Low-Dimensional Disordered Electronic Systems (Experimental Aspects)

or

“Галопом по Разупорядоченным Системам”

Michael Gershenson

Dept. of Physics and Astronomy
Rutgers, the State University of New Jersey

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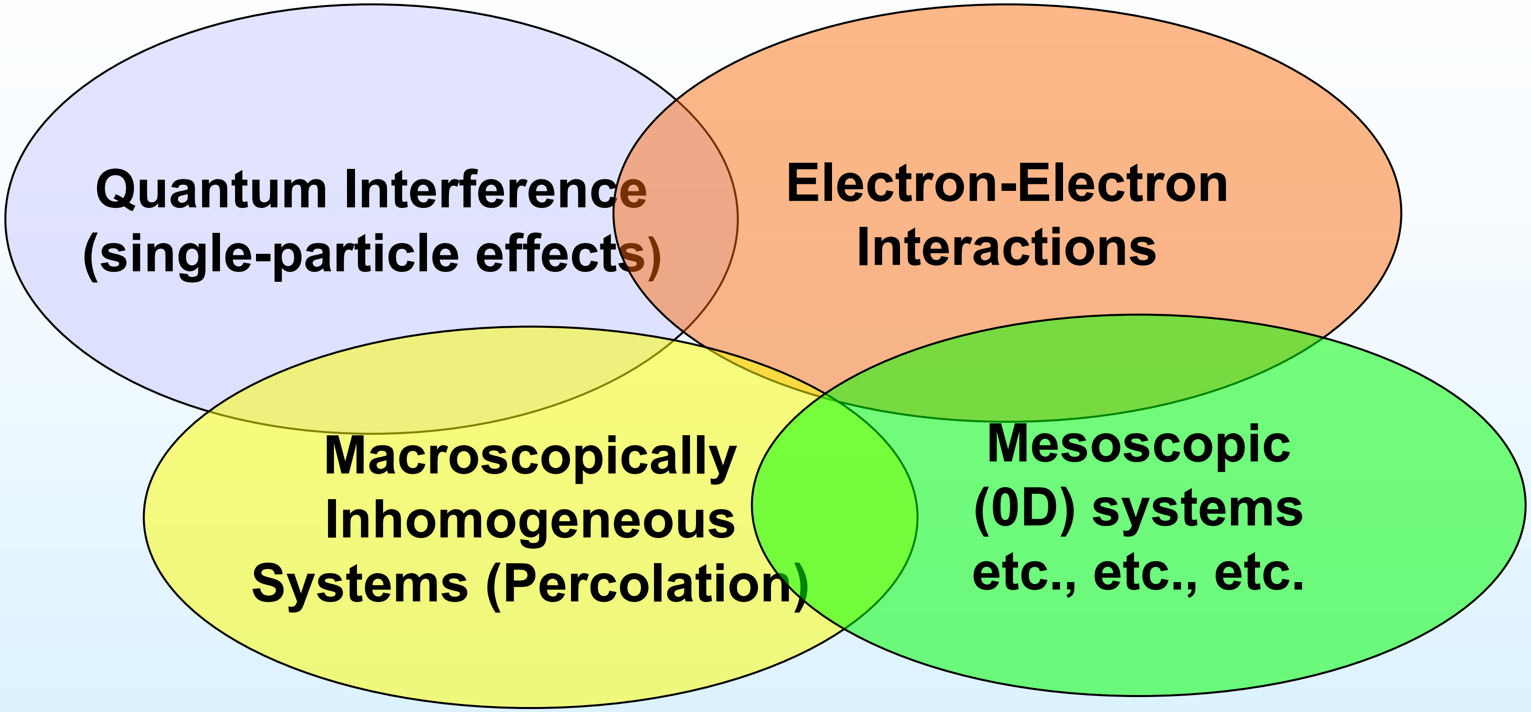
The State University of New Jersey
Rutgers
Department of Physics and Astronomy

Roadmap



Lecture 1. Weak and Strong Localization in 1D and 2D Conductors (single-particle effects).

Lecture 2. Electron-electron Interactions and Dephasing Processes in Disordered Conductors

Lecture 3. Quantum Corrections in the Conductivity of Si MOSFETs

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Lecture 1: WL and SL in 1D and 2D conductors (non-interacting electrons)

- Intro: Anderson Localization, Scaling Theory
- Weak Localization Corrections
  - Non-locality of WL effects
  - Interplay between WL and percolation
- WL-SL crossover
- Hopping in Systems with Large Localization Length

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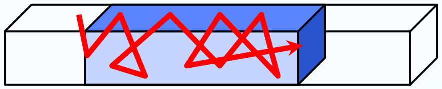
Disordered Conductors: Characteristic Scales

Quenched disorder, degenerate Fermi gas



Structure size: L

Fermi wavelength: λ_F < 1 nm (metals), ~10 nm (semiconductors)

Elastic mean free path: $l = v_F \tau$ 10-100 nm (metals), 0.1-1μm (semiconductors)

Localization length: ξ $l \ll \xi$

Phase coherence length: $L_\varphi = \sqrt{D\tau_\varphi}$ $l < L_\varphi < \xi$, typically 0.1-10 μm

