Charges and Spins in Quantum Dots

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Outline

• Confined (0D) Fermi liquid: Electron-electron interaction and ground state properties of a quantum dot

• Confined (0D) Fermi liquid: Transport across a quantum dot

• Kondo effect in quantum dots
Bulk Fermi Liquids

\[ H = \sum_k \xi_k \psi_{k\sigma}^\dagger \psi_{k\sigma} \]

\[ \xi_k = \frac{k^2}{2m} - \mu \]

\[ H_{\text{int}} = \sum_q V(q) : \hat{n}_q \hat{n}_{-q} : \]

\[ \hat{n}_q = \sum_\sigma \psi_{p+q,\sigma}^\dagger \psi_{p,\sigma} \]

\[ r_s = \frac{e^2}{\hbar v_F} \]

\[ \xi_k \rightarrow \tilde{\xi}_k \quad V(q) \rightarrow \tilde{V}(q) \]

interaction remains weak

(Landau 1956)
Droplets of a Fermi Liquid

small, symmetric

descendants of “dirty” bulk Fermi liquid

$L, l_{tr} \gg \lambda_F$

allow for statistical description

Marcus; Chang ~1996

physics of “artificial atoms”

Kouwenhoven+Tarucha ~1996
Quantum Chaos and Interactions in Quantum Dots: Energy Scales

Single-particle level spacing: \( \delta E = \frac{1}{\nu L^d} \)

Thouless energy: \( E_T = \frac{\hbar v_F}{L} \) or \( E_T = \frac{\hbar D}{L^2} \)

Charging energy: \( E_C = \frac{e^2}{2C} \sim \frac{e^2}{L} \)

\( g \equiv \frac{E_T}{\delta E} = (k_F L)^{d-1} \)

Good conductor: \( g \gg 1 \) (Thouless1972)

2D: \( g = k_F L \sim \sqrt{N} \)

\( \frac{E_C}{E_T} \sim r_s \leq 1 \)

(ratios for the ballistic case)
Quantum Chaos and Interactions in Quantum Dots: Universal Hamiltonian – Single-Particle Part

\[ |E| \leq E_T, \quad g \gg 1 \quad : \quad \text{Random Matrix Theory (RMT) limit} \]

\[ H = \sum_k \xi_k \psi_{k\sigma}^\dagger \psi_{k\sigma} \quad \Rightarrow \quad H_0 = \sum_i \xi_i \psi_{i\sigma}^\dagger \psi_{i\sigma} \]

\[ \xi_i \quad \text{are random,} \quad \xi = |\xi_{i+1} - \xi_i| \sim \delta E \quad \text{repel each other} \]

at \( \xi \ll \delta E \):

\[ P(\xi) \propto \xi^\beta; \quad \beta = 1(\text{GOE}) \text{ or } 2(\text{GUE}) \]

Random eigenfunctions \( \varphi_i(r) \):

\[ \langle \varphi_i^*(r_1) \varphi_j(r_2) \rangle \propto \delta_{ij} \quad \text{GUE, GOE} \]

+ \[ \langle \varphi_i(r_1) \varphi_j(r_2) \rangle \propto \delta_{ij} \quad \text{GOE} \]
Quantum Chaos and Interactions in Quantum Dots: Universal Hamiltonian of Interactions

\[ H_{\text{int}} = \sum_q V(q) \hat{n}_q \hat{n}_{-q} : \]

\[ \hat{n}_q = \sum_{\sigma} \psi_{p+q,\sigma}^{\dagger} \psi_{p,\sigma} \]

\[ h_{ijkl} = \int dr dr' \varphi_i^*(r) \varphi_i(r) V(r - r') \varphi_j^*(r') \varphi_k(r') \]

Random eigenfunctions \( \varphi_i(r) : \) GUE, GOE

\[ \langle \varphi_m^*(r_1) \varphi_n(r_2) \rangle \propto \delta_{mn} \]

\[ \langle \varphi_i(r_1) \varphi_j(r_2) \rangle \propto \delta_{ij} \] GOE

Important only in supercond. dots, \( \Lambda < 0 \)

\[ E_C \sim \frac{e^2}{L} \] charging

\[ E_S \sim r_s \delta E \] exchange

\[ |\Lambda| \ll \delta E \] Cooper
Quantum Chaos and Interactions in Quantum Dots: Universal Hamiltonian of Interactions

