Michael Gershenson
Dept. of Physics and Astronomy
Rutgers, the State University of New Jersey

“Low-Temperature Nano-Physics”
Chernogolovka, August 2007
Lecture 1. Weak and Strong Localization in 1D and 2D Conductors (single-particle effects).

Lecture 2. Electron-electron Interactions and Dephasing Processes in Disordered Conductors

Lecture 3. Quantum Corrections in the Conductivity of Si MOSFETs
Lecture 1:  WL and SL in 1D and 2D conductors (non-interacting electrons)

- Intro: Anderson Localization, Scaling Theory
- Weak Localization Corrections
  - Non-locality of WL effects
  - Interplay between WL and percolation
- WL-SL crossover
- Hopping in Systems with Large Localization Length
Disordered Conductors: Characteristic Scales

Quenched disorder, degenerate Fermi gas

Structure size: \( L \)

Fermi wavelength: \( \lambda_F \) 
\(< 1 \text{ nm (metals), } \sim 10 \text{ nm (semiconductors)} \)

Elastic mean free path: \( l = v_F \tau \) 
10-100 nm (metals), 0.1-1 \( \mu \text{m} \) (semiconductors)

Localization length: \( \xi \) 
\( l << \xi \)

Phase coherence length: \( L_\phi = \sqrt{D \tau_\phi} \) 
\( l < L_\phi < \xi \), typically 0.1-10 \( \mu \text{m} \)

Thermal dephasing length: \( L_T = \sqrt{\hbar D / k_B T} \) 
\( l < L_T < \xi \), typically 0.03-1 \( \mu \text{m} \)

The magnetic length: \( L_H = \sqrt{\Phi_0 / 2\pi H} \) 
1 \( \mu \text{m} @ 20 \text{G} \)

Percolation correlation length \( \xi_P \) (macroscopically non-homogeneous systems with percolation)

Dimensionality: with respect to the quantum lengths \( \xi, L_\phi, L_T, L_H \)
Anderson Localization (3D)

Non-interacting electrons in a 3D ordered system ⇒ delocalized electron states

What happens if we crank up the disorder?

**P.W. Anderson** ('58): Quantum interference can completely suppress the diffusion of a particle in random potential (disorder can localize a particle despite of tunneling).

At $T=0$, there is a quantum phase transition in the system at a critical strength of disorder, $x_C$. At $x > x_C$, the electron states at the Fermi level are localized and look like that:

More realistically, like that:

*Multifractal wave function at the critical point (courtesy of F Evers, A Mildenberger, and A Mirlin, unpublished)*
Anderson MIT

“Anderson” insulator: finite density of states at the Fermi level.

Realization of the Anderson MIT – by tuning either disorder or electron concentration

\[
\sigma = \frac{e^2}{h} n \frac{1}{k_F}, \quad n = \frac{k_F^3}{3\pi^2}, \quad l \sim k_F^{-1}
\]

\[
\sigma_{3D} \sim \frac{e^2}{h} n^{1/3} \sim 2 \cdot 10^{-4} \quad \Omega^{-1} n^{1/3}
\]

\[
\sigma_{2D} \sim \frac{e^2}{h} \sim (26k\Omega)^{-1}
\]

Anderson MIT – continuous, no min. metallic conductivity!

**Wegner** ('76, '79): a close connection between the Anderson transition and the scaling theory of critical phenomena; field description of the localization problem in terms of a nonlinear \(\sigma\)-model.

**Efetov** ('80): a microscopic derivation of the \(\sigma\)-model.
Localization Length in 3D

The length scale: **the localization length** $\xi$

$$|\psi^2(\vec{r})| \propto \exp\left(-\frac{|\vec{r} - \vec{r}_0|}{\xi}\right)$$

Scaling at the Anderson MIT:

$$\xi \propto (E_C - E)^\nu \quad \sigma \propto (E - E_C)^s \quad s = \nu(d - 2)$$

Wigner (‘51) and Dyson (‘62): classification of ensembles of random Hamiltonians on the basis of invariance of the system under *time reversal and spin rotations*: unitary, orthogonal, and symplectic symmetries.

**Important:** $\xi$ depends on the **disorder** (proximity to the critical point) and the underlying **symmetry** of the system.
Scaling Ideas

The conductivity $\sigma$ is convenient when it is a “local” quantity that does not depend on the sample dimensions.