Thermal dephasing length: $L_T = \sqrt{\hbar D / k_B T}$ $l < L_T < \xi$, typically 0.03-1 μm

The magnetic length: $L_H = \sqrt{\dfrac{\Phi_0}{2\pi H}}$ 1 μm @ 20G

Percolation correlation length ξ_P (macroscopically non-homogeneous systems with percolation)

Dimensionality: with respect to the quantum lengths ξ, L_φ, L_T, L_H

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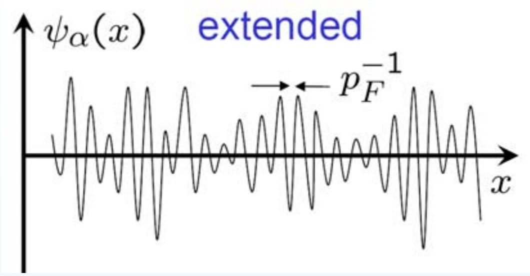
Anderson Localization (3D)

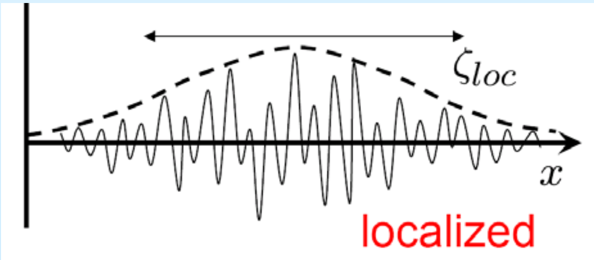
Non-interacting electrons in a 3D ordered system $\Rightarrow$ delocalized electron states



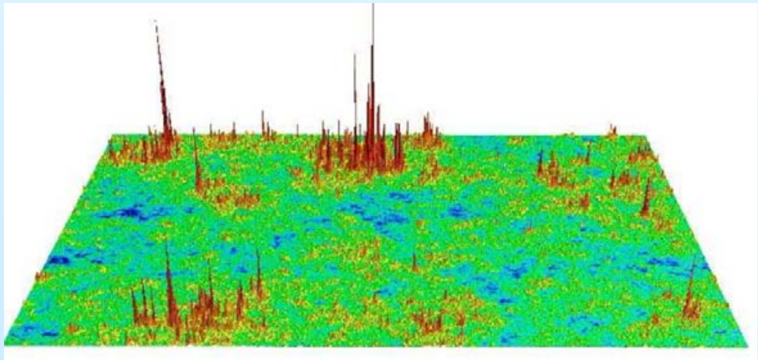
What happens if we crank up the disorder?

P.W. Anderson ('58): Quantum interference can completely suppress the diffusion of a particle in random potential (disorder can localize a particle despite of tunneling).

At $\boldsymbol{T}$=0, there is a quantum phase transition in the system at a critical strength of disorder, $\boldsymbol{x_c}$. At $\boldsymbol{x > x_c}$, the electron states at the Fermi level are localized and look like that:



More realistically, like that:



Multifractal wave function at the critical point (courtesy of F Evers, A Mildenberger, and A Mirlin, unpublished)

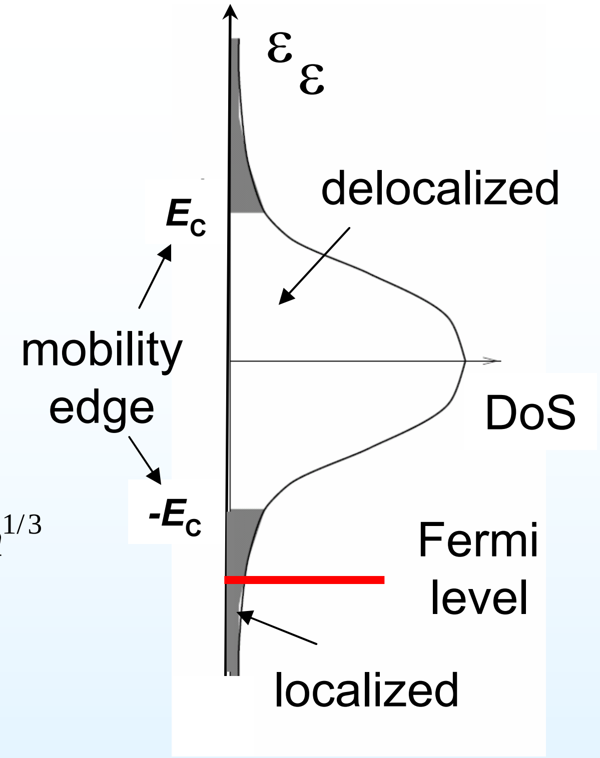
Anderson MIT

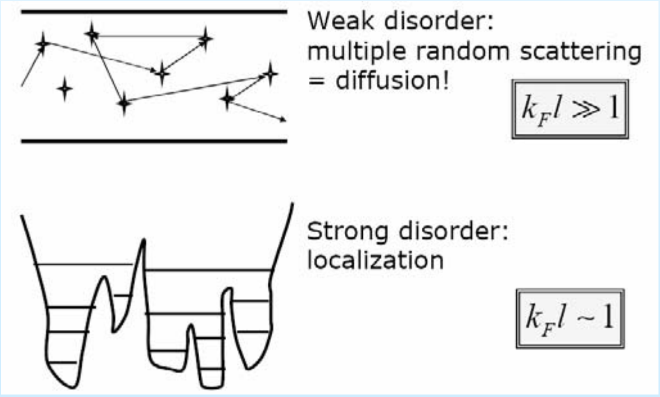
"Anderson" insulator: finite density of states at the Fermi level.

