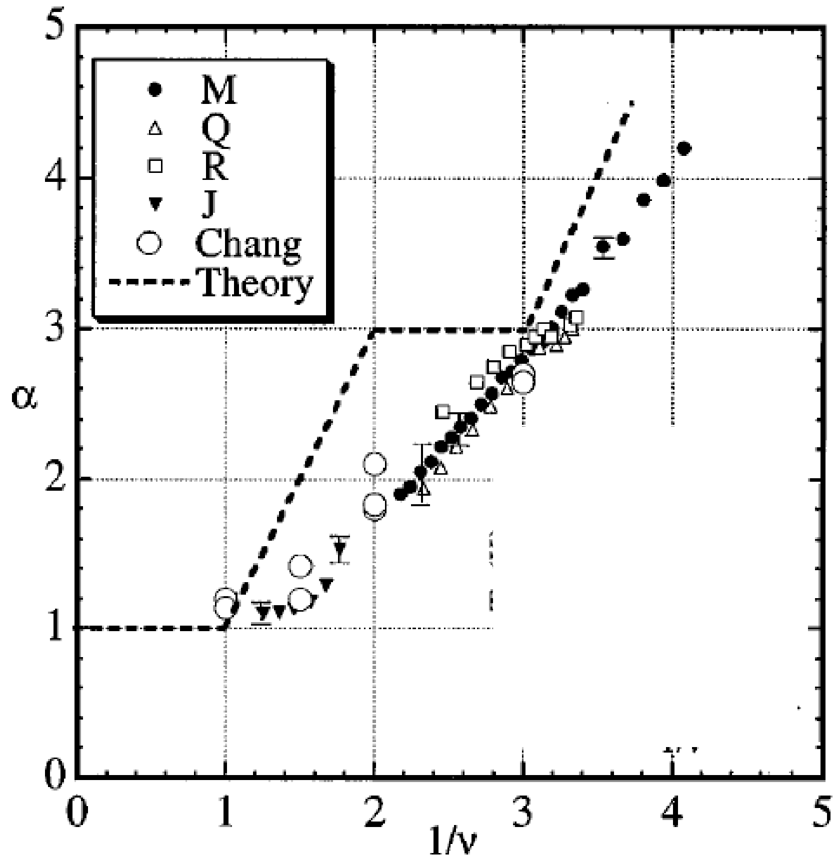


There is no predicted qualitative difference between the compressible and incompressible states; continuous evolution of current-voltage characteristics with ν.

[M. Grayson et al (1998)]

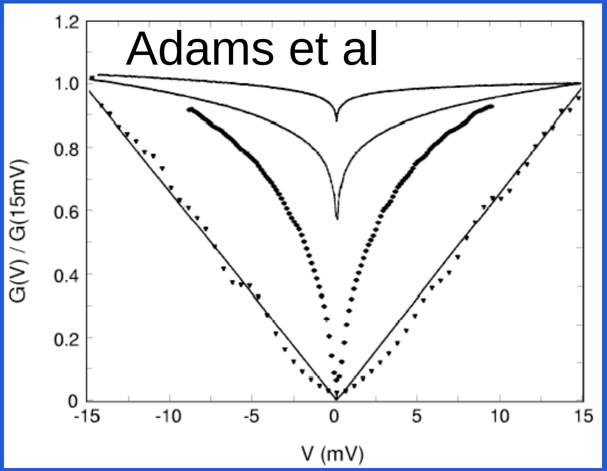
I(V)~V$^\alpha$

continuous set of α



Similar ideas in other dimensions

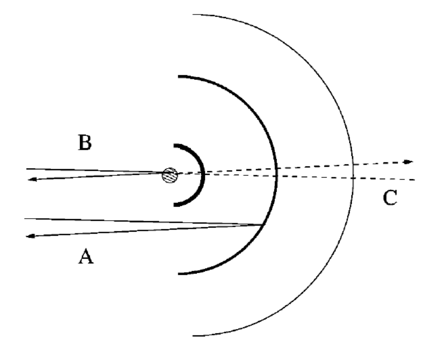
Scattering off a Friedel oscillation in D=2



Interaction anomaly in tunneling into a diffusive conductor (Altshuler,Aronov, Lee 1980s)

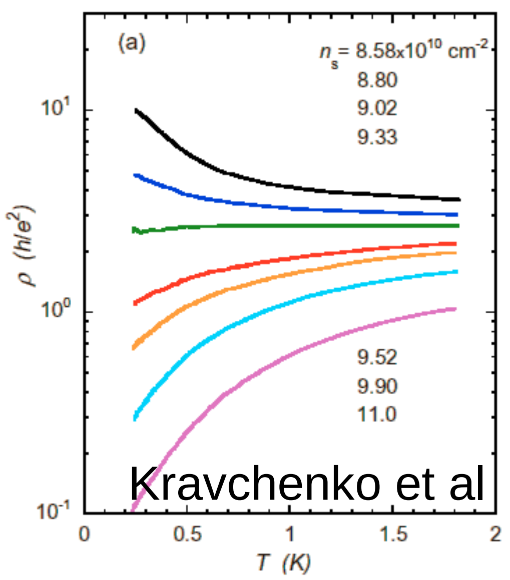
$$eV\tau/\hbar \ll 1$$

Back to 1D systems



Anomalies in quasi-ballistic conductors

arbitrary $eV\tau/\hbar$ Rudin, Aleiner, LG, 1997



Zala, Narozhny, Aleiner, 2001

Electron density waves = waves of classical fluid

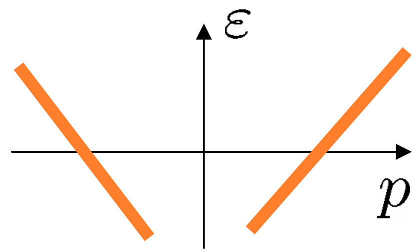
Electron tunneling = quantum motion of fluid

Quantized electron fluid = Luttinger liquid

Linear (hydro)dynamics of density waves

=

crucial simplification:



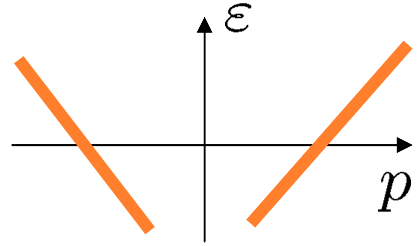
$$\varepsilon(p) = v_F(|p| - k_F)$$

Tomonaga (1950);
Luttinger (1963)

Tomonaga-Luttinger model

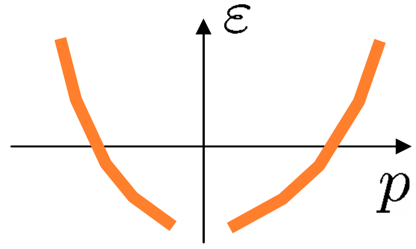
Beyond the Tomonaga-Luttinger model

crucial simplification:



$$\varepsilon(p) = v_F(|p| - k_F)$$

Tomonaga (1950);
Luttinger (1963)

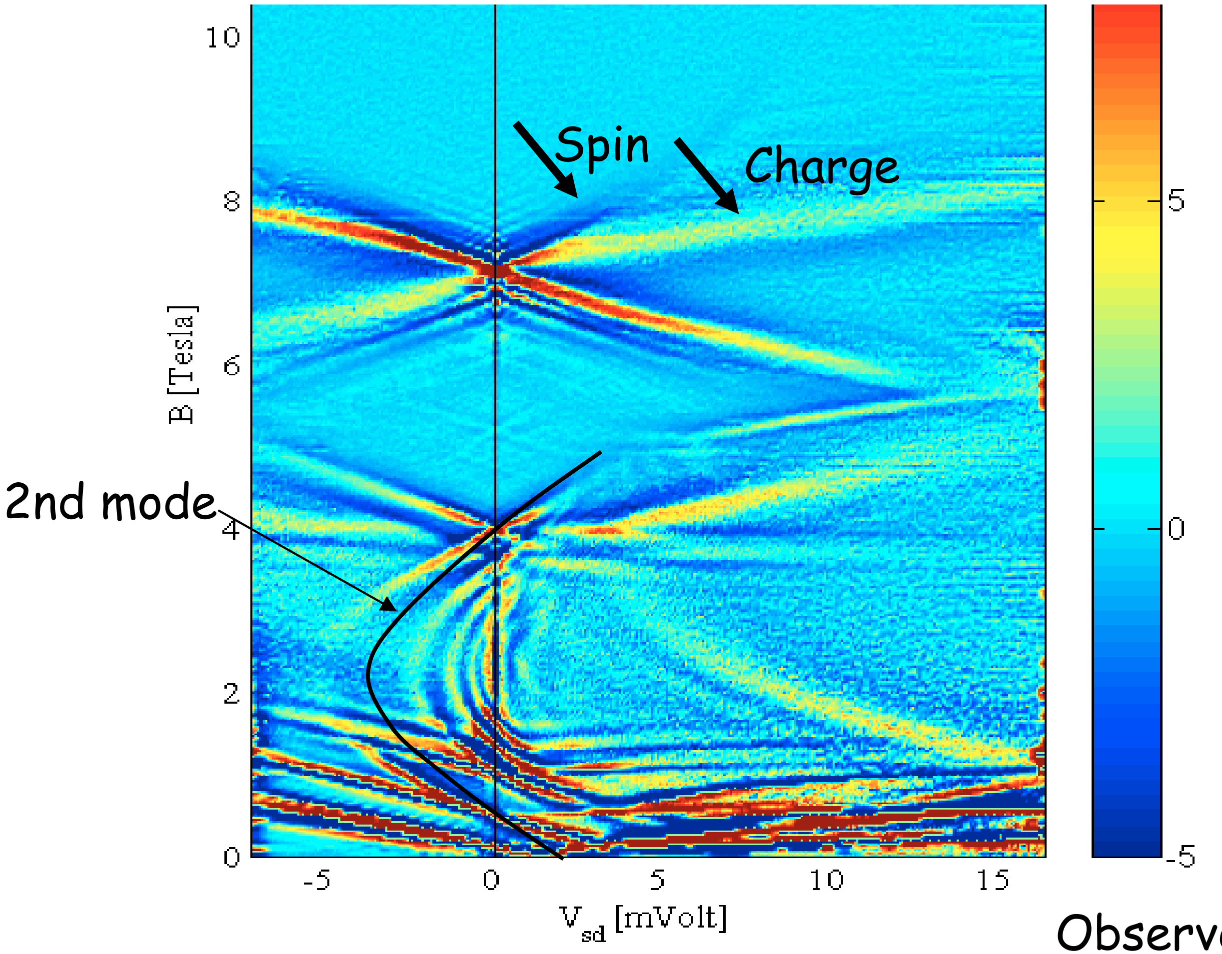


$$\varepsilon(p) = \frac{p^2}{2m} - \varepsilon_F$$

Non-linear dynamics of *quantum* waves

Spectral Function – big picture

$$A(\varepsilon, p)$$



Spin

$$v_\sigma \approx v_F$$

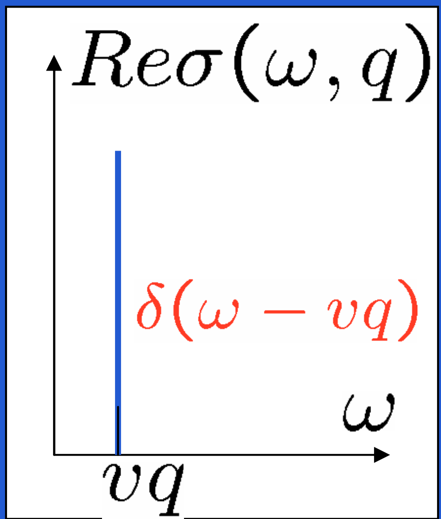
Charge

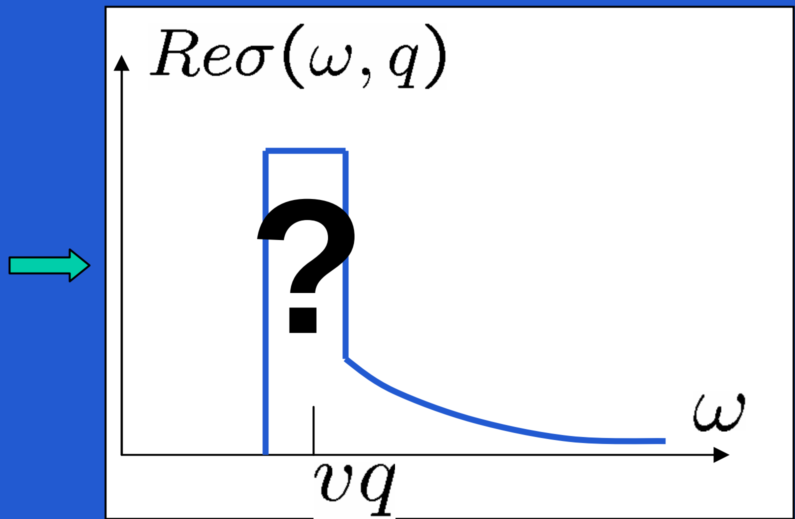
$$v_c \approx \frac{v_F}{0.7}$$

Observe 30% deviations

Dissipative part of conductivity

"free" plasmons





Structure factor, susceptibility, conductivity

