# Lecture 3: Quantum Effects in the Conductivity of High-Mobility Si MOSFETs

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# **Outline:**

- Intro: quantum corrections in Si MOSFETs (the most ubiquitous 2D structure) ⇒ 25-year-old puzzle, and the work is still in progress
- Ingredients essential for better understanding of interaction effects in Si MOSFETs:
  - interaction parameters in high-mobility Si MOSFETs
  - valley splitting and inter-valley scattering
- Analysis of  $\Delta \sigma(T, B_{II})$ : (semi)-quantitative agreement with the theory
- The crossover from "metallic" to "insulating" conductivity: role of large-scale potential fluctuations?



### Si MOSFET timeline



Shockley and Brattain attempted to make FET, but failed...

attributed the failure of Shockley's efforts to the surface states

RUTGE

F. Heiman



THE STATE UNIVERSITY OF NEW JERSEY

#### Low-µ devices, high carrier densities



#### Qualitatively – in line with the QC theory, quantitatively – not quite...

Bishop, Dynes, and Tsui, *Phys. Rev.* B **26**, 773 (1982) Dolgopolov, Dorozhkin, Shashkin, *Sol. State Commun.* **50**, 273 (1984) Burdis and Dean, *Phys. Rev.* B **38**, 3269 (1988)

unreasonably large values of the triplet-channel parameter F, lack of B/T scaling in MR, etc.



#### High- $\mu$ devices, high carrier densities

of

#### very limited supply of the devices!

(instead of "resistance")

**<u>25+year-old puzzle</u>**:  $\sigma$  (T,B<sub>II</sub>)

the most ubiquitous 2D system

predictions of the WL+Int. theory

electrons in Si MOSFETs - defied

#### Reluctance of two-dimensional systems

A. I. Larkin

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

(Submitted 17 January 1980)

Pis'ma Zh. Eksp. Teor. Fiz. 31, No. 4, 239-243 (20 February 1980)







 $\mu = 1.7 \times 10^4 \, \text{cm}^2/\text{Vs}$ 



#### High- $\mu$ devices, low carrier densities

The discrepancy became more dramatic as lower densities became accessible in high-mobility structures ( $\mu \sim 20,000 \text{ cm}^2/\text{Vs}$ ).



Observation of an apparent 2D MIT –

no excitement yet...



Письма в ЖЭТФ, том 45, вып. 10, стр. 476 - 480

25 мая 1987 г.

*n*- and *p*-type Si MOSFETs Zavaritskaya and Zavaritskaya ('86)

#### ПЕРЕХОД МЕТАЛЛ — ДИЭЛЕКТРИК В ИНВЕРСИОННЫХ КАНАЛАХ КРЕМНИЕВЫХ МДП СТРУКТУР

Т.Н. Заварицкая, Э.И. Заварицкая



# High-µ devices, low carrier densities (cont'd)

Finally, some excitement...





EX

# "Metallic" Behavior of 2D Holes

density  $p=1 \times 10^{10}/\text{cm}^2$   $r_s = 30$ 

Fermi temperature  $T_F = 0.77 \text{ K} \text{ at "2D MIT"}$ mobility  $\mu \sim 1 \times 10^6 \text{ cm}^2/\text{Vs}$ 

Loren Pfeiffer, Ken West and the MBE chamber at Bell Labs



*T*<sub>o</sub> = temperature at which
 "metallic" behavior begins



Gao *et al.*, PRL **89**, 016801 ('02)



### Other high- $\mu$ , low-*n* systems

The "metallic" behavior has been observed in many *high-mobility* systems at *low densities* ( > 500 publications over the last 10 years ).



Lai *et al.*, *PRB* **72**, 081313 ('05) *strained Si quantum well* μ = **1.9x10<sup>5</sup> cm<sup>2</sup>/Vs**  Proskuryakov *et al.*, *PRL* **89**, 076406 ('02) *p*-type GaAs/GaAlAs μ = **5.6x10<sup>5</sup> cm<sup>2</sup>/Vs** 





No upturn down to at least 50 mK Kravchenko and Klapwijk, '00:



Ultra-low-T conductivity



Klimov et al. ('07) unpublished



Huang et al., PRL 98, 226801 ('07)

# Anomalous Magnetoconductivity near the apparent 2D MIT



Simonian *et al.*, *PRL* **79**, 2304 ('97) *"the metallic state is suppressed by an arbitrarily small magnetic field..."*. Conclusion: spin effects play prominent role





distance to the gate >  $1/\sqrt{n}$ 

 $\Rightarrow$  Coulomb potential 1/r

# Interactions as the primary suspect

Strength of interactions is characterized by the parameter  $r_s$ 

$$r_s = \frac{E_C}{E_F} \left(\frac{\propto \sqrt{n}}{\propto n}\right) = \frac{a_B}{\sqrt{\pi n}}$$