Realization of the Anderson MIT – by tuning either disorder or electron concentration

$$\sigma = \frac{e^2}{h}\frac{nl}{k_F}, \quad n = \frac{k_F^3}{3\pi^2} \qquad l \sim k_F^{-1} \qquad \sigma_{3D} \sim \frac{e^2}{h} n^{1/3} \sim 2\cdot 10^{-4}\,\Omega^{-1} n^{1/3}$$

$$\sigma_{2D} \sim \frac{e^2}{h} \sim (26k\Omega)^{-1}$$





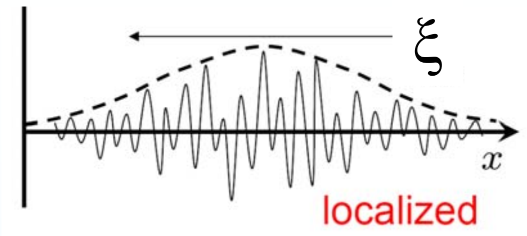
Anderson MIT – continuous,
no min. metallic conductivity!

Wegner ('76, '79): a close connection between the Anderson transition and the scaling theory of critical phenomena; field description of the localization problem in terms of a nonlinear σ-model.
Efetov ('80): a microscopic derivation of the σ-model.

Localization Length in 3D

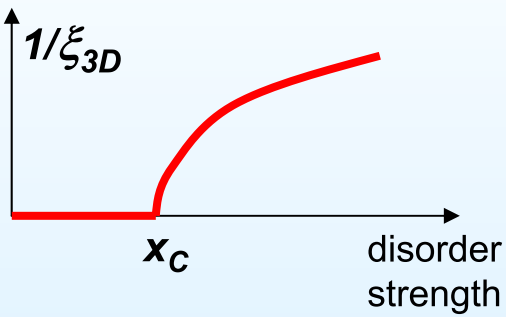
The length scale: ***the localization length*** ξ

$$\left|\psi^2(\vec{r})\right| \propto \exp\left(-\left|\vec{r}-\vec{r}_0\right|/\xi\right)$$



Scaling at the Anderson MIT:

$$\xi \propto \left(E_C - E\right)^{-\nu} \quad \sigma \propto \left(E - E_C\right)^{s} \quad s = \nu(d-2)$$



Wigner ('51) and ***Dyson*** ('62): classification of ensembles of random Hamiltonians on the basis of invariance of the system under ***time reversal and spin rotations***: unitary, orthogonal, and symplectic symmetries.

Important: ξ depends on the ***disorder*** (proximity to the critical point) and the underlying ***symmetry*** of the system.

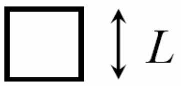
Scaling Ideas

The conductivity σ is convenient when it is a "local" quantity that does not depend on the sample dimensions.

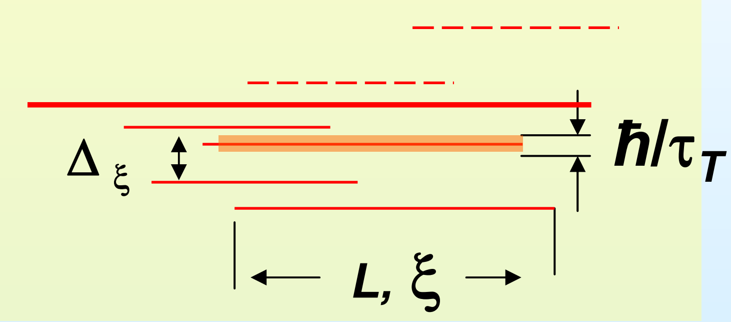
The development of scaling ideas (Thouless, '74,77, Abrahams, Anderson, Licciardello, Ramakrishnan, '79, Wegner, '79) showed that more useful quantity is the dimensionless ***conductance g(L)*** of a system of the size ***L*** (in units of e^2/h).