The development of scaling ideas (Thouless, '74,77, Abrahams, Anderson, Licciardello, Ramakrishnan, '79, Wegner, '79) showed that more useful quantity is the dimensionless conductance $g(L)$ of a system of the size $L$ (in units of $e^2/h$).

Scaling of the conductance $g$ with the size of a disordered system ($T=0$, non-interacting electrons):

**The idea:** to investigate how the conductance changes if we increase the size of the system

![Diagram showing scaling of conductance](image)

**Put together:** Level hybridization

<table>
<thead>
<tr>
<th>$L$</th>
<th>$\uparrow$</th>
<th>$L$</th>
</tr>
</thead>
</table>

Discrete (“localized”) levels
Level spacing: $\Delta_L = (\nu L^d)^{-1}$

Shift: $\delta E \sim \frac{E_T^2}{\Delta_L}$

- Thouless energy

(inverse time to diffuse across the system)

(i) $\delta E \ll \Delta_L$

(ii) $\delta E \ll \Delta_L$

The state is localized in one of the pieces

The state is extended

$g = 2\pi E_T / \Delta_L$ - dimensionless conductance of a piece $L$

**Thouless criterion:**

$g \gg 1$ - metal  $g \ll 1$ - insulator
Scaling Theory

**Abrahams et al.** (’79): a scaling theory of localization which describes the flow of the dimensionless conductance \( g \) with the system size \( L \).

Evolution of the conductance with the system’s size is determined by the conductance itself:

\[
\frac{d \ln g}{d \ln L} = \beta(g)
\]

**MIT:** \( \beta(g=g_{\text{crit}})=0 \)

3D (orthogonal and unitary symmetry classes): Anderson MIT

2D and 1D: all states are **localized**, even if the disorder is weak.

"**Metal**": Ohm’s law \( g \propto L^{d-2} \Rightarrow \beta(g) = d - 2 \)

2D: the geometrical factor in the conductance, \( g \sim (L/l)^{d-2} \), disappears, and the equation \( \beta(g) \) does not depend explicitly on \( L \).
Localization Length in 1D and 2D

Quasi-1D conductors:

\[ \lambda_F \ll W \ll \xi \]

\[ N_{1D} = \frac{2W}{\lambda_F} \gg 1 \]

- the # of conducting channels

Berezinsky, '74; Efetov and Larkin, '83
Dorokhov, ‘83

Orthogonal
quenched disorder, no mag. field
(time reversal symmetry)

Unitary
strong mag. field breaks
the time reversal symmetry

\[ \xi_{1D}^O = N_{1D} \cdot l \]

\[ \xi_{1D}^U = 2N_{1D} \cdot l \]

2D conductors:

Orthogonal

\[ \xi_{2D}^O = l \cdot \exp\left(\frac{\pi}{2} k_F l\right) \]

\[ B >> \frac{\Phi_0}{\xi W} \]

- flux quantum in
the area of a
localized state

Unitary

\[ \xi_{2D}^U = l \cdot \exp\left[\left(\frac{\pi}{2} k_F l\right)^2\right] \]

\[ B >> \frac{\Phi_0}{\xi^2} \]

d \ll \xi

- strong mag. field breaks
the time reversal symmetry

\( \Phi_0 \) - flux quantum in
the area of a
localized state

\( \lambda_F \) - Fermi wavelength

\( W \) - sample width

\( \xi \) - coherence length

\( N_{1D} \) - number of conducting channels

\( k_F \) - Fermi wave number
Finite Temperatures ⇒ Dephasing Processes

All electronic states in 1D and 2D are localized. However, some of these systems are very good conductors at high temperatures. Why?

If the disorder is not too strong and the temperature is not too low, the electron wavefunction will be “mutilated” before an electron has a chance to diffuse over the localization length. Because of these dephasing processes, an electron will be frequently scattered between localized states, and it will diffuse ALMOST as if its wavefunction is not localized.