\[ H_{\text{int}} = \frac{1}{2} \sum_{ijkl} h_{ijkl} : \hat{n}_{il} \hat{n}_{jk} : \]

\[ \hat{n}_{il} = \sum_{\sigma} \psi_{i,\sigma}^\dagger \psi_{l,\sigma} \]

\[ h_{ijkl} = \int dr dr' \phi_i^*(r) \phi_l(r) V(r - r') \phi_j^*(r') \phi_k(r') \]

\[ \langle h_{ijkl} \rangle = A \delta_{il} \delta_{jk} + B \delta_{ik} \delta_{jl} + C \delta_{ij} \delta_{kl} \]

\[ h_{ijkl} = \langle h_{ijkl} \rangle + \delta h_{ijkl} \]

No other matrix elements with non-zero averages in the limit \( g \equiv E_T / \delta E \gg 1 \)

finite part, does not fluctuate

fluctuates, average is zero, variance is \( \propto 1/g \)
Full Form of the Universal Hamiltonian

\[ H = H_0 + H_{\text{int}} \]

\[ H_0 = \sum_i \xi_i \psi_i^{\dagger} \psi_i \sigma \]

\[ \xi = |\xi_{i+1} - \xi_i| \sim \delta E \]

not random

\[ H_{\text{int}} = E_C (\hat{N} - N_0)^2 - E_S \hat{S}^2 + \Lambda \hat{T}^{\dagger}\hat{T} \]

\[ \hat{N} = \sum_{i,\sigma} \psi_i^{\dagger} \psi_i \sigma \]

number of electrons

\[ E_C = e^2/2C \]

\[ \hat{S} = \sum_{\sigma' \sigma} s_{\sigma \sigma'} \sum_i \psi_i^{\dagger} \psi_i \sigma \sigma' \]

spin of the dot

\[ E_S \sim r_s \delta E \]

\[ \Lambda = \Lambda (E_T) \]

Cooper pairs operator

\[ \hat{T} = \sum_i \psi_{i,\uparrow}^{\dagger} \psi_{i,\downarrow}^{\dagger} \]

Kurlyand, Aleiner, Altshuler, '00; Aleiner, Brouwer, LG (review) 02

\[ N_0 = C_g V_g / e \]

\[ C = \sum C_i \]

C = total capacitance

2D electron gas

quantum dot

gate

V_g
Corrections to the Universal Hamiltonian

Leading correction (only planar dots):

\[ H_1^{1/g} \propto \frac{\delta E}{\sqrt{g}} \quad (g \gg 1) \]

does not limit quasiparticle lifetime

Blanter, Mirlin, Muzykantskii, 1997

Smaller correction (planar dots, 3D grains):

\[ H_2^{1/g} \propto \frac{\delta E}{g} \]

causes transitions between the levels

\[ \frac{1}{\tau \varepsilon} \sim \delta E \cdot \left( \frac{\varepsilon}{E_T} \right)^2 \quad (E_T \gg \varepsilon \gg \delta E) \]

Sivan, Imry, Aronov, 1994; Blanter 1996
Equilibrium Properties of a Quantum Dot

\[ H_0 = \sum_i \xi_i \psi_i^\dagger \psi_i \]

\[ H_{\text{int}} = E_C (\hat{N} - N_0)^2 - E_S \hat{S}^2 + \Lambda \hat{T}^\dagger \hat{T} \]

Charge vs. gate voltage

\[ \delta E \ll T \]

Electrostatics

\[ \Lambda = 0 \]

\[ \Lambda \neq 0 \]

P. Lafarge et al (SACLAY) 1991-1993
Matveev, LG, Shekhter (theo review) 1994

\[ E_C \gg \Delta \sim \delta E \]

\[ \frac{\Delta}{e} = 180 \mu V \]

\[ \frac{\Delta}{(\Delta - E)} = 10 \mu V \]

\[ T = 28 \text{ mK} \]

P. Lafarge et al (SACLAY) 1991-1993
Matveev, LG, Shekhter (theo review) 1994

\[ E_C \gg \Delta \sim \delta E; \ T = 0 \]