Scaling of the conductance ***g*** with the size of a disordered system (T=0, non-interacting electrons):

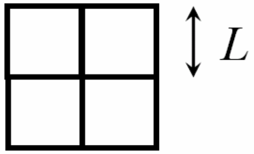
The idea: to investigate how the conductance changes if we increase the size of the system



Discrete ("localized") levels
Level spacing: $\Delta_L = (\nu L^d)^{-1}$



Put together: Level hybridization



Shift: $\delta E \sim \dfrac{E_T^2}{\Delta_L}$

$E_T \sim \hbar D / L^2$ - Thouless energy

(inverse time to diffuse across the system)

(i) $\delta E \ll \Delta_L$ The state is localized in one of the pieces

(ii) $\delta E \ll \Delta_L$ The state is extended

$g = 2\pi E_T / \Delta_L$ - dimensionless conductance of a piece L

Thouless criterion:

$g \gg 1$ - metal $\qquad g \ll 1$ - insulator

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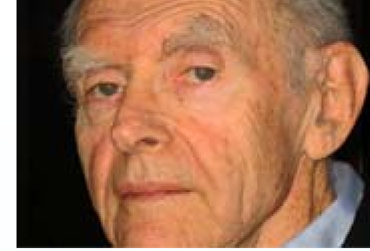
Scaling Theory



Abrahams et al. ('79): a scaling theory of localization which describes the flow of the dimensionless conductance ***g*** with the system size ***L***.

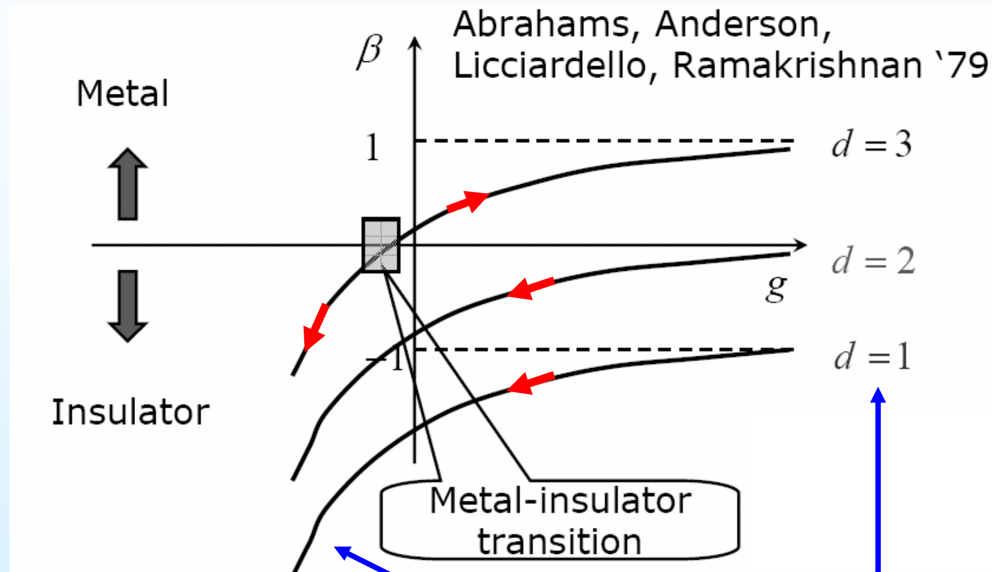
Evolution of the conductance with the system's size is determined by the conductance itself:

$$\boxed{\frac{d\ln g}{d\ln L} = \beta(g)}$$

MIT: $\beta(g=g_{crit})=0$

3D (orthogonal and unitary symmetry classes): Anderson MIT

2D and 1D: all states are ***localized***, even if the disorder is weak.



Insulator: $g \propto e^{-L} \Rightarrow \beta(g) \propto \ln g$

" Metal": Ohm's law $g \propto L^{d-2} \Rightarrow \beta(g) = d-2$

2D: the geometrical factor in the conductance, $\boldsymbol{g \sim (L/l)^{d-2}}$, disappears, and the equation $\beta(\boldsymbol{g})$ does not depend explicitly on ***L***.

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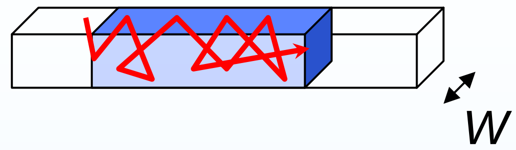
Localization Length in 1D and 2D

Quasi-1D conductors:



$$\lambda_F << W << \xi$$

Berezinsky, '74; *Efetov and Larkin*, '83
Dorokhov, '83

$N_{1D} = 2W/\lambda_F >> 1$ –
the # of conducting channels

Orthogonal
quenched disorder, no mag. field
(time reversal symmetry)

$$\xi_{1D}^{O} = N_{1D} \cdot l$$

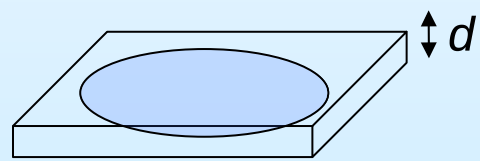
Unitary
strong mag. field breaks
the time reversal symmetry

$$\xi_{1D}^{U} = 2N_{1D} \cdot l$$

$$B >> \frac{\Phi_0}{\xi W}$$

- flux quantum in the area of a localized state

2D conductors:



$$d << \xi$$

Orthogonal

$$\xi_{O}^{2D} = l \cdot \exp\left(\frac{\pi}{2} k_F l\right)$$

Unitary

$$\xi_{U}^{2D} = l \cdot \exp\left[\left(\frac{\pi}{2} k_F l\right)^2\right]$$

$$B >> \frac{\Phi_0}{\xi^2}$$

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Finite Temperatures $\Rightarrow$ Dephasing Processes

All electronic states in 1D and 2D are localized. However, some of these systems are very good conductors at high temperatures. Why?