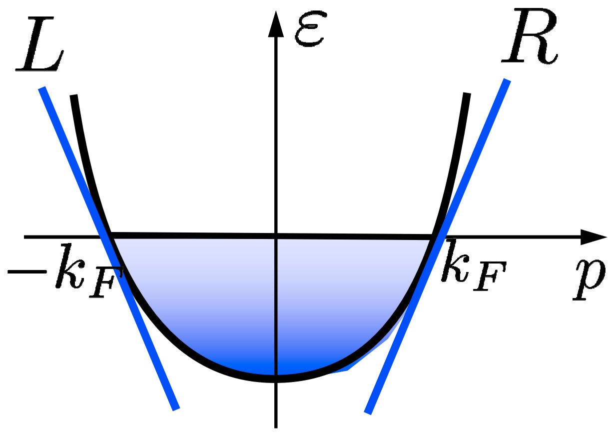
dynamic structure factor:

$$S(q,\omega) = \int dx\, dt\, e^{i(\omega t - qx)} \langle \hat{n}(x,t)\hat{n}(0,0)\rangle \;=\; 2\,\mathrm{Im}\chi(q,\omega)$$

$$\text{at } T = 0 \;\; (\text{FDT})$$

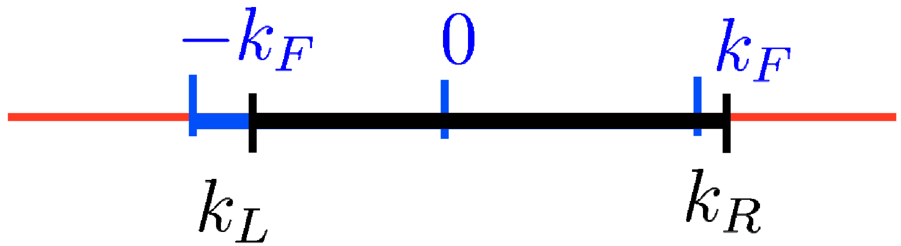
$$\mathrm{Re}\,\sigma(q,\omega) \propto \frac{1}{\omega} S(q,\omega)$$

Revisiting Tomonaga-Luttinger Model



$$\xi_k = \pm v(p \mp k_F)$$

Bosonisation:



$$k_{L,R}(x) = \pm k_F + 2\pi n_{R,L}(x) \;, \quad n_{L,R}(x) = \partial_x \varphi_{L,R}$$

$$H_{kin}(x) = \int_{k_L(x)}^{k_R(x)} \xi_k \, dk = \frac{v}{2}\left[(\partial_x\varphi_L)^2 + (\partial_x\varphi_R)^2\right]$$

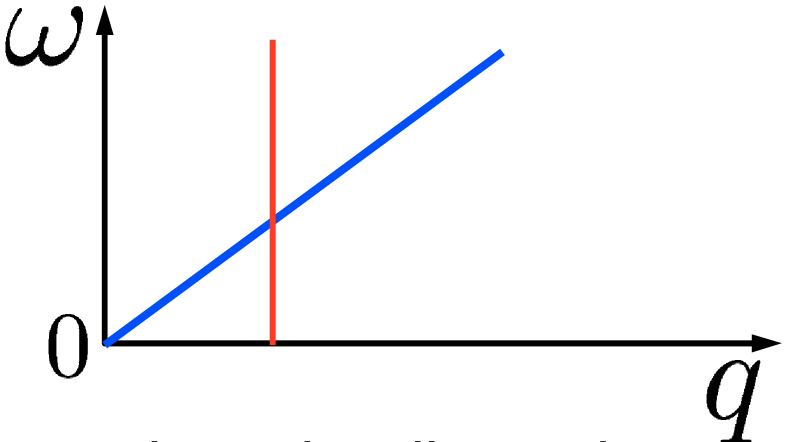
$$H_{int}(x) = V_{LL}(\partial_x\varphi_L)^2 + V_{RR}(\partial_x\varphi_R)^2 + 2V_{LR}(\partial_x\varphi_L)(\partial_x\varphi_R)$$