Matveev, Larkin, 1997

Delft, Ralph – Phys Rep (review) 2001
Spin States of a Quantum Dot

$$H_0 = \sum_i \xi_i \psi_i^\dagger \psi_i \sigma$$

$$H_{\text{int}} = E_C (\hat{\mathcal{N}} - N_0)^2 - E_S \hat{S}^2$$

$$\delta E \gg T$$

$$E_S = 0$$

$$E_S \sim r_s \delta E > 0$$

$$\xi \equiv \xi_{i+1} - \xi_i \sim \delta E,$$
random (large dots)
or controllable (small dots)

theory:
Oreg, Brouwer, Halperin`99;
Ullmo, Baranger, LG 2000;
Usaj, Barabger, 2002

experiments:
Tarucha group (small dots) 2001
Ensslin group (larger dots) 2001
Marcus group (large dots) 2001
Transport through a Quantum Dot: “Classical” Coulomb Blockade

\[ \delta E \ll T \]

\[ N \]

\[ \begin{align*}
N_0 \propto V_{\text{gate}} \\
3 \quad 2 \quad 1 \quad 0 \quad 1/2 \quad 3/2 \quad 5/2
\end{align*} \]

Tunneling rate

\[ t_{kp} \]

\[ \text{rate: } \frac{1}{t_{0*1}} \]

\[ V \]

\[ G_L, G_R \]

Probability of no extra electron on the dot: \( W_0 \)

\[ W_0 + W_1 = 1 \]

\[ \frac{1}{t_{0*1}} = W_0 \cdot \frac{2\pi}{h} \sum \frac{1}{t_{kp}} \frac{1}{n_K} (1-n_p) \delta (E_k + eV + E_0 - e_p - E_1) \]

\[ = W_0 \cdot \frac{1}{e^2} \cdot G_L \int f (E_1 - E_0 - eV) \]

\[ f(x) = \frac{x}{e^x - 1} \]

\[ x = E_1 - E_0 - eV \]

electrostatic energies
"Classical" Coulomb Blockade

Balance Equations (currents through L and R junctions equal each other):

\[ G_L \left[ W_0 f(E_0 - E_1 - ev) - W_1 f(E_0 - E_1 + ev) \right] = \]
\[ = G_R \left[ W_1 f(E_0 - E_1) - W_0 f(E_1 - E_0) \right] \]

Linear conductance (ev \to 0):

\[ G = \frac{G_L G_R}{G_L + G_R} \frac{f(\varepsilon)f'(-\varepsilon) + f'(\varepsilon)f(-\varepsilon)}{f(\varepsilon) + f(-\varepsilon)} \]

\[ \varepsilon = E_1 - E_0 \text{ depends on } V_g \]
“Classical” Coulomb Blockade: Linear Conductance

\[ G = \lim_{V \to 0} \frac{dI}{dV} = G_\infty \frac{E_C (N_0 - N_0^*)/T}{\sinh[2E_C (N_0 - N_0^*)/T]} \]

\[ N_0 = N_0^* \text{ corresponds to the Coulomb blockade peak} \]

\[ \delta E \ll T \ll E_C \]

Peak width:

\[ |N_0 - N_0^*| \lesssim T/E_C \]

P. Joyez et al. (SACLAY) PRL 1997
Transport through a Quantum Dot: Resonances

\[ \delta E \gg T \]

separate-level resonances

\[ \Gamma_i^l \propto |\varphi_i(x_l)|^2 \]
\[ \Gamma_i^r \propto |\varphi_i(x_r)|^2 \]

\[ T(\varepsilon) = \frac{4\Gamma_i^l\Gamma_i^r}{(\varepsilon - \varepsilon_i)^2 + (\Gamma_i^l + \Gamma_i^r)^2} \]

\[ G(\varepsilon_i) = \frac{2e^2}{2\pi\hbar} \int d\varepsilon \frac{\partial f_F}{\partial \varepsilon} T(\varepsilon) \]

\[ G_{\max}^i \sim \frac{e^2}{\pi\hbar T} \frac{\Gamma_i^l\Gamma_i^r}{\Gamma_i^l + \Gamma_i^r} \]

\[ G_{\max} > G_{\infty} \]

\[ \mathcal{P}(r) = \int d\ell_x d\ell_r \rho_{rt}(\ell_x) \rho_{tr}(\ell_r) \delta(\frac{\ell_x - \ell_r}{\ell^* + \ell_r}) \]

Folk et al 1996
Transport through a Quantum Dot: Statistics of Coulomb Blockade Peaks