If the disorder is not too strong and the temperature is not too low, the electron wavefunction will be "mutilated" before an electron has a chance to diffuse over the localization length. Because of these dephasing processes, an electron will be frequently scattered between localized states, and it will diffuse ALMOST as if its wavefunction is not localized.

Sources of dephasing:

Internal ('*the enemies within*'): phonons, other electrons, all dynamic degrees of freedom ("…ons")

External ('*the enemies without*'): spin-spin scattering, external high-frequency E&M fields

(to be discussed at Lecture 2)

Length scale: ***the phase coherence length*** $L_\varphi = \sqrt{D\tau_\varphi}$

Typical value of τ_φ(1K) in metals ~10^{-8}-10^{-7} s, $\boldsymbol{D}$ ~ 1-100 cm^2/s, $\boldsymbol{L}_\varphi$(1K) ~ 0.1-10μm

$$L_\varphi(T) \Leftrightarrow \xi$$

Weak Localization: $\lambda_F << \ell < L_\varphi(T) << \xi$

Strong Localization: $\lambda_F < \ell < \xi < L_\varphi(T)$

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Weak Localization Corrections

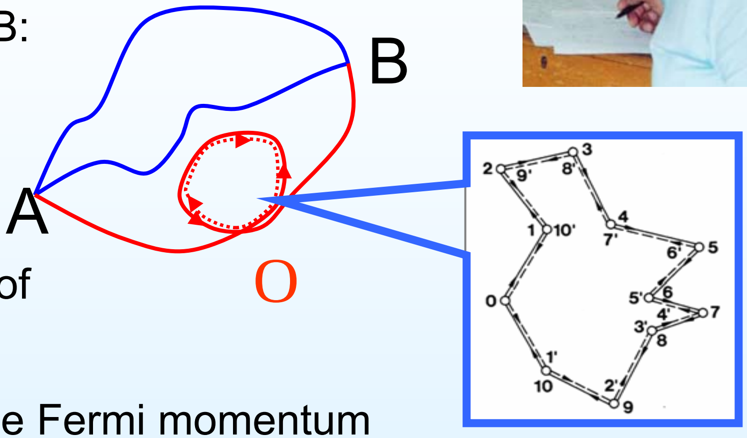
Gor'kov, Larkin, Khmelnitskii ('79): resummation of singularities in perturbation theory in 2D and formulation of the scaling in the systematic form of a renormalization group theory.

The total probability to get from A to B:

$$w_{AB}=\left|\sum_i A_i\right|^2=\sum_i\left|A_i\right|^2+\sum_{ij}\left|A_iA_j\right|$$

probabilities

interference of amplitudes



The phase gain: $\Delta\varphi=\frac{1}{\hbar}\int_A^B \vec{p}d\vec{l}>>1$ $\quad$ p – the Fermi momentum

The interference term vanishes $\left\langle\sum_{ij}\left|A_iA_j\right|\right\rangle=0$ due to the disorder averaging, but NOT for the time-reversed paths (loops)

The self-intersecting (loop-like) trajectories: $\vec{p}\Rightarrow-\vec{p}\quad \vec{l}\Rightarrow-\vec{l}\quad \Delta\varphi=0$

$$w_0=\left|A_1\right|^2+\left|A_2\right|^2+2\,\mathrm{Re}\,A_1A_2=4\left|A_1\right|^2$$

As a result, the total scattering probability at site O *increases*, and the conductance – *decreases*. The corresponding corrections to the transport properties are known as the ***weak localization*** corrections.

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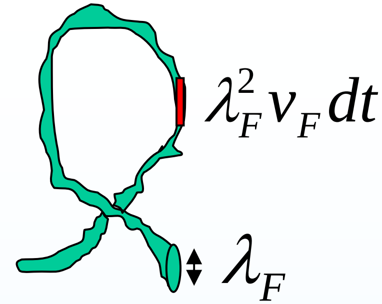
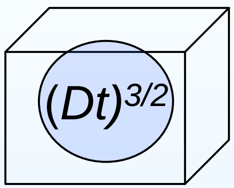
Different dimensionalities:

The dimensionality of a conductor: ***L*** $\Leftrightarrow$ ***L***$_{\varphi}$**(*T*)**   (<< ξ)



the upper cutoff - τ_{φ}, the loops with ***L* > *L***$_{\varphi}$**(*T*)** do not contribute to $\Delta\sigma_{WL}$

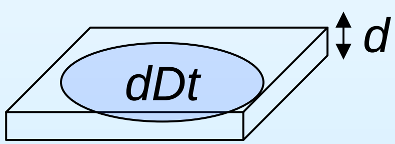
3D:



$L_{\varphi} <$ all dimensions, ξ

$$\Delta\sigma_{WL} \propto \int_{\tau}^{\tau_{\varphi}} \frac{\lambda_F^2 v_F}{(Dt)^{3/2}} dt \propto \frac{e^2}{h}\left[\frac{1}{L_{\varphi}(T)} - const\right]$$