**Sources of dephasing:**

**Internal (‘the enemies within’):** phonons, other electrons, all dynamic degrees of freedom (“…ons”)

**External (‘the enemies without’):** spin-spin scattering, external high-frequency E&M fields

Length scale:  the phase coherence length  \( L_\varphi = \sqrt{D\tau_\varphi} \)

Typical value of \( \tau_\varphi(1K) \) in metals \( \sim10^{-8}-10^{-7} \) s, \( D \sim 1-100 \) cm\(^2\)/s, \( L_\varphi(1K) \sim 0.1-10\mu\text{m} \)

\[ L_\varphi(T) \Leftrightarrow \xi \]

**Weak Localization:**  \( \lambda_F << \ell < L_\varphi(T) << \xi \)

**Strong Localization:**  \( \lambda_F < \ell < \xi < L_\varphi(T) \)
**Weak Localization Corrections**

**Gor’kov, Larkin, Khmelnitskii** (1979): resummation of singularities in perturbation theory in 2D and formulation of the scaling in the systematic form of a renormalization group theory.

The total probability to get from A to B:

$$w_{AB} = \left| \sum_i A_i \right|^2 = \sum_i |A_i|^2 + \sum_{ij} |A_i A_j|$$

probabilities interference of amplitudes

The phase gain: \[ \Delta \varphi = \frac{1}{\hbar} \int_A^B \tilde{p} d\tilde{l} \gg 1 \quad p – the\ Fermi\ momentum \]

The interference term vanishes \( \left\langle \sum_{ij} |A_i A_j| \right\rangle = 0 \) due to the disorder averaging, but **NOT** for the time-reversed paths (loops)

The self-intersecting (loop-like) trajectories:

\[ \tilde{p} \Rightarrow -\tilde{p} \quad \tilde{l} \Rightarrow -\tilde{l} \quad \Delta \varphi = 0 \]

\[ w_0 = |A_1|^2 + |A_2|^2 + 2 \text{Re} \ A_1 A_2 = 4|A_1|^2 \]

As a result, the total scattering probability at site O increases, and the conductance – decreases. **The corresponding corrections to the transport properties are known as the weak localization corrections.**
Different dimensionalities:

The dimensionality of a conductor: $L \leftrightarrow L_\phi(T) \ll \xi$

3D:

$$\lambda_F v_F dt \propto \Delta\sigma_{WL} \propto \int_\tau^{\tau_\phi} \frac{\lambda_F^2 v_F}{(Dt)^{3/2}} dt \propto \frac{e^2}{h} \left[ \frac{1}{L_\phi(T)} - \text{const} \right]$$

$L_\phi < \text{all dimensions, } \xi$

2D:

$$\int_\tau^{\tau_\phi} \frac{\lambda_F^2 v_F}{dDt} dt \propto -\frac{e^2}{h} \ln \left[ \frac{L_\phi(T)}{l} \right]$$

$d < L_\phi \ll \xi$

Quasi-1D:

$$\int_\tau^{\tau_\phi} \frac{\lambda_F^2 v_F}{dW(Dt)^{1/2}} dt \propto -\frac{e^2}{h} L_\phi(T)$$

$d, W < L_\phi \ll \xi$

Though these QC are small at $L_\phi \ll \xi$, they diverge in low dimensions as $T$ decreases (and $L_\phi$ increases), and eventually they drive the system into the SL regime.
Observation of the WL corrections

\[ \Delta \sigma_{WL} \ll \sigma_0 \]

Still, WL effects are observable because \( \Delta \sigma_{WL} \) depends (in a non-trivial way) on \( T \) and \( B \), in contrast to the «residual» Drude conductivity \( \sigma_0 \).

\( \Delta \sigma_{WL}(T) \) – due to the \( T \)-dependence of phase relaxation (the lower \( T \), the slower the relaxation). However, \( \Delta \sigma_{WL} \) is not the only \( T \)-dependent quantum correction, the interaction effects result in a similar dependence (Lectures 2,3).

Curiously, the first observations of the increasing resistivity in 1D and 2D metal films with cooling, which have been considered as a support for the WL theory, were in fact the observations of INT corrections (the WL correction was \( T \)-independent because of a strong spin-spin scattering).

It is easier to analyze \( \Delta \sigma_{WL}(B) \) because INT corrections aren’t sensitive to weak magnetic fields.

Giordano, *PRB* 22 ('80)  
Dolan and Osheroff, *PRL* 43 ('79)
WL in external magnetic field

Classically weak magnetic fields: no trajectory “bending”.