#### The lower *n*, the stronger the effective interactions

In particular, the most pronounced "metallicity" is observed in Si MOSFETs at  $r_s$ ~ 2-7



# Not-too-low densities ( $n > 2 \cdot 10^{11} \text{ cm}^{-2}$ , $R_{\Box} < 5 \text{k}\Omega$ , $r_{S} < 5$ ): is the low- $T \sigma(T,B)$ in line with the theory of quantum corrections?

#### We seek an explanation for:

- "metallic"  $\partial \rho / \partial T$  over a wide *T* range down to ~ 0.5K, upturn at lower *T*
- positive MR in parallel magnetic fields

#### What should be taken into account:

- the most pronounced "matellicity" in the ballistic regime Tτ > 1 (Zala, Narozhny, Aleiner, '01)
- two valleys (Punnoose, Finkelstein, '02)
- independent measurements of *F*<sub>0</sub><sup>σ</sup>
  (Okamoto *et al.*, '99, Pudalov, MG *et al.*, '02, Shashkin, Kravchenko *et al.*, '02)



#### The main message:

At not-too-low densities ( $n > 2 \cdot 10^{11} \text{ cm}^{-2}$ ,  $\sigma > 10e^2/h$ ), the anomalous "metallic" behavior of Si MOSFETs is consistent with the theory of interaction corrections.

The interaction corrections in Si MOSFETs are enhanced by the valley degeneracy and the interaction-driven renormalization of Fermi-liquid parameters.



#### Can "metallicity" be explained by the interaction effects?

"Metallicity" is observed mostly in the ballistic regime  $(T\tau > 1)$  – modification of the Altshuler-Aronov theory of interaction corrections was required.

#### Two (complimentary) theoretical models based on the Fermi liquid theory

#### Stern ('80), Gold & Dolgopolov ('86), Das Sarma ('86), Das Sarma & Hwang ('99,'04)

The growth of  $\rho$  with T (or with  $B_{II}$ ) is due to the weakening of screening with increasing T (or increasing spin-polarization).

The leading order in interactions, all orders in *T* <u>Disorder</u>: screened long-range Coulomb disorder

Applicable at  $T_{\rm D} << T << T_{\rm F}$  ( $T_{\rm D} = \hbar/\tau \sim 0.5$ K)

# "Metallicity" - if this term becomes **positive** with strengthening of the interactions ( $IF_0^{\sigma}I > 0.15$ )

#### Zala, Narozhny, & Aleiner ('01)

The interaction corrections to the conductivity are due to the interference between the waves backscattered off an impurity (a short-range potential) "dressed" by Friedel oscillations.

All orders in interactions, the leading order in *T*, <u>Disorder</u>: short-range disorder



$$\Delta \sigma_{INT} \equiv f(F_0^{\sigma}) \frac{T\tau}{\hbar} + g(F_0^{\sigma}) \ln\left(\frac{\hbar}{T\tau}\right)$$

Applicable at  $T << T_F (1 + F_0^{\sigma})^2$  - potentially, can describe the crossover between the ballistic to diffusive regimes at  $T \sim T_D$ 



#### Interaction effects in a multi-valley system

**Ballistic regime:** Zala, Narozhnii & Aleiner ('01)

*T*τ >> 1

$$\partial \sigma_{\text{int}} = -\frac{e^2}{\pi \hbar} \left\{ 1 + \frac{15}{\hbar} \frac{F_0^{\sigma}}{1 + F_0^{\sigma}} \right\} \frac{T\tau}{\hbar}$$

Valley degeneracy [Punnoose and Finkelstein, '02]

2 spins  $\otimes$  2 degenerate valleys  $\Rightarrow$  **16 (1 singlet +15 triplets)** 

Diffusive regime: Altshuler, Aronov, & Lee ('80); Finkelstein ('83)

$$\partial \sigma_{\text{int}} = -\frac{e^2}{2\pi^2 \hbar} \left\{ 1 + \mathbf{15} \left[ 1 - \frac{\ln(1 + F_0^{\sigma})}{F_0^{\sigma}} \right] \right\} \ln\left(\frac{\hbar}{k_B T \tau}\right)$$