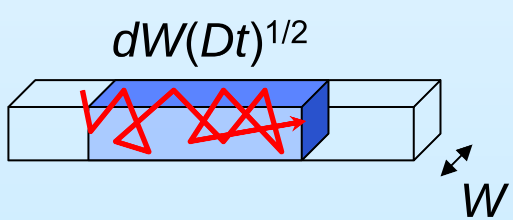
2D:



$d < L_{\varphi} << \xi$

$$\int_{\tau}^{\tau_{\varphi}} \frac{\lambda_F^2 v_F}{dDt} dt \propto -\frac{e^2}{h}\ln\left[\frac{L_{\varphi}(T)}{l}\right]$$

Quasi-1D:



$d, W < L_{\varphi} << \xi$

$$\int_{\tau}^{\tau_{\varphi}} \frac{\lambda_F^2 v_F}{dW(Dt)^{1/2}} dt \propto -\frac{e^2}{h} L_{\varphi}(T)$$

Though these QC are small at L_{φ} << ξ, they diverge in low dimensions as ***T*** decreases (and L_{φ} increases), and eventually they drive the system into the SL regime.

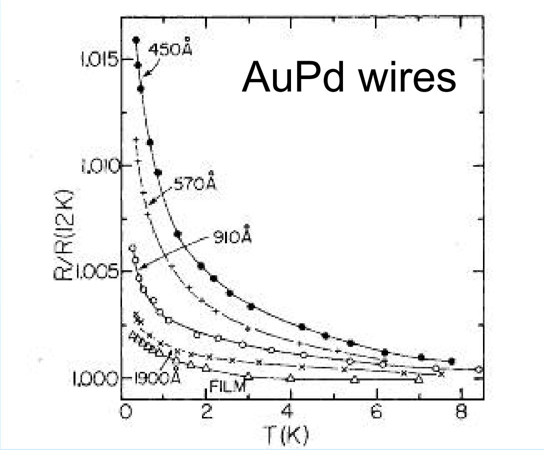
Observation of the WL corrections

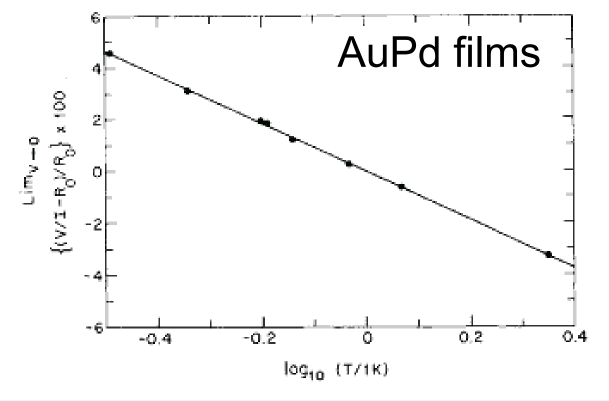
$$\Delta\sigma_{WL} << \sigma_0$$

Still, WL effects are observable because $\Delta\sigma_{WL}$ depends (in a non-trivial way) on ***T*** and ***B*** , in contrast to the «residual» Drude conductivity σ_0.

$\Delta\sigma_{WL}(T)$ – due to the T-dependence of phase relaxation (the lower T, the slower the relaxation). However, $\Delta\sigma_{WL}$ is not the only T-dependent quantum correction, the interaction effects result in a similar dependence (Lectures 2,3)

Curiously, the first observations of the increasing resistivity in 1D and 2D metal films with cooling, which have been considered as a support for the WL theory, were in fact the observations of INT corrections (the WL correction was T-independent because of a strong spin-spin scattering).

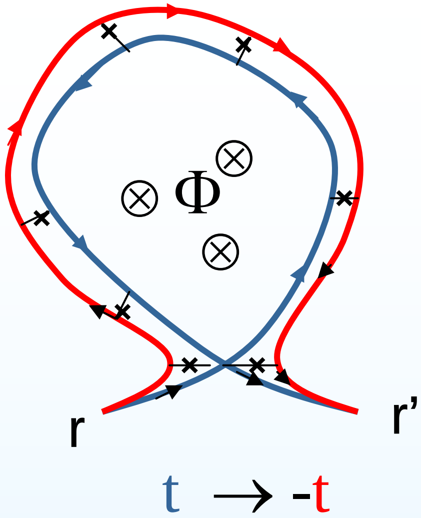




It is easier to analyze $\Delta\sigma_{WL}(B)$ because INT corrections aren't sensitive to weak magnetic fields.

Giordano, *PRB* **22** ('80)

Dolan and Osheroff, *PRL* **43** ('79)



WL in external magnetic field

Classically weak magnetic fields: no trajectory "bending".