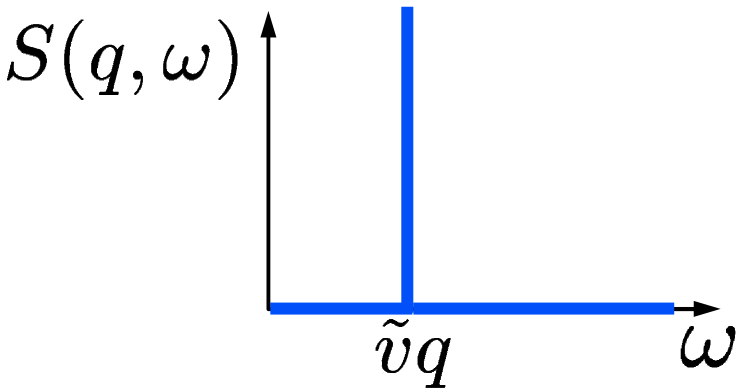
Structure factor for the linear spectrum

$$n(x) = \partial_x \varphi \qquad \text{“phonons”}$$

$$S(q,\omega) = \Big\langle n(q,\omega) n(-q,-\omega) \Big\rangle \propto \Big\langle \varphi(q,\omega)\varphi(-q,-\omega) \Big\rangle$$

$$\sim q\,\delta(\omega - \tilde{v} q)$$

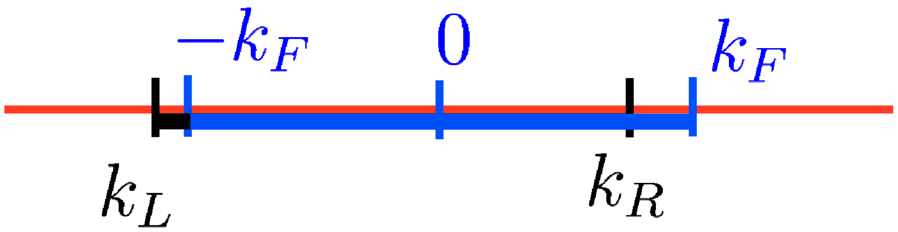




How does the dispersion curvature affect the structure factor ?

$$\xi_k = \pm v k + \frac{k^2}{2m}$$

Spectrum curvature: anharmonic bosons



$$k_{L,R}(x) \to \partial_x \varphi_{L,R}$$

$$\xi_k = \pm v(p \mp k_F)$$

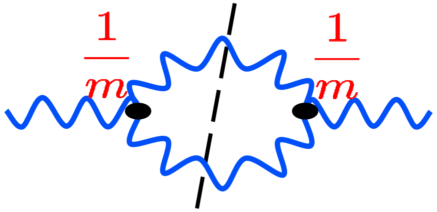
$$\xi_k = \pm vk + \frac{k^2}{2m}$$

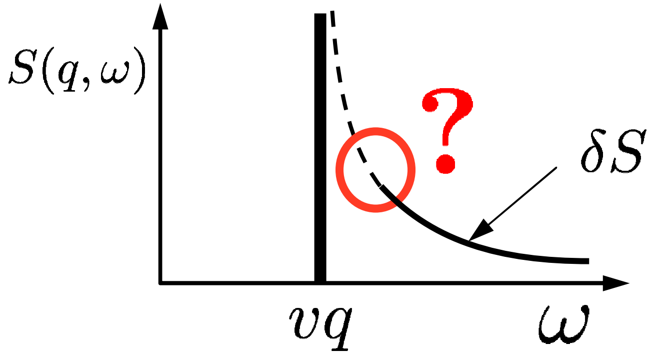
$$H_{kin}(x) = \int_{k_L(x)}^{k_R(x)} \xi_k \, dk = \frac{v}{2}(\partial_x \varphi_R)^2 + \underline{\frac{1}{m}(\partial_x \varphi_R)^3} + (R \leftrightarrow L)$$

$$H_{int}(x) = V_{LL}(\partial_x \varphi_L)^2 + V_{RR}(\partial_x \varphi_R)^2 + 2V_{LR}(\partial_x \varphi_L)(\partial_x \varphi_R)$$

Curvature in perturbation theory

$$\delta H = \frac{1}{m}(\partial_x \varphi_R)^3 + (R \leftrightarrow L)$$





$$\delta S \propto \frac{(q^2/m)^2}{\omega^2 - (vq)^2}$$

$$\chi(q,\omega) = \left\langle -i\theta(t)\left[\hat{n}(x,t), \hat{n}^\dagger(0,0)\right]\right\rangle_{q,\omega} \to \frac{q}{\omega - vq - \Sigma_{\mathrm{Boson}}(\omega,q)}$$

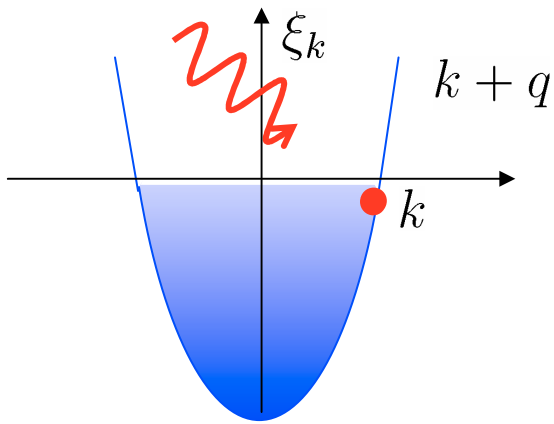
$$\Sigma^{(2)}(\omega = vq, q) = \frac{1}{m^2} \cdot \infty$$

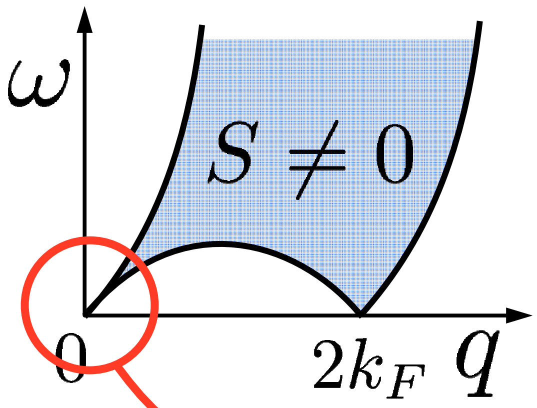
Divergent at $\omega = vq$

Free fermions, curved spectrum

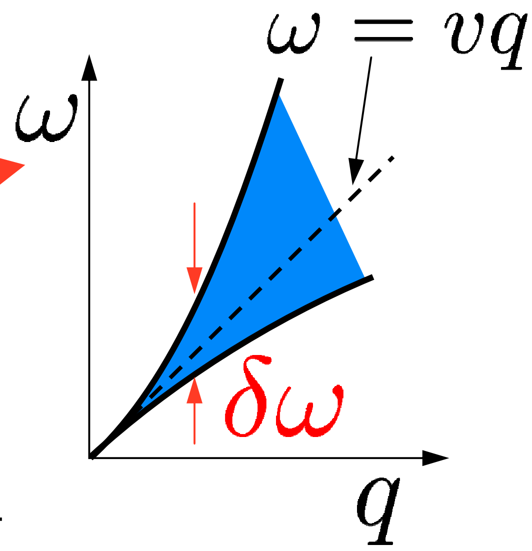
Lehmann (Golden rule – like) representation

$$S(q,\omega) = 2\pi \sum_{k=k_F-q}^{k_F} \delta\left[\omega - (\xi_{k+q} - \xi_k)\right]$$