We have observed a strongly non-Gaussian distribution of Coulomb blockade conductance peak heights for tunneling through quantum dots. At zero magnetic field, a low-conductance spike dominates the distribution; the distribution at nonzero field is distinctly different and still non-Gaussian. The observed distributions are consistent with theoretical predictions based on single-level tunneling and the concept of "quantum chaos" in a closed system weakly coupled to leads.

\[ G_{\text{max}} = \frac{e^2}{h} \frac{\pi \Gamma}{2kT} \alpha \]

\[ P(\alpha) = \sqrt{\frac{2}{\pi \alpha}} e^{-2\alpha} \quad (B=0) \]

\[ P(\alpha) = 4\alpha [K_0(2\alpha) + K_1(2\alpha)]e^{-2\alpha} \quad (B > B_0) \]

(Jalabert, Stone, Alhassid, 1992)

\[ \Phi_c > \Phi/\sqrt{E_{\text{c}}/E_0} \]

(Falko, Efetov, 1996)
Coulomb Blockade Valleys: Inelastic Transport

\[ A_{\text{in}} \propto \frac{t_{LR}}{E_C} \]

Initial electron energy: \( \delta E \ll \varepsilon \ll E_C \)

Available phase space: \( \propto \varepsilon^2 \)

Linear conductance: \( \varepsilon \sim T \)

\[ G_{\text{in}} \sim |A_{\text{in}}|^2 T^2 \sim \frac{G_L G_R}{e^2/\hbar} \left( \frac{T}{E_C} \right)^2 \]

Averin, Odintsov 1988

\[ G_{\text{act}} \sim G_\infty \exp(-E_C/T) \lesssim G_{\text{in}} \rightarrow T \lesssim \frac{E_C}{\ln[G_q/(G_L + G_R)]} \]
Coulomb Blockade Valleys: Elastic Transport

\[ A_j \propto \frac{\varphi_j(x_l)\varphi_j^*(x_r)}{E_C + |\delta \xi_j|} \]

Partial amplitudes are random

\[ \langle |A_j|^2 \rangle \propto \frac{1}{E_C^2} \frac{\Gamma_L \Gamma_R}{E_C^2} \]

Small, fluctuates

\[ G_{el} = \frac{e^2}{\pi \hbar} T(\epsilon_F) \]

Landauer formula

\[ A = \sum_j A_j \]

\[ 1 \lesssim j \lesssim \frac{E_C}{\delta E} \]

Survives averaging

Averin, Nazarov 1990; Aleiner, LG 1996; Cronnenwett et al (exp) 1997
Mesoscopic Fluctuations of Elastic Transport

\[ T(\epsilon_F) = |A|^2 = \left| \sum_j A_j \right|^2 = \sum_j A_j A_j^* + \sum_{i \neq j} A_i A_i^* \]

\[ \text{var } G_{\text{el}} \sim \sqrt{N^2 - N \langle |A_j|^2 \rangle} \]

\[ N \sim E_C/\delta E \text{ terms} \]

\[ N^2 - N \text{ terms} \]

\[ \langle G_{\text{el}} \rangle \sim N \langle |A_j|^2 \rangle \]

\[ \text{var } G_{\text{el}} \sim \langle G_{\text{el}} \rangle \sim \frac{\hbar}{e^2 G_L G_R} \frac{\delta E}{E_C} \]

\[ G_{\text{act}} \sim G_\infty \exp(-E_C/T) \lesssim G_{\text{in}} \rightarrow T \lesssim \frac{E_C}{\ln[G_q/(G_L + G_R)]} \]

\[ G_{\text{in}} \sim G_{\text{el}} \text{ at } T \sim \sqrt{E_C \delta E} \]

Odd valley:

looks like Anderson Impurity Model → Kondo effect
Summary: Conductance through a blockaded dot

Valleys:
- Kondo: $G \sim \frac{e^2}{h} f\left(\frac{T}{T_K}\right)$
- Elastic cotunneling: $G \sim \frac{h}{e^2} G_L G_R \frac{\Delta E}{E_C}$
- Inelastic cotunneling: $G(T) \sim \frac{h}{e^2} G_L G_R \left(\frac{T}{E_C}\right)^2$

Peaks:
- Quantum limit: $G \sim \frac{e^2}{h}$, fluctuates
- $G_L = G_R \equiv 2G_\infty \ll \frac{e^2}{h}$
Small quantum dots (~ 500 nm)

M. Kastner, Physics Today (1993)
E.B. Foxman et al., PRB (1993)

\[ G \propto e^{-E_C / T} \]
Even smaller quantum dots (~ 200 nm)