Aharonov-Bohm phase acquired by the loops:

$$\vec{p} \Rightarrow \vec{p} - \frac{e}{c}\vec{A} \qquad \Psi \Rightarrow \Psi \exp\left(i \frac{e}{c\hbar} \oint \vec{A} d\vec{l} \right)$$

A - the vector potential

$$\Delta\varphi_H = \frac{2e}{c\hbar} \oint \vec{A} d\vec{l} = \frac{2e}{c\hbar} \oint \vec{H} d\vec{s} = 2\pi \frac{\Phi}{\Phi_0}$$

- the phase difference between CW and counter-CW loops increases with field.

$$\Phi_0 = \frac{hc}{2e} = 2.07 \cdot 10^{-7}\,\mathrm{G} \cdot \mathrm{cm}^2$$

- the flux quantum (~20G in 1μm^2)

$$w_0 = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\Delta\varphi_H) = 2A^2[1 + \cos(\Delta\varphi_H)]$$

$$L_H = \sqrt{\frac{\Phi_0}{2\pi H}}$$

- the magnetic length (the characteristic length at which two interfering waves propagating along a loop in opposite directions acquire the phase difference ~ 2π)

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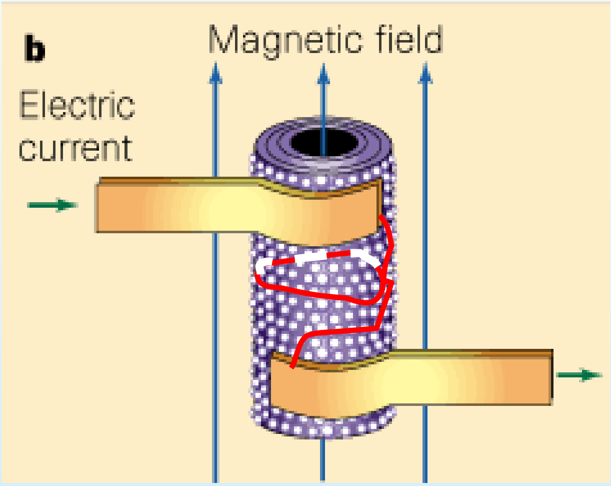
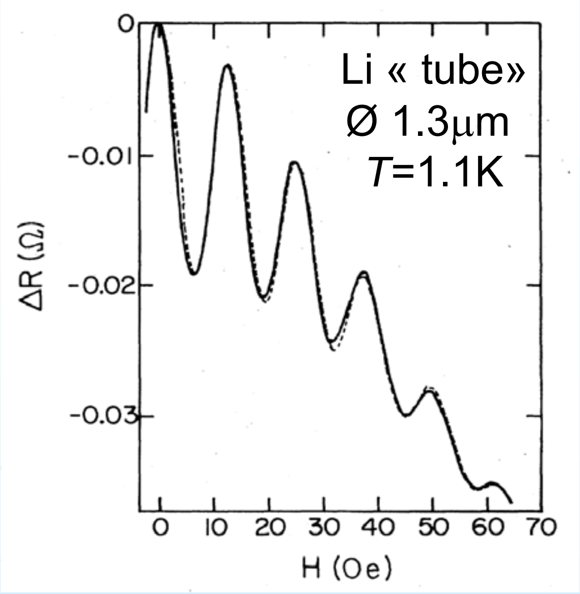
YURIĬ VASIL'EVICH SHARVIN

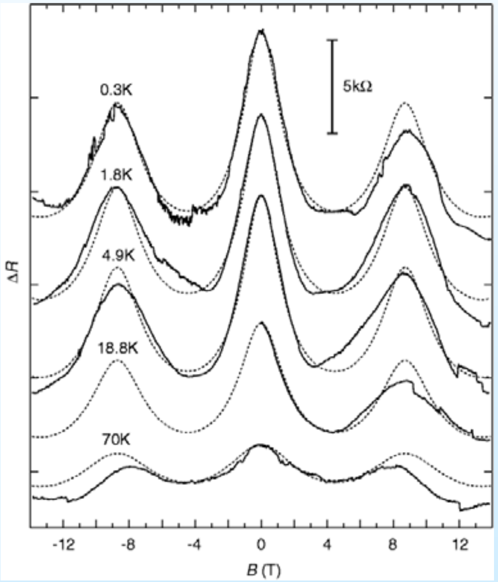
Aharonov-Bohm effect in disordered conductors

$$w_0 = 2A^2\left[1+\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\right]$$





Sharvin and Sharvin, ’81
Altshuler *et al*., ‘82



multi-wall carbon nanotubes r_e = 8.6 nm
Bachtold *et al.* (’98)