Aharonov-Bohm phase acquired by the loops:

\[ \vec{p} \Rightarrow \vec{p} - \frac{e}{c} \vec{A} \]

\[ \Psi \Rightarrow \Psi \exp \left( i \frac{e}{\sqrt{2} \hbar} \oint \vec{A} \, d\vec{l} \right) \]

\( \vec{A} \) - the vector potential

\[ \Delta \varphi_H = \frac{2e}{\sqrt{2} \hbar} \oint \vec{A} \, d\vec{l} = \frac{2e}{\sqrt{2} \hbar} \oint H \, d\vec{s} = 2\pi \frac{\Phi}{\Phi_0} \]

- the phase difference between CW and counter-CW loops increases with field.

\[ \Phi_0 = \frac{hc}{2e} = 2.07 \cdot 10^{-7} \text{ G cm}^2 \]

- the flux quantum (~20G in 1\( \mu \text{m}^2 \))

\[ w_0 = \left| A_1 \right|^2 + \left| A_2 \right|^2 + 2 \left| A_1 \right| \left| A_2 \right| \cos(\Delta \varphi_H) = 2A^2 \left[ 1 + \cos(\Delta \varphi_H) \right] \]

\[ L_H = \sqrt{\frac{\Phi_0}{2\pi H}} \]

- the magnetic length (the characteristic length at which two interfering waves propagating along a loop in opposite directions acquire the phase difference ~ 2\( \pi \)).
Aharonov-Bohm effect in disordered conductors

\[ w_0 = 2A^2 \left[ 1 + \cos \left( 2\pi \frac{\Phi}{\Phi_0} \right) \right] \]

Sharvin and Sharvin, '81
Altshuler et al., '82

Li « tube»
\[ \Phi = 1.3\mu m \]
\[ T = 1.1K \]

multi-wall carbon nanotubes \[ r_e = 8.6 \text{ nm} \]
Bachtold et al. (’98)
WL magnetoresistance

When the magnetic length $L_H \sim (\Phi_0/B)^{1/2}$ becomes smaller than $L_\varphi(T) \Rightarrow WL$ magnetoresistance

WL magnetoresistance:
- negative (no SOI so far)
- anisotropic in 1D and 2D (purely orbital effect)
- observed in classically weak magnetic fields

WL MR measurements are extensively used to measure $L_\varphi$.

MG et al., '81

Cu film
$d=7$ nm
$T=10K$
Weak Anti-Localization

So far, we’ve considered spinless electrons. However, the electrons have spin 1/2, and the spin scattering modifies the WL corrections. Two scattering processes:

(a) spin-spin scattering (e.g., by paramagnetic impurities) – destroys constructive interference at $t > \tau_S$ ($L_\phi(T) \Rightarrow L_S$).

(b) spin-orbit interaction (as a relativistic effect, strongly depends on $\nu$)

When electrons are scattered by a strong electric field of nuclei, they “feel” (in their reference frame) an effective magnetic field

$$\vec{B}_{SO} = \frac{1}{2c^2}(\vec{v} \times \vec{E})$$

Because the interfering electron waves propagate in the opposite directions ($\nu \leftrightarrow -\nu$), the $B_{SO}$ vectors are also oppositely directed, and the spins “diffuse” away from each other on the Bloch sphere.

The spin-orbit length: $L_{SO} = \sqrt{D \tau_{SO}}$ - $T$-independent, a strong function of the nucleus charge (Abrikosov & Gorkov, '62)

$$\tau_{SO} = \tau(\alpha Z)^{-4}$$

After averaging over all trajectories with dimensions $L_{SO} < L < L_\phi$, the WL correction becomes positive – “weak anti-localization”
WL and WAL magnetoresistance

Only WL

WL + WAL

Only WAL

Si MOSFET

2D Ag film

1D Ag wire

WL MR

\[ \phi_L \]

\[ \phi_{SO} \]

\[ L^2 \]

\[ \ell^2 \]

\[ L_{SO}^2 \]

\[ L_{\phi}^2 \]

\[ W = 70 \text{ nm} \]

\[ T = 0.2 \text{K} \]

\[ L_H = W \]

\[ L_H = L_{\phi}(T) \]
Non-locality of Quantum Corrections

2D and 3D: due to the non-locality of QC, the disorder is averaged over a large length (in comparison with the defect size) ⇒ "universality" of QC.