Should work for all  $T < (1+F_0^{\sigma})^2 E_F$ :

$$\delta\sigma_{ee}^{T}(T) = 2\left\{\frac{T\tau}{\hbar}\right\} \left[ \left(1 - \frac{3}{8}f(T\tau)\right) + 15\frac{F_{0}^{\sigma}}{1 + F_{0}^{\sigma}} \left(1 - \frac{3}{8}t(T\tau;F_{0}^{\sigma})\right) \right] - \left[1 + 15\left(1 - \frac{\ln(1 + F_{0}^{\sigma})}{F_{0}^{\sigma}}\right)\right] \left\{\frac{1}{\pi}\ln\left(\frac{E_{F}}{T}\right)\right\}$$

Crossover  $\Rightarrow$  $T_{\tau} \sim (1+F_0^{\sigma})/2\pi$ 



### Mechanisms of "pre-factor 15" reduction:

**Magnetic field:**  $k_B T \ll g * \mu B_{||} \ll E_F \quad (15 \Rightarrow 7)$ 

 $\Rightarrow$  spin-induced positive magnetoresistance in parallel fields

Valley splitting:

$$\Delta_V > k_B T \quad (15 \Longrightarrow 7)$$

Inter-valley scattering:

$$\hbar/\tau_V >> k_B T \quad (15 \Longrightarrow 3)$$

single-valley result

The ZNA theory takes into account  $B_{II}$  and  $\Delta_V$ , but ignores  $\tau_V$ .



#### **Analysis of Experimental Data**

*The goal:* to compare data with the theory without using any fitting parameters.



 $\sigma(T, B_{\parallel}) - \sigma_D =$  $\partial \sigma_{WL}(T) + \partial \sigma_{C} + \partial \sigma_{T}$  $\partial \sigma_{INT}(T)$ 

#### Strategy:

- find all INT-relevant parameters in independent measurements (including the study of WL in weak fields)
- suppress WL by weak perp. magnetic field and study INT effects as functions of temperature and parallel magnetic field



### **Analysis of Experimental Data (cont'd)**



#### **Electron Temperature**





#### **Damping:**



Dingle temperature:

$$k_B T_D = \frac{\hbar}{2\pi\tau_q}$$

 $\tau_q$  - the "all-angle" scattering time, almost the same as the transport time for a short-range disorder (Si MOSFETs)

 $T_D \sim 0.3-0.4$ K in our samples

This "thermometer" works down to ~50mK.





# **Valley Splitting**



 $\Delta_V = 0.7 K$ 





Valley splitting: "minority" and "majority" carriers



The interaction-driven renormalization of *m*<sup>\*</sup>, *g*<sup>\*</sup>,  $\chi^*$  is controlled by the Fermiliquid parameters  $F_0^{\sigma}$  and  $F_1^{s}$ :

#### Interaction parameter $F_0^{\sigma}$

$$\chi^* = g^* m^* \frac{g_b \mu_B^2}{\pi \hbar^2} \quad m^* = m_b (1 + F_1^s) \quad g^* = \frac{g_b}{1 + F_0^\sigma}$$

All parameters can be found from the SdH measurements in crossed magnetic fields



 $IF_0^{\sigma}I$  increases up to ~ 0.3 at large  $r_s$ (still very far from Stoner instability ⇒  $F_0^{\sigma}$  = -1) sufficient for explanation of "metallicity"





### $\tau_{\rm V}$ - from the WL MR

#### **Two-valley system:**

 $\partial \sigma_{WL} = \alpha \frac{e^2}{\pi h} f(B, \tau_{\varphi})$ weak inter-valley scatt. ( $\tau_V >> \tau_{\alpha}$ ):  $\alpha = 2$ strong inter-valley scatt. ( $\tau_V << \tau_{o}$ ):  $\alpha = 1$ 



#### Intervalley scattering in Si MOSFETs



Inter-valley scattering is *T*-independent and the ratio  $\tau_V/\tau$  decreases with *n*: *roughness of the Si-SiO*<sub>2</sub> *interface is the dominant factor*.

 $\tau_{\rm V} \sim 20 \text{ps} \implies \hbar/\tau_{\rm V} \sim 0.4 \text{K}$  - close to this *T*, the " $3F_0^{\sigma} \Leftrightarrow 15F_0^{\sigma}$ " crossover in the INT corrections is expected



# **Dephasing in Si MOSFETs**

The dephasing rate due to both singlet and triplet channels (2D): Narozhny, Zala, Aleiner, '02

$$\frac{1}{\tau_{\varphi}} = \left[1 + \frac{15(F_0^{\sigma})^2}{(1 + F_0^{\sigma})(2 + F_0^{\sigma})}\right] \frac{T}{g} \ln\left[g(1 + F_0^{\sigma})\right] + \frac{\pi}{4} \left[1 + \frac{15(F_0^{\sigma})^2}{(1 + F_0^{\sigma})^2}\right] \frac{T^2}{E_F} \ln\left(E_F^{\sigma}\tau\right)$$

Solid lines – 15 triplet components (two degenerate valleys) Dashed lines – 3 triplet components (single valley)





Despite strong interactions ( $r_{\rm S}$  up to ~ 5), Si MOSFETs behave as a Fermi-liquid system.