1998

D. Goldhaber-Gordon et al. (MIT-Weizmann)
S.M. Cronenwett et al. (TU Delft)
J. Schmid et al. (MPI @ Stuttgart)

van der Wiel et al. (2000)
Low-$T$ Conductance Anomaly

$$G \propto \ln\left(\frac{E_C}{T}\right)$$

Activation conductance theory fails qualitatively in every other valley.
Anomalous low-temperature behavior

\[ G \propto \ln \left( \frac{E_C}{T} \right) \]

**T- dependence**

- \( N = \text{even}: \) normal (decrease at \( T \rightarrow 0 \))
- \( N = \text{odd}: \) anomalous (increase at \( T \rightarrow 0 \))

\[ S \neq 0 \]

\[ G \propto \ln \left( \frac{E_C}{T} \right) \]

for \( N = \text{odd} \) all possible electronic configurations have \( S \approx 0 \)

Kondo physics
Anomalous behavior of metallic resistivity

de Haas et al. (1934)

$\frac{10^4 R(T)}{R(273)}$

resistivity of a clean Au sample

expected saturation
Kondo effect

Jun Kondo (1964)

\[ H_{\text{Kondo}} = H_0 + J(s \cdot S) \]

local spin density of conduction electrons
magnetic impurity

correction to resistivity grows at \( T \to 0 \)

\[ \delta \rho \propto n_{\text{imp}} \left[ J^2 + J^3 \ln(\epsilon_F/T) \right] \]

A problem: how to deal with singularities at \( T \to 0 \)?
The Origin of Exchange Interaction

**Anderson impurity model**

- Strong on-site repulsion: $H_d = U(N-1)^2$, $N = n_\uparrow + n_\downarrow$
- Impurity level is *singly* occupied: $\langle N \rangle = 1$

$t = 0$: doubly degenerate ground state

$\Psi_\uparrow = |\text{electron gas}\rangle \otimes |\uparrow\rangle$

$\Psi_\downarrow = |\text{electron gas}\rangle \otimes |\downarrow\rangle$

Finite $t$: tunneling $\rightarrow$ exchange

$H_{\text{exchange}} = J (\mathbf{s} \cdot \mathbf{S})$

$J \propto \frac{t^2}{U} > 0$
Electron Scattering in the Perturbation Theory

\[ \mathcal{H} = \frac{J_0}{2} \sum_{\mathbf{p}\mathbf{p}'\sigma\sigma'} \mathbf{S} \cdot \mathbf{s}_{\sigma\sigma'} C_{\mathbf{p},\sigma}^{\dagger} C_{\mathbf{p}',\sigma'} \]

Scattering amplitude in the Born approximation:

\[ A_{p\sigma \to p'\sigma'}^{(1)} \propto J_0 \]

\[ w_{p\sigma \to p'\sigma'}^{(1)} \propto J_0^2 \delta(\varepsilon_p - \varepsilon_{p'}) \]

Kondo (1964) correction:

\[ A_{p\sigma \to p'\sigma'}^{(2)} \propto 2 \int_{-D}^{0} d\varepsilon'' \frac{\nu J_0^2}{\varepsilon - \varepsilon''} - \int_{0}^{D} d\varepsilon'' \frac{\nu J_0^2}{\varepsilon'' - \varepsilon} \]

\[ \propto \nu J_0^2 \ln \frac{D}{|\varepsilon|} \]

\[ w_{p\sigma \to p'\sigma'}^{(1)+(2)} \propto \left| A_{p\sigma \to p'\sigma'}^{(1)} + A_{p\sigma \to p'\sigma'}^{(2)} \right|^2 \cdot \delta(\varepsilon_p - \varepsilon_{p'}) \]
Electron Scattering in the Leading-Log Approx

Logarithmic scaling, Abrikosov (1965); log—RG, Anderson (1970):

\[ \nu J_0 \to \nu J(\varepsilon) = \frac{1}{\ln |\varepsilon/T_K|} \]

\[ T_K = D \exp \left(-\frac{1}{J_0 \nu}\right) \]

\[ \delta \rho_K(T \gg T_K) \propto \frac{n_s}{\nu} \frac{1}{\ln^2(T/T_K)} \]

\[ \sigma(\varepsilon \to \varepsilon') \sim \lambda_F^2 \left[ \frac{1}{\ln |\varepsilon/T_K|} \right]^2 \delta(\varepsilon - \varepsilon') \]
Kondo singlet

\[ H_{Kondo} = H_0 + J (s \cdot S) \]

local spin density of conduction electrons
magnetic impurity

Cartoon: 2 spins-1/2

\[ H_{exchange} = J (S \cdot S) \]

\[ S = 1 \quad S = 0 \]

\[ E_{\text{triplet}} - E_{\text{singlet}} = J \]