0D and 1D: Because of non-locality, QC can be strongly influenced by the phase coherent regions extending beyond the classical current paths.

Umbach et al., '87
MG et al., '95

Distance between side branches:

T=0.1K
$L_\varphi=2.3\mu m$

$\infty$
$3.6\mu m$
$1.6\mu m$
$0.4\mu m$
$0.2\mu m$
WL in macroscopically non-homogeneous conductors (2D and 3D)

To be specific, we’ll consider WL (though many conclusions apply to the INT corrections as well).

New scale: **the percolation correlation length** \( \xi_P \)

At \( L \gg \xi_P \) the conductivity scales with the conductor dimensions as in a homogeneous system \( (R_{\text{macro}}) \).

At \( L < \xi_P \) - the anomalous diffusion, the diffusion constant \( D(L) \) depends on the scale \( L \).

Far from the percolation threshold \( (L_\phi \gg \xi_P) \) - “universal” corrections, \[ \Delta \sigma_{WL} = \frac{\Delta \rho_{WL}}{\rho_{\text{macro}}^2} \]

Close to the threshold \( (L_\phi < \xi_P) \): an interplay between QC and percolation.
Consequences of macroscopic non-homogeneity

Near the percolation threshold \((L_\varphi, L_H < \xi_P)\), the quantum corrections are much smaller than that expected for a homogeneous system with the same total resistance.

\[
\frac{\Delta \rho_{WL}(T, B)}{\rho^2_{macro}} = "\Delta \sigma_{WL}" \ll \Delta \sigma_{WL}
\]

Aronov, MG, and Zhuravlev ('84) - experiment + theory
Palevski and Deutscher ('86)
Dumpich and Carl ('91)

Using the theory of WL corrections in conductors with percolation, one can extract \(\xi_P\) from the WL magnetoresistance.

One should be careful about interpretation of "\(L_\varphi\)" in systems with percolation \([L_\varphi \Rightarrow (L_\varphi \xi_P)^{1/2}]\).

In particular, \(\tau_\varphi\) may be insensitive to \(R_{macro}\) and the temperature dependence of "\(\tau_\varphi\)" is weakened.

[see also Germanenko et al. PRB 64, 165404 ('01)]
Man-made 2D percolation networks

2D network made of 0.05 μm-wide Au wires. The unit cell size – 0.4 μm, # of cells ~ 10^6. In the process of e-beam writing, the probability of skipping a bond varied from 0 to 0.5.

<table>
<thead>
<tr>
<th>Sample</th>
<th>x</th>
<th>ξ (μm)</th>
<th>R_□(ξ) (Ω)</th>
<th>R^MR_□</th>
<th>R^TD_□</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>0.4</td>
<td>14</td>
<td>14</td>
<td>20</td>
</tr>
<tr>
<td>b</td>
<td>0.3</td>
<td>1.4</td>
<td>50</td>
<td>38</td>
<td>~50</td>
</tr>
<tr>
<td>c</td>
<td>0.4</td>
<td>3.4</td>
<td>130</td>
<td>47</td>
<td>—</td>
</tr>
<tr>
<td>d</td>
<td>0.45</td>
<td>8.6</td>
<td>527</td>
<td>51</td>
<td>—</td>
</tr>
<tr>
<td>e</td>
<td>0.47</td>
<td>17</td>
<td>1039</td>
<td>78</td>
<td>—</td>
</tr>
</tbody>
</table>

The percolation network made of 1D wires behaves like a homogeneous 2D film with respect to the quantum interference effects if \( L_\phi, L_H > \xi_P \).

Close to the percolation threshold, when \( \xi_P \) becomes much larger than \( L_\phi, L_H \), the QC become quasi-1D.

These 1D QC are affected by the presence of dead ends of the infinite percolation cluster when \( L_\phi > a \) (a – the unit cell size).
What happens when $\sigma_{2D}$ approaches $e^2/h$?