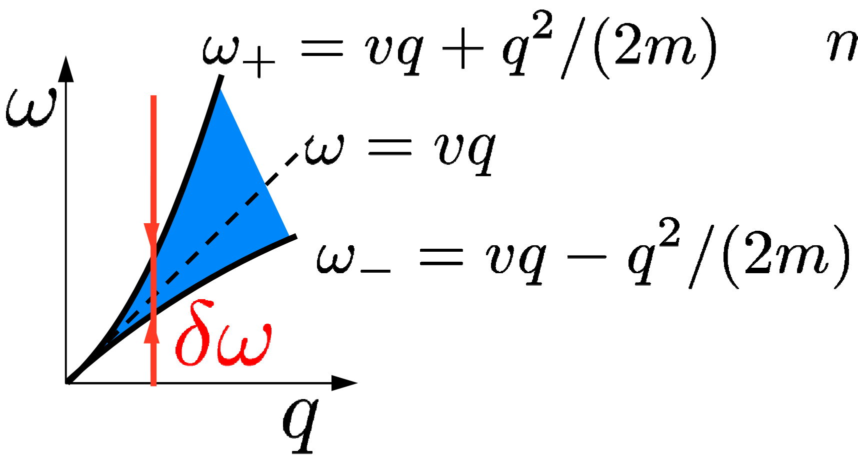


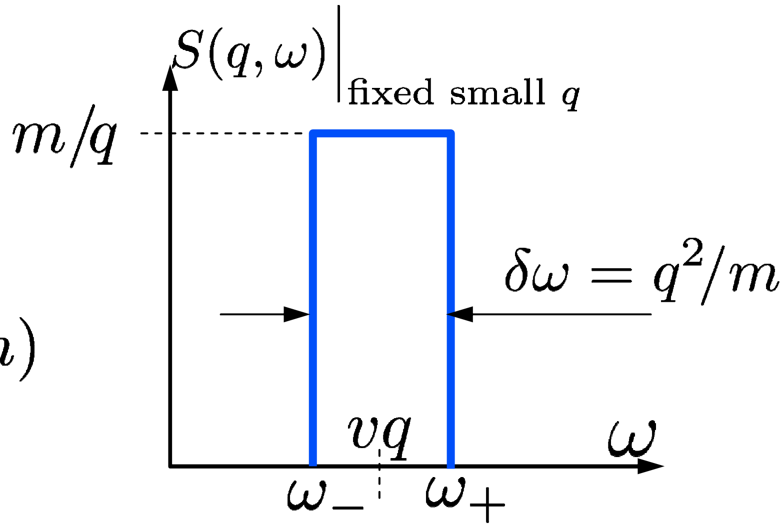


$$\delta\omega = \frac{q^2}{2m}$$



Curvature: free fermions perspective





$$\delta\omega = q^2/m \sim \omega^2/\epsilon_F$$

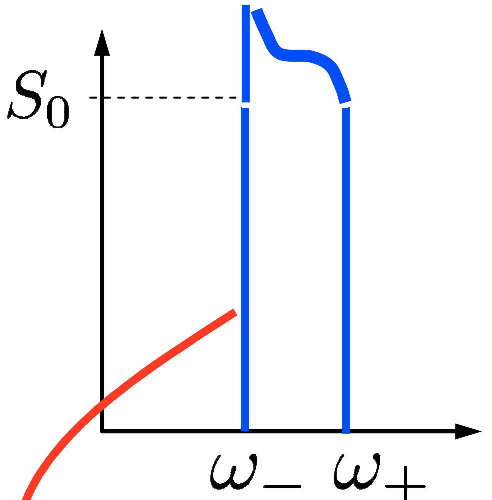
$\Longrightarrow$

- **the peak is narrow**

but…

- **it is not a Lorentzian**
- $\delta\omega \propto 1/|m|$ **(non-perturbative in curvature)**

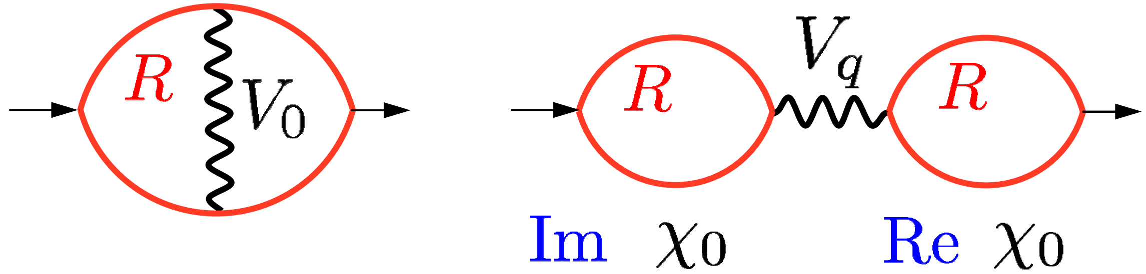
Perturbation theory: near the shell



$\omega_\pm = vq \pm q^2/2m$

$\omega_- < \omega < \omega_+$

leading corrections in the interaction V_q



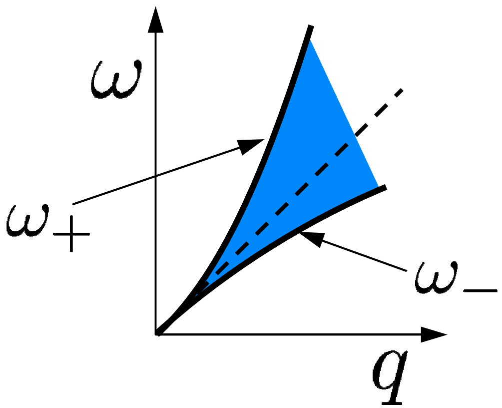
$$\delta S \propto \delta \mathrm{Im}\chi(q,\omega) \propto [V_0 - V_q]\, \mathrm{Im}\chi_0(q,\omega)\, \mathrm{Re}\chi_0(q,\omega)$$