**GS** is a singlet for antiferromagnetic exchange \((J > 0)\)

unlike the cartoon, the conduction electrons are delocalized
Kondo singlet

\[ H_{\text{Kondo}} = H_0 + J (\mathbf{s} \cdot \mathbf{S}) \]
- favors \[ \uparrow \downarrow \] for \( J > 0 \)

local spin density of conduction electrons
magnetic impurity

ground state:
\[ \Psi_{\text{Kondo}} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

characteristic energy scale:
Kondo temperature \[ T_K \sim \epsilon_F e^{-1/J} \]

~ \( \hbar \nu_F / T_K \)
small interaction radius

Kondo effect = lifting of the ground state degeneracy
**Digression: Resonant tunneling**

Transmission coefficient: \( T(\varepsilon) = \sin^2 \delta(\varepsilon) \)

scattering phase shift

\[
T(\varepsilon) = \left| t \right|^2 = \frac{\Gamma^2}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2}
\]

(Breit - Wigner)

\( T(\varepsilon) = 1 \iff \delta(\varepsilon) = \pi/2 \)
Resonant tunneling

density of states:

\[ \rho(\varepsilon) = \frac{1}{\pi} \frac{\partial \delta(\varepsilon)}{\partial \varepsilon} \]

resonance at \( \varepsilon = \varepsilon_F \) \( \Rightarrow \) localized level is half-occupied

ground state expectation value

\[ N = \int_{-\infty}^{\varepsilon_F} \rho(\omega) d\omega = \frac{1}{2} \]

\[ T(\varepsilon_F) = 1 \iff \delta(\varepsilon_F) = \frac{\pi}{2} \iff \rho(\varepsilon_F) = \text{max} \iff N = 1/2 \]
Friedel Sum Rule

\[ N \text{ and } \delta(\varepsilon_F) \text{ are related!} \]

**Spinless fermions:**

\[ N = \frac{1}{\pi} \delta(\varepsilon_F) \]

\[ N = 2 \implies \delta_{\uparrow}(\varepsilon_F) = \delta_{\downarrow}(\varepsilon_F) = \pi \]

- **No resonance**

**Anderson impurity:**

\[ \varepsilon_0 < \varepsilon_F \quad \text{but} \quad \varepsilon_F - \varepsilon_0 < U \]

- Impurity level is singly occupied: \( N = 1 \)

\[ \implies \delta_{\uparrow}(\varepsilon_F) + \delta_{\downarrow}(\varepsilon_F) = \pi \]

- **Singlet ground state**

\[ \delta_{\uparrow}(\varepsilon_F) = \delta_{\downarrow}(\varepsilon_F) = \pi/2 \]

\[ T(\varepsilon_F) = \sin^2 \delta(\varepsilon_F) = 1 \]

- **Resonance!**
From Scattering to Transport

\[ I = e \int \frac{dp}{h} \left( \frac{\partial \epsilon_p}{\partial p} \right) T(\epsilon_p) = \frac{e}{h} \int_{\mu_R}^{\mu_L} d\epsilon \, T(\epsilon) = \frac{e}{h} T(\epsilon_F) eV \]

\[ \mu_R < \epsilon_p < \mu_L \]

velocity

transmission coefficient

reservoir

reservoir

leads

dot

Landauer formula: \[ G = \frac{e^2}{h} \left( T_\uparrow + T_\downarrow \right) \]

\[ T_\uparrow = T_\downarrow = 1 \text{ (resonance)} \quad \Rightarrow \quad G = 2e^2/h \]

conductance quantum: \[ e^2/h \approx (25 \text{ k}\Omega)^{-1} \]
Transport in the Kondo regime

Isolated dot: doubly-degenerate ground state

\[ \uparrow = \begin{array}{c}
\uparrow \\
\uparrow
\end{array} \quad \downarrow = \begin{array}{c}
\downarrow \\
\downarrow
\end{array} \]