# **Analysis of Experimental Data**

*The goal:* to compare data with the theory without using any fitting parameters.

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 $\sigma(T, B_{\rm H}) - \sigma_{\rm D} =$  $\partial \sigma_{WL}(T) + \partial \sigma_{C} + \partial \sigma_{T}$  $\partial \sigma_{INT}(T)$ 



0.0

0.2

**B**. T

0.4

#### Strategy:

- find all INT-relevant parameters in independent measurements (including the study of WL in weak fields)
  - suppress WL by weak perp. magnetic field and study INT effects as functions of temperature and parallel magnetic field

 $B_{\perp}$  separates the WL and interaction corrections (orbital effects)

**B**<sub>II</sub> elucidates the structure of interaction corrections (spin effects)



# **Zero-B\_{II}** $\sigma(T)$ : fitting

 $F_0^{\sigma}$  - let's find from the ballistic regime ( $T >> \hbar/\tau, \Delta_V, \hbar/\tau_V$ ) and compare with the SdH data



Black curves: Δσ<sub>INT</sub>(Δ<sub>V</sub><*T*, *h*/τ<sub>V</sub><*T*) Red curves:

 $\Delta \sigma_{\rm INT} (\Delta_{\rm V}, h/\tau_{\rm V} < T)$ 

Blue curves:

 $\Delta \sigma_{\rm INT}(h/\tau_V >> T)$ 

The theory captures all essential features of the observed  $\sigma(T)$ ; for further improvement, the theory consider a finite intervalley scattering rate.

Klimov et al., unpublished





# $\sigma(T)$ in strong $B_{\rm II}$



Klimov et al., unpublished

As soon as the Zeeman energy exceeds *T*, the "metallicity" is weakened. (the number of triplet components is reduced from 15 to 7).



# **Magnetoconductance** $\Delta \sigma(B_{II})$

**Experimental difficulty**: small misalignment of  $B_{\parallel}$  with respect to the 2DEG plane leads to a non-zero  $B_{\perp}$  and orbital (WL) magnetoconductance.

**Solution**: compensation of  ${\pmb B}_{\!\!\perp}$  in the set-up with crossed magnetic fields



MG et al., Physica E **12**, 585 ('02)



### The Fermi-liquid parameter $F_0^{\sigma}$ : comparison





At not-too-low conductances ( $\sigma > 10e^{2/h}$ ), the anomalous "metallic" behavior of Si MOSFETs (as well as in the other high-mobility and low-density systems) is consistent with the theory of interaction corrections. The interaction corrections in Si MOSFETs are enhanced by the valley degeneracy and the interaction-driven renormalization of Fermi-liquid parameters.

#### Agreement with the ZNA theory:

- *n*-type Si: Shashkin *et al.*, *PRB* 66, 73303 ('02); Kvon *et al.*, *PRB* 65, 161304 ('02); Vitkalov *et al.*, *PRB* 67, 113310 ('03); Pudalov *et al.*, *PRL* 91, 126403 ('03)
- *p*-type GaAs: Proskuryakov *et al.*, *PRL* **89**, 076406 ('02); Noh *et al.*, *PRB* **68**, 165308 ('03); Yasin *et al.*, cond-mat/0403411
- n-type GaAs: Li et al., PRL 90, 076802 ('03); Yasin et al., cond-mat/0403411
- *p*-type SiGe: Coleridge *et al.*, *PRB* 65, 125328 ('02)





What is the nature of the apparent 2D MIT?

# (At least) two schools of thought:

2D MIT - a quantum phase transition between a true 2D metal and an insulator.

A new "metallic" phase is stabilized by the strong electronelectron interactions. "2D MIT" - a classical (percolation) transition driven by the emerging macroscopic fluctuations at low *n*.

Anomalous "metallic" behavior – a finite-*T* effect associated with the interaction contribution to the resistivity.



#### What is so special about this crossover?



The WL-SL crossover has been observed in many quasi-1D and 2D systems (this does not mean, however, that we know how to describe R(T) at the crossover quantitatively...).