With diminishing $\sigma$, the second-loop corrections become important (approx. at $R > 5k\Omega$). Because of these corrections, $\Delta\sigma_{WL}(B)$ is strongly diminished whereas $\Delta\sigma_{WL}(T)$ remains “universal” with pre-factor $\sim e^2/h$.

$$\Delta\sigma_{WL}(B) = \alpha \frac{e^2}{2\pi^2 h} f(B)$$

$\alpha = 1 - \frac{2G_0}{\sigma}$  
$G_0 = \frac{e^2}{(2\pi^2 h)}$

Caution: “vanishing” WL MR at $\sigma \Rightarrow e^2/h$ is expected for both homogeneous and macroscopically inhomogeneous systems. In the latter case, however, $\Delta\sigma_{WL}(T)$ should also be diminished (similar to the behavior of high-mobility Si MOSFETs).
“WL ⇔ SL” crossover in 1D and 2D

What happens when \( L_\phi(T) \) becomes of the order of \( \xi \) with cooling?

**WL:** \( L_\phi < \xi \)  
**SL:** \( L_\phi > \xi \)

The QC become of the order of the Drude conductivity:

**2D:** \[ \sigma_0 = \frac{e^2}{h} k_F l \quad \sigma \approx \frac{e^2}{h} \left( k_F l - \ln \frac{L_\phi}{l} \right) \Rightarrow 0 \quad \text{at} \quad L_\phi \approx \xi_{2D} \quad [\xi_{2D} \approx l \exp(k_F l)] \]

**1D:** \[ \sigma_0 = \frac{e^2}{h} k_F lW \quad \sigma \approx \frac{e^2}{h} \left( k_F lW - L_\phi \right) \Rightarrow 0 \quad \text{at} \quad L_\phi \approx \xi_{1D} \quad [\xi_{1D} \approx lk_F W] \]

\[ L_\phi, \xi \]

\[ T \]

\( \phi \)
Electrons are localized over many impurity states ($\xi >> n^{1/2}l$), and there is a diffusive motion within the localization envelope. The localized states strongly overlap in space [e.g., in our Si:GaAs samples, there are $10^3$ localized electrons confined within an area of a size of $\xi \cdot W$].

Quasi-1D wires in Si $\delta$-doped GaAs

$\xi_{1D} \approx L_{\varphi}(T_{\xi})$, Nyquist dephasing $\Rightarrow T_{\xi} \approx \frac{1}{D W^2 (m^*)^2}$

Electrons are localized over many impurity states ($\xi >> n^{1/2}l$), and there is a diffusive motion within the localization envelope. The localized states strongly overlap in space [e.g., in our Si:GaAs samples, there are $10^3$ localized electrons confined within an area of a size of $\xi \cdot W$].

$E_F \sim 10^3 K$

the inter-level spacing within the localization length $\sim E_F/(nW\xi)$ in quasi-1D
At the crossover:

1. \( L_\varphi(T_\xi) \sim \xi \)

2. The conductance of a wire segment of the length \( \xi \) \( \Rightarrow \frac{e^2}{h} \)

3. At the crossover temperature \( T_\xi \), all energy scales are \textit{of the same order of magnitude}:

\[
\frac{\hbar}{\tau_\varphi(T_\xi)} \sim \Delta_\xi \sim k_B T_\xi \sim \frac{\hbar D}{\xi^2}
\]

- level broadening
- inter-level distance
- crossover temperature
- Thouless time \( \xi^2/D \)
2D "WL ⇔ SL" crossover

The WL-SL crossover is "sharper" for 1D samples.

Qualitatively – similar to the 1D "WL-SL" crossover

The 2D crossover can be observed only if \( k_F l \sim 1 \)

(more room for that in 1D)

\[
L_\varphi (T) \leftrightarrow \xi \approx l \exp (k_F l)
\]

\( h/e^2 \)

\( R \Omega \)

\( T (K) \)

\( k_F l \)

\( \xi \)

\( \varphi \)

\( \phi \)

Gated Si:GaAs
MG et al., unpublished

Rutgers University of New Jersey
Limits of 2D WL Description

Minkov et al. ‘07:
δ-doped GaAs structures, 
\( n \sim 1.5 - 2.5 \times 10^{12} \text{cm}^{-2} \)

\[ \sigma \leftrightarrow \frac{e^2}{h} \quad \text{where should we stop using the WL theory?} \]

\[ \frac{1}{\tau_\phi^*} = \frac{1}{\tau_\phi} + \frac{\xi^2}{D} \]
Minkov et al. '07: “...down to the very low conductivity value, $\sigma \sim 10^{-2} e^2/h$, the experimental results are described by the same expressions which are valid in the WL regime. One should only replace the dephasing rate $\tau_\phi^{-1}$ by the quantity $\tau_\phi^{-1} + \tau_\xi^{-1}$.”
Hopping in Systems with Large Localization Length