Dot in contact with leads: Kondo singlet

\[ \Psi_{\text{Kondo}} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

scattering phase shifts

Conductance:

\[ G = \frac{e^2}{h} (T_\uparrow + T_\downarrow) = \frac{e^2}{h} \left( \sin^2 \delta_\uparrow + \sin^2 \delta_\downarrow \right) \]

Friedel sum rule:

\[ \delta_\uparrow = \pi N_\uparrow, \quad \delta_\downarrow = \pi N_\downarrow \]

ground state expectation values

number of electrons on the dot

\[ N_\uparrow = \langle \Psi_{\text{Kondo}} | \hat{N}_\uparrow | \Psi_{\text{Kondo}} \rangle = N_\downarrow = \frac{N}{2} \]
Transport in the Kondo regime

\[ G = \frac{e^2}{h} (T_\uparrow + T_\downarrow) = \frac{e^2}{h} (\sin^2 \delta_\uparrow + \sin^2 \delta_\downarrow) \]

\[ \delta_\uparrow = \delta_\downarrow = \frac{\pi N}{2} \]

\[ \rightarrow \quad G = \frac{2e^2}{h} \sin^2 \left( \frac{\pi N}{2} \right) \]

**odd N:** \( G = \frac{2e^2}{h} \) - perfect transmission

**even N:** \( G = 0 \) - perfect blockade
What is necessary for the Kondo effect to occur?

- degeneracy
- interaction
- electron gas

Zeeman energy:

\[ E_{\uparrow} - E_{\downarrow} = B = g \mu_B H \]

Control parameter:

\[ B/T_K \]

Thermal fluctuations have similar effect:

\[ G \sim 2e^2/h \times \left( 1 - \left( \frac{B}{T_K} \right)^2 \right) \]

\[ \frac{\pi^2}{16} \times \frac{1}{\left[ \ln \left( \frac{B}{T_K} \right) \right]^2} \]
Other local degeneracies—more Kondo effects

Kondo effect recovers at $B = \delta E$ and even electron number!

review: M. Pustilnik et. al. cond-mat/0010336; LNP 579, 3 (2001)
Summary: Kondo effect in quantum dots

Strong effect: lifting of the Coulomb blockade at low $T$

Ubiquity in nanostructures:
- interaction
- degeneracy
- electron gas

always present
can be tuned
Other stuff: “Quantum Impurity” systems

- Kondo effects: $S > 1/2$, Multi-channel, SU(4), out-of-equilibrium

Kondo in a carbon nanotube quantum dot (G. Finkelstein et al, 2006):
4 states, 2 channels
FIG. 1: (a) A schematic potential and energy level diagram for a single quantum dot in which one electron is confined to the low energy spectrum of a three dimensional potential. Only the ground and first-excited states, each a Kramer's doublet, are shown. (b) The lowest orbital state has a spin-1/2 electron interacting with the lattice nuclear spins. (c) Effective magnetic field due to both external field and the nuclear field. When the external field is large, the transverse components of the nuclear field are neglected in a rotating wave approximation.
Other stuff: Inelastic electron scattering
mediated by magnetic impurity

1. Simplest inelastic process in a toy model

\[
\mathcal{H} = \mathcal{H}_0 + \hat{V}_{\text{toy}} \quad \hat{V}_{\text{toy}} = J_0 \sum_{p_1p_2} \left( S^+ \sigma^- c_{p_2 \downarrow}^\dagger c_{p_1 \uparrow} + S^- \sigma^+ c_{p_2 \uparrow}^\dagger c_{p_1 \downarrow} \right)
\]

\[
\mathcal{H}_0 = \sum_{p\sigma} \xi_p c_{p\sigma}^\dagger c_{p\sigma}
\]

only two electrons in the band, \( \Psi_{\text{in}} = |p' \uparrow, p \downarrow, \uparrow\rangle \)

\[
\Psi_{\text{out}} \equiv \hat{T}\Psi_{\text{in}} \quad \text{T-matrix:} \quad \hat{T} = \hat{V} + \hat{V} \frac{1}{\varepsilon - \mathcal{H}_0} \hat{V} + \ldots
\]