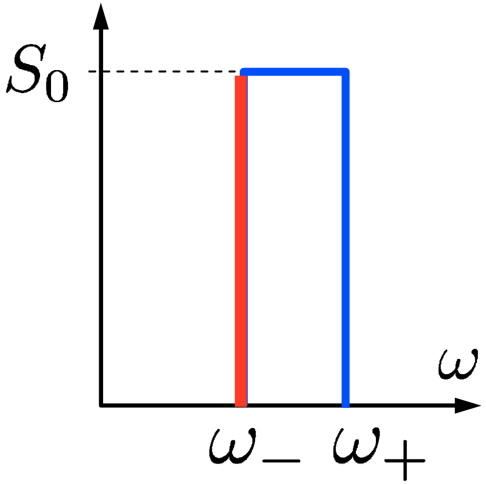
Kramers-Kronig

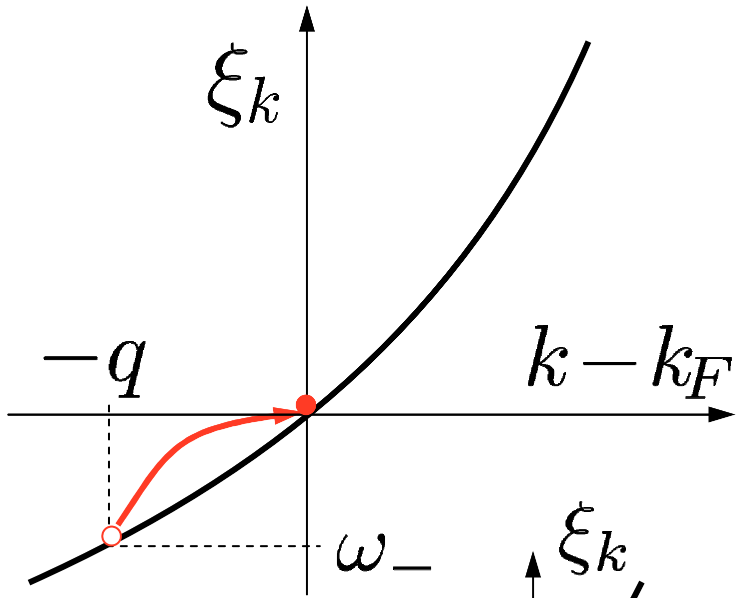
$$\Rightarrow \quad \frac{\delta S}{S_0} = \frac{v}{2\pi q}[V_0 - V_q] \ln\left[\frac{\omega_+ - \omega}{\omega - \omega_-}\right]$$

Beyond perturbation theory, $\omega \rightarrow \omega_-$

free electrons

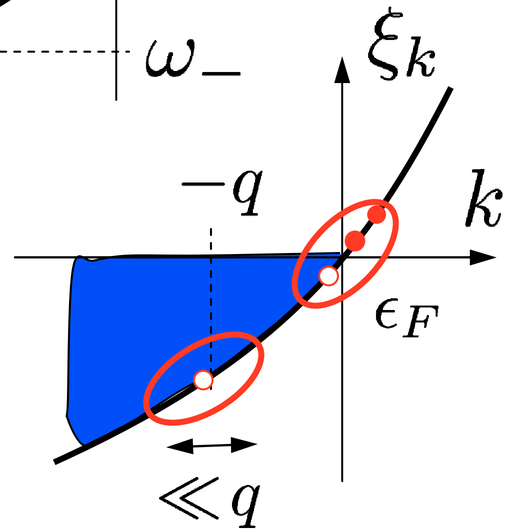






Analogy: X-ray edge singularity

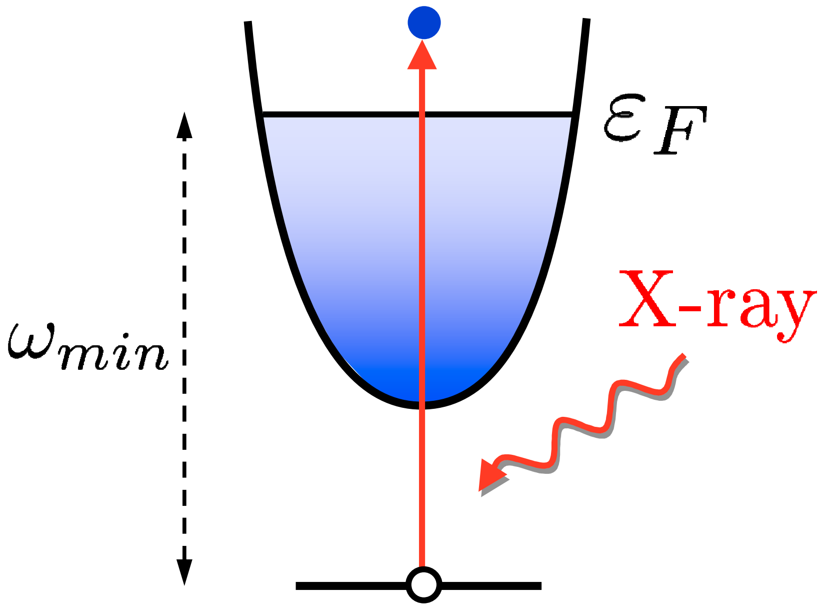
important states:

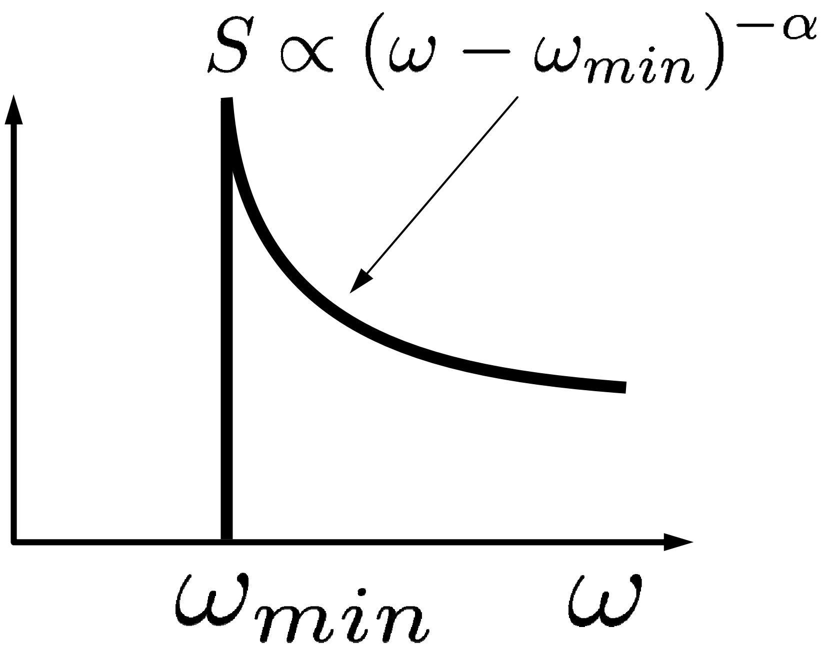


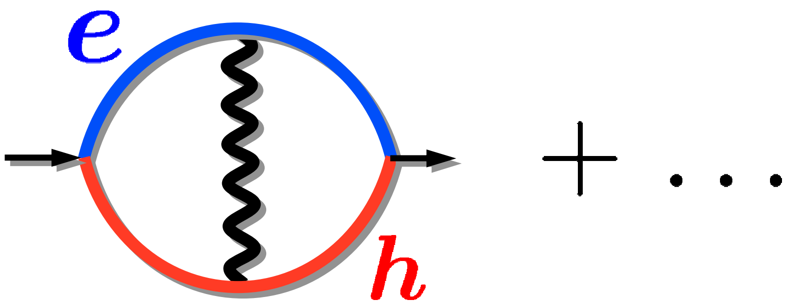
interaction with the “core hole”

singularity $[\ln(\omega - \omega_-)]^n$ in each order of perturb. theory

Fermi edge singularity in metals





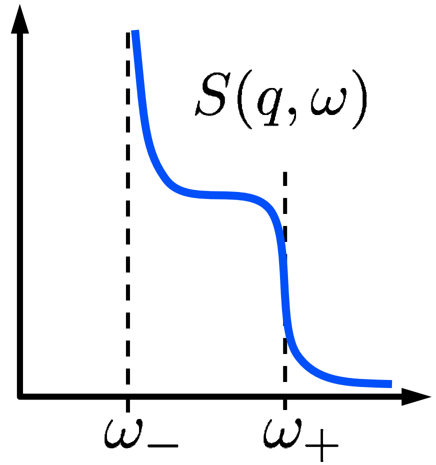


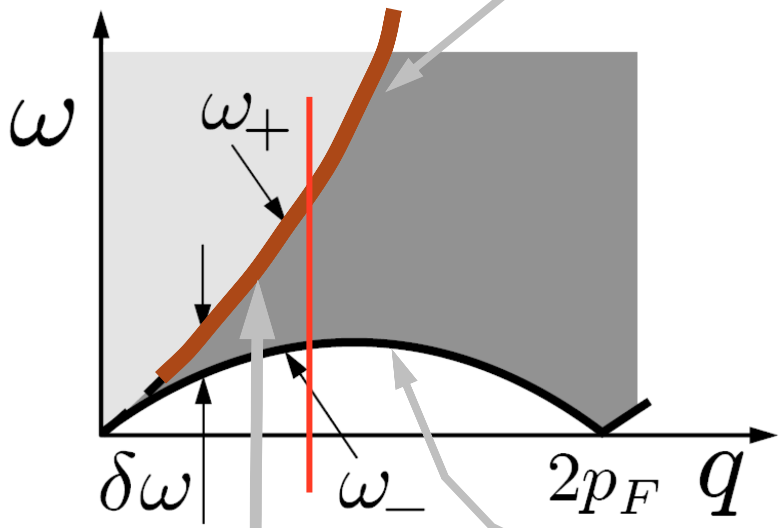
Mahan 67
Nozieres, DeDominicis 69

threshold + interactions = power law

Effect of Curvature on Structure Factor

$$\frac{S(q,\omega)}{S_0} = f(q) + \mathrm{sign}(\omega_+ - \omega)\left|\frac{\omega_+ - \omega}{\delta\omega}\right|^{\mu}$$