Low-\(T\) WL-SL crossover - a unique opportunity to explore electron hopping with a large \(\xi\), up to a few \(\mu\)m in 1D and \(~0.5\mu\)m in 2D.

Hopping with large \(\xi\) in 1D and 2D systems – a very interesting transport regime, which is qualitatively different in many respects from the conventional hopping with small \(\xi\) in lightly doped semiconductors - the electron motion is still diffusive within the localization length

\[ \lambda_F < \ell < \xi < L_\phi(T) \]

Signatures of hopping with large \(\xi\): orbital magnetoresistance, hot-electron effects in strong electric fields.

Several experimental facts (“universality” of \(R(T)\), hot-electron effects in SL regime) suggest that this hopping might be driven by electron-electron interactions rather than electron-phonon interactions.
The “Arrhenius” dependences $R(T)$ have been observed for both 1D and 2D conductors in the SL regime.

The activation energy is close to the mean level spacing within the localization envelope, $\sim (0.2-1) \Delta_\xi$.

**Puzzle:** for different 2D systems, the pre-factor is “universal” – not expected if hopping is phonon-assisted.

Gated *quasi-1D wires* in Si $\delta$-doped GaAs

*Khavin, MG, Bogdanov, PRB 58, 8009 ('98)*

*2D Si:GaAs*

*MG et al, PRL 85, 1718 ('00)*

Similar dependences – in ultra-thin metal films (e.g., Goldman et al.)

*Holes in GaAs/AlGaAs*

*Leturcq et al. ('02)*
MR in strongly-localized 1D systems

\[ R(T, B) = R_0 \exp \left[ \frac{\Delta_\xi(B)}{T} \right] \]

the pre-factor \( R_0 \) does not depend on \( B \)

\[ \Delta_\xi = (v_{1D} \xi)^{-1} \]

Doubling of \( \xi \) \( \Rightarrow \) Cutting \( \Delta_\xi = (v_{1D} \xi)^{-1} \) in half
What does the theory say?

- Kolesnikov and Efetov, *PRL* 83, 3689 ('99)
- Schomerus and Beenakker, *PRL* 84, 3927 ('00)
- Kettemann, *PRB* 62, R13282 ('00)
- Kettemann and Mazzarello, *PRB* 65, 085318 ('02)

- direct measurements of $\xi$ in 1D (and 2D) conductors with a large $\xi$
MR in the 2D SL regime

The activation energy is decreased in the magnetic field \( \perp \) to the plane of a 2D sample:

\[
R(T, B) = R_0 \exp \left[ \frac{\Delta \xi(B)}{T} \right] \quad \Delta \xi = \left( \nu_{2D} \xi^2 \right)^{-1}
\]

\( R_0 \approx \frac{h}{e^2} \) - does not depend on B

In 2D, one might expect more radical drop of the activation energy in \( B \):

\[
\xi_{2D}^O = l \cdot \exp \left( \frac{\pi}{2} k_F l \right) \quad \xi_{2D}^U = l \cdot \exp \left( \frac{\pi}{2} k_F l \right)^2
\]

However, \( k_F l \sim 1 \) for the 2D conductors which demonstrate the SL-WL crossover at low T (= large \( \xi \)).

\( \xi \sim 0.1 \mu m \ (\sim 10l) \)
Examples of strong orbital MR in 2D

Butko and Adams, *Nature* 409, 161 ('01)

"Quantum metallicity" – strong reduction of the activation energy due to the growth of $\xi$ in $B>\Phi_0/\xi^2$?