Born

2nd order

\[
\Psi^{(2)}_{\text{out}} \propto J_0^2 \sum_{p_1p_2} S^+ \sigma^- c_{p_1 \downarrow}^\dagger c_{p_2 \uparrow} \frac{1}{\varepsilon - \mathcal{H}_0} \sum_{p_3p_4} S^- \sigma^+ c_{p_3 \uparrow}^\dagger c_{p_4 \downarrow} |p' \uparrow, p \downarrow, \uparrow\rangle
\]

\[
= \frac{J_0^2}{\xi_p - \xi_{p_3}} |p_1 \downarrow, p_3 \uparrow, \uparrow\rangle
\]
Inelastic electron scattering

Energy transferred in the collision:
\[ \xi_p - \xi_{p_3} \equiv E \]

Scattering cross-section:
\[ \left| A^{(2)}_{p,p'\rightarrow p_1,p_3} \right|^2 \sim \frac{J_0^4}{E^2} \delta(\xi_p + \xi_{p'} - \xi_{p_1} - \xi_{p_3}) \]

2. Full 2\textsuperscript{nd} order perturbation theory result

Total cross-section \( \varepsilon, \varepsilon' \rightarrow \varepsilon - E, \varepsilon' + E \)
averaged over \( S \):

\[ K(E) = \frac{\pi n_s}{2 \nu} (J \nu)^4 S(S + 1) \frac{1}{E^2} \]
Experiments on Energy Relaxation: Cu

Energy Distribution Function of Quasiparticles in Mesoscopic Wires

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(Received 25 April 1997)

Experimental layout:

Results:

Data for $f(\varepsilon)$ scales with $E/eU \rightarrow K(E, U) \propto \frac{1}{E^2} g\left(\frac{eU}{E}\right)$

If $K$ is $U$-independent, then

$$K(E) \approx \frac{1}{\tau_0 E^2}$$

with $\tau_0 \approx 1 \text{ns}$;

$1/\tau_\varepsilon$ does not go to zero at $\varepsilon \to 0$!

FIG. 3. Continuous lines in all four panels: distribution functions, for $U$ ranging from 0.05 to 0.3 mV by steps of 0.05 mV, plotted as a function of the reduced energy $E/eU$, for the same positions as in Fig. 2. Open symbols are best fits of the data to the solution of the Boltzmann equation with an interaction kernel $K(x, x', \varepsilon) = \tau_0^{-1} \delta(x - x')/\varepsilon^2$; in top panel, open circles correspond to the calculated distribution function in the middle of wires 1 and 2 ($x = 0.5$), with $\tau_0/\tau_D = 2.5$ and $\tau_0/\tau_D = 0.3$, respectively (both compatible with $\tau_0 \sim 1 \text{ ns}$). In bottom panels, open diamonds are computed at $x = 0.5$ and $x = 0.25$ with $\tau_0/\tau_D = 0.08$ ($\tau_0 \sim 0.5 \text{ ns}$).
Experiments on Energy Relaxation: Ag

Energy Redistribution Between Quasiparticles in Mesoscopic Silver Wires

F. Pierre, H. Pothier, D. Esteve, and M.H. Devoret

We have measured with a tunnel probe the energy distribution function of quasiparticles in silver diffusive wires connected to two large pads ("reservoirs"), between which a bias voltage was applied. From the dependence in energy and bias voltage of the distribution function we have inferred the energy exchange rate between quasiparticles. In contrast with previously obtained results on copper and gold wires, these data on silver wires can be well interpreted with the theory of diffusive conductors...

Distribution functions for \( U = 0.1, 0.2, 0.3, \) and \( 0.4 \) mV, plotted as a function of the reduced energy \( E/eU \).

Left panel: Ag sample D20a; right panel: Cu sample, \( L = 5 \mu m \).

“In silver samples we have assumed that the interaction kernel still obeys a power law \( \tilde{K}(\varepsilon) = \kappa_\alpha \varepsilon^{-\alpha} \), with \( \kappa_\alpha \) and \( \alpha \) taken as fitting parameters... the best fits obtained with the exponent set at its predicted value \( \alpha = 3/2 \).”
Energy Relaxation in Ag, Cu, and Au wires

F. Pierre et al., JLTP 118, 437 (2000) and NATO Proceedings (cond-mat/0012038)

\[ K(E) \approx \frac{1}{\tau_0 E^{3/2}} \]
Energy Relaxation in Cu and Au Wires: Spins Rule!


The diagrams show the behavior of energy relaxation in Cu and Au wires under different conditions. The graphs illustrate the relationship between the Fermi energy and the magnetic field, highlighting the conditions under which spins are dominant.