Gated GaAs/AlGaAs structures,  $n = (0.65-6) \times 10^{15} \text{ m}^{-2}$ Van Keuls *et al.*, *Phys. Rev*.B **56**, 13263 ('97)



Unusual feature of the "metallic"-"insulating" crossover in high-mobility Si MOSFETs:

the WL and INT corrections seem to be suppressed near the crossover (at large R) rather than enhanced...

#### The WL MR near the apparent 2D MIT

Up to ~ 5 kΩ/ , the WL corrections can be well described by the conventional theory. However, at larger *R* , the magnitude of the WL MR (but not  $\Delta \sigma_{WL}(T)$ !!) is *smaller* than the predictions for a *homogeneous* system with  $\sigma >> e^2/h$ .

high-µ Si MOSFET 10 (a) 8 6 σ(e²/h) 0.98 4 0.942 0.90 0.87 0.82 -0.4-0.20.2 0.4 B (tesla) Rahimi et al. ('03)

"Vanishing" WL MR at  $\sigma \Rightarrow e^2/h$  is expected for both homogeneous and macroscopically inhomogeneous systems:

homogeneous – owing to the second-loop corrections

$$lpha = 1 - rac{2G_0}{\sigma}$$
 Gornyi ('04)  
 $G_0 = e^2/(2\pi^2\hbar)$ 

macro inhomogeneous – owing to the percolation effects Aronov, MG, Zhuravlev ('86)



This scenario seems more likely - it explains suppression of both  $\Delta \sigma_{WL}(T)$  and  $\Delta \sigma_{WL}(B)$ 



#### Entertaining the non-homogeneous scenario...



In our samples, separation between dopants ~ 0.1  $\mu$ m.



Emerging macroscopic inhomogeneity at a macroscopic scale  $L > L_{\varphi}$ The screening becomes weaker and strongly nonlinear as the density is lowered  $\Rightarrow$  a nominally uniform 2DEG breaks up into isolated puddles.

Efros ('88), Nixon and Davies ('90), Meir, '99 Das Sarma and Hwang, ('83, '99, '04), MG ('02), Fogler ('03), etc.



### What would be the consequences of percolation?

The amplitude of WL corrections is suppressed:

$$\Delta \sigma_{WL} = \frac{\Delta \rho_{WL}}{\rho_{macro}^2} \qquad \text{but } R_{\text{metallic}} << R_{\text{total}} \qquad \checkmark$$

- in the conductors with percolation, the quantum corrections are smaller than one might expect for a homogeneous system with the same total resistance.

Aronov, MG, and Zhuravlev, *JETP* **60**, 554 (1984) MG *et al.*, *Phys.Rev.Lett.* **74**, 446 (1995).

• At the same time,  $\tau_{q}$  would not depend on  $R_{macro}$ 

• The same conclusion would be applicable to the INT corrections – this percolation "reduction" of the quantum corrections might explain why the slope dR/dT in the vicinity of the apparent MIT becomes smaller instead of getting bigger ( $IF_0^{\sigma}I$  remains large near the apparent 2D MIT).





D. Popovic et al., R. Leturcq et al. PRL 90, 076402 (2003)

• The amplitude of SdH oscillations becomes anomalously large:  $\tau_a$  becomes greater than  $\tau_{tr}$ 



(The WL corrections would be reduced by a large factor  $\tau_{\rm q}$  /  $\tau_{\rm tr}$ ).



# Hysteresis effects at lower densities/higher resistances



Pudalov, MG, Klimov, and Kojima *JETP Lett.* **82**, 412 (2005). The same device, cooled down to 4K at different fixed values of  $V_{\rm g}$ .

**Important:** different  $V_g^{cool}$  did not affect significantly the parameters which control the interaction effects  $(\tau \text{ and } F_0^a)$ 

Universal behavior  $\Rightarrow$  sufficiently far from the apparent 2D MIT ( $\rho \le 0.1h/e^2$ , or  $n > 1.3x10^{11}$  cm<sup>-2</sup>).



# **Part II: Conclusion**

The crossover from "metallic" to "insulating" conductivity in highmobility Si MOSFETs seems to be consistent with a classical percolation scenario.

Close to the crossover, the dilute 2D systems may become nonhomogeneous. If this is the case, the temperature and magnetic-field dependences of  $\sigma$  are still due to the quantum corrections, but the magnitude of these corrections is diminished by development of percolation at low electron densities.

In the percolation scenario, the "apparent 2D MIT" has nothing to do with interactions.

Probing the macroscopic homogeneity of dilute 2D systems – the key issue for better understanding of the apparent 2D MIT.