Interesting parallel:
strong-field MR of 2D systems close to the disorder-driven Superconductor-Insulator Transition

Baturina *et al.*, TiN films
Shahar *et al.*, InO$_x$ films

$\Phi_0/B \sim \xi \sim \xi_{SC}

Baturina *et al.*, '06

$R(T, B) = \frac{h}{e^2} \exp \left[ \frac{\Delta_{\xi}(B)}{k_B T} \right]

\Delta_{\xi}(B)$ – mean level spacing within $\xi^2 (B)$
Aharonov-Bohm Effect in Hopping

Poyarkov et al. ‘86: disordered PbTeO$_x$ with persistent low-T photoconductivity

**FIG. 30.** $R(H)$ dependence at $T = 1.12$ K for the same sample as in Fig. 29 (average of 24 runs). The $R(\Omega)$ scale for the single-run curve (dotted line) is shown on the right (Poyarkov et al., 1986).

Never reproduced!
Non-linearity of the hopping conductivity of 2D systems with a large $\xi$ is due to hot-electron effects rather than the field effects; the electric field, which is sufficient to overheat electrons, does not depend on $\xi$.

The thermal conductivity between electrons and phonons, $G_{e-ph} = C_e/\tau_{e-ph}$, is the same on both sides of the 2D WL-SL crossover (this might be expected, since the el. motion is still diffusive at $L<\xi$, and $q_T\xi \sim 1$ even at $T=50$ mK).
Non-Linear Effects in Conductors with a large $\xi$

In the systems with a large $\xi$, *electron heating is expected to dominate over the field effects*. Let’s compare two values of the power $P$ dissipated in a conductor:

- $P_{NL}$, the Joule heat power at which the $E$-induced non-linearity develops:

$$
P_{NL} = \frac{(EL)^2}{R} = \frac{\left(\frac{k_B T L}{e \xi}\right)^2}{h / e^2 \exp\left(\frac{1}{k_B T \nu_{2D} \xi^2}\right) \cdot \xi^2 \exp\left(\frac{1}{\xi^2}\right)}
$$

- decreases exponentially with decreasing $\xi$ for $\exp(...)>> 1$

- $P_{E-PH}$, the power that can be removed from an electron system due to electron-phonon coupling without significant overheating of electrons

$$
P_{E-PH} \approx e E \xi \approx k_B T
$$

- does not depend on $\xi$

$P$ vs $\xi$ diagram:
- el. field nonlinearity
- hot-electron nonlinearity

The State University of New Jersey
RUTGERS
What have we learned from the hot-electron experiments:

2D systems: the $\rho(E)$ dependence can be attributed to carrier heating all along the crossover from the diffusive to the VRH regimes [n-GaAs – MG et al. ('00), SiGe – Leturcq et al., ('03)].

This tendency has been observed for $\sigma$ as small as $10^{-4}$ e²/h. Recall: Minkov et al. ('04) considered the hot-electron regime as a signature of “diffusiveness” rather than hopping down to $\sigma \sim 2 \times 10^{-2}$ e²/h.

3D Strongly Localized systems: strong evidence for the existence of separate temperatures for the electron and phonon systems analogous to the hot-electron effects in metals [doped Si and Ge - Zhang et al., ('98); doped Ge – Wang et al., ('90,'99); Marnieros et al., ('00); doped Si - Galeazzi et al, '07].

Hopping conductivity depends only on the electron temperature when electrons are removed from equilibrium with phonons. The effective electron temperature is established via interactions among the localized electrons. This suggests that electron-electron interactions play crucial role in hopping in conductors with a large localization length (electron-assisted hopping rather than phonon-assisted one).

**In the WL regime:** the temperature dependences of $\Delta \sigma_{WL}$ and $\Delta \sigma_{EEI}$ are governed by electron-electron interactions

**In the SL regime:** “universal” hopping $R(T)$; hot-electron effects when electrons are removed from equilibrium with phonons

*Temptation: to develop a unifying (EEI-based) approach that would describe $R(T)$ over a wide range of resistances (WL+SL).*
Single-particle effects in disordered conductors: quantum interference leads to WL corrections to the Drude conductivity in the regime of weak disorder/high $T$ ($L_\phi < \xi$) or SL in the regime of strong disorder/low $T$ ($L_\phi > \xi$).

Due to non-locality, the WL corrections are very universal; WL magnetoresistance provides unique opportunity to study inelastic and spin scattering in disordered conductors.

Realization of the low-T WL-SL crossover provides an access to the regime of (unconventional) hopping with a large localization length.