$$\mu = \frac{mv}{\pi q}[V_0 - V_q] \ \propto q$$

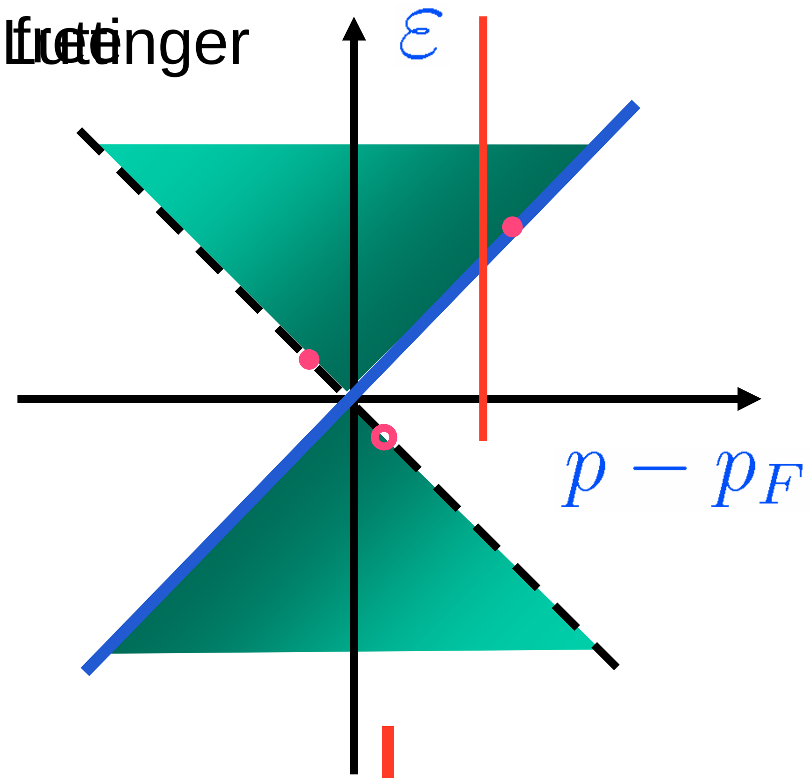
exponent: positive for repulsion

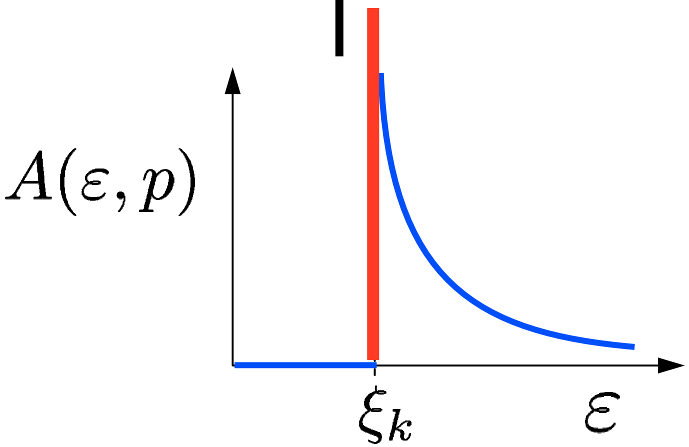
broadening of anomaly

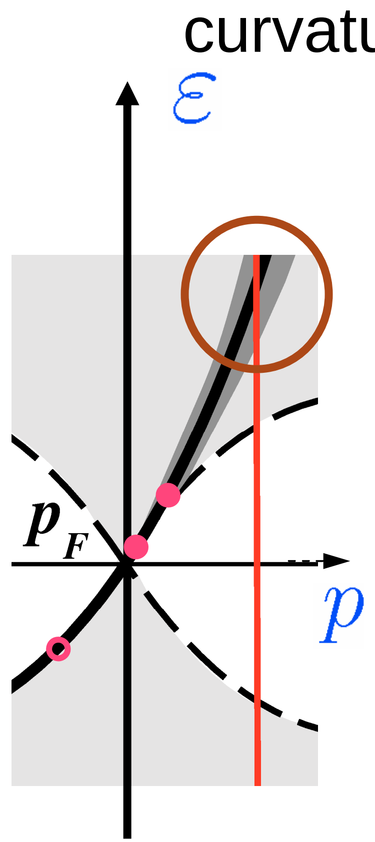
$$\propto V^4 \frac{q^8}{m^3}$$

$$\frac{S(q,\omega)}{S_0} = \left[\frac{\delta\omega}{\omega - \omega_-(q)}\right]^{\mu}$$

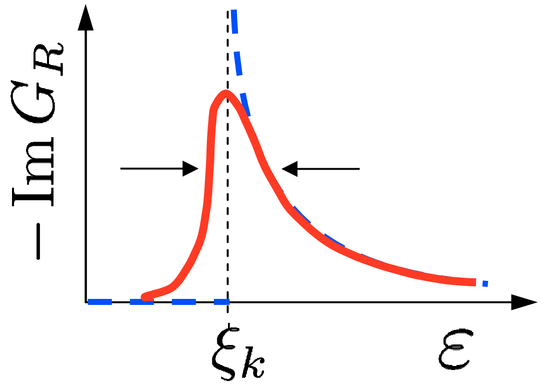
Spectral Function in 1D







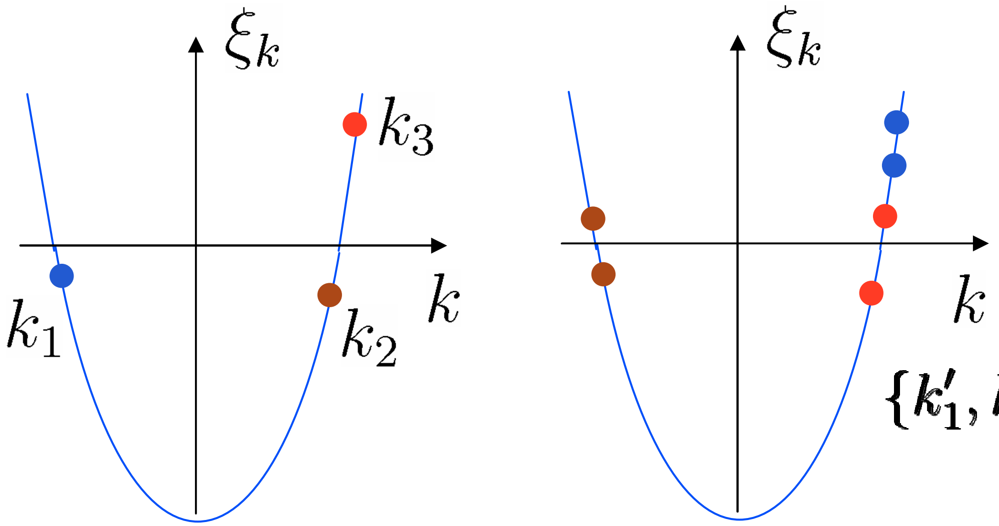
$$\frac{1}{\tau(\varepsilon)} \propto \frac{V^4 \varepsilon^8}{m^3}$$



$$\varepsilon = \xi_k$$

mass shell $\neq$ edge of $A(\varepsilon, p)$

Relaxation rate $1/\tau(k)$



Integrable models

$$\{k_1', k_2', k_3'\} \neq \text{Permutation}\{k_1, k_2, k_3\}$$

Luttinger Liquids – other stuff

- Shot noise – “charge fractionalization”
- Resonant tunneling
- Quantum fluctuations of charge in large quantum dots
- Coulomb drag, thermopower
- Dynamics of cold atoms confined to 1D
- Inelastic neutron scattering off S=(odd/2) spin chains
- Spin-incoherent Luttinger liquid