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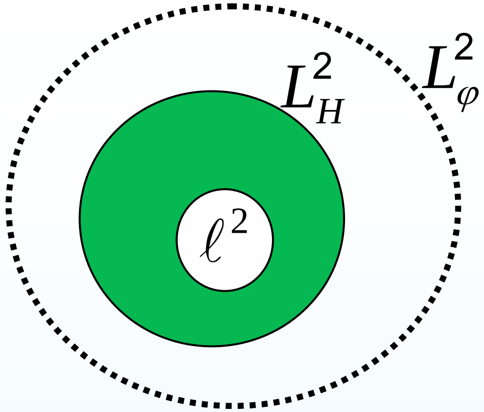
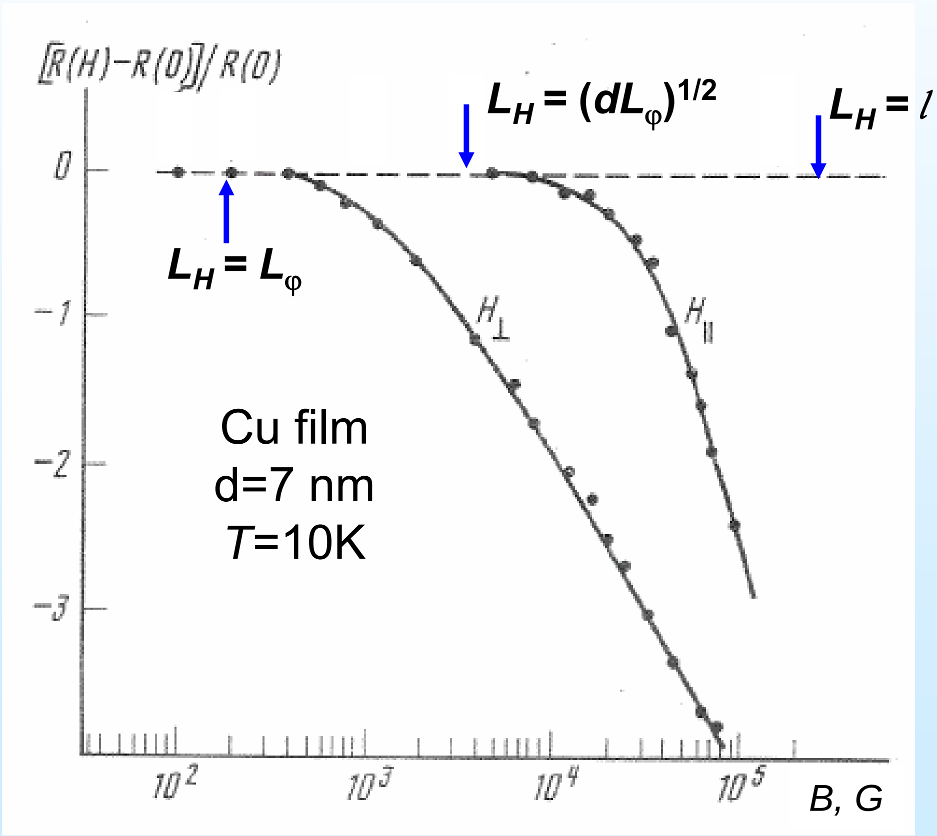


WL magnetoresistance

When the magnetic length $L_H \sim (\Phi_0/B)^{1/2}$ becomes smaller than $L_\varphi(T)$ $\Rightarrow$ WL magnetoresistance

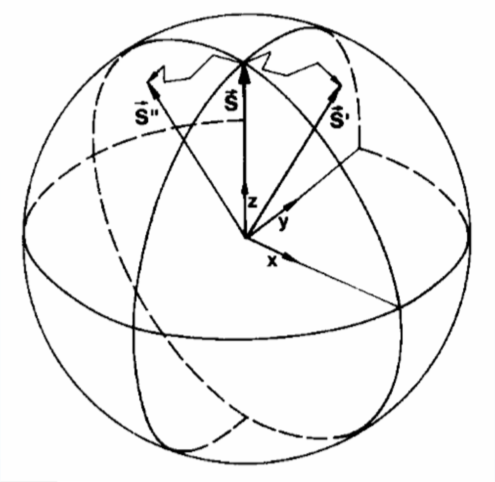


MG *et al.*, '81

WL magnetoresistance:

- negative (no SOI so far)
- anisotropic in 1D and 2D (purely orbital effect)
- observed in classically weak magnetic fields

WL MR measurements are extensively used to measure L_φ.



Weak Anti-Localization

So far, we've considered ***spinless*** electrons. However, the electrons have spin 1/2, and the spin scattering modifies the WL corrections. Two scattering processes:

(a) spin-spin scattering (e.g., by paramagnetic impurities) – destroys constructive interference at $t > \tau_S$ $\left(L_\varphi(T) \Rightarrow L_S \right)$.

(b) spin-orbit interaction (as a relativistic effect, strongly depends on ***v***)

When electrons are scattered by a strong electric field of nuclei, they "feel" (in their reference frame) an effective magnetic field

$$\vec{B}_{SO} = \frac{1}{2c^2}\left(\vec{v} \times \vec{E}\right)$$

Because the interfering electron waves propagate in the opposite directions (***v*** ⇔ -***v***), the ***B***$_{SO}$ vectors are also oppositely directed, and the spins "diffuse" away from each other on the Bloch sphere.

The spin-orbit length: $$L_{SO} = \sqrt{D\tau_{SO}}$$

- ***T***-independent, a strong function of the nucleus charge (Abrikosov & Gorkov, '62)

$$\tau_{SO} = \tau\left(\alpha Z\right)^{-4}$$

After averaging over all trajectories with dimensions $L_{SO}<L<L_\varphi$, the WL correction becomes ***positive*** – "**weak anti-localization**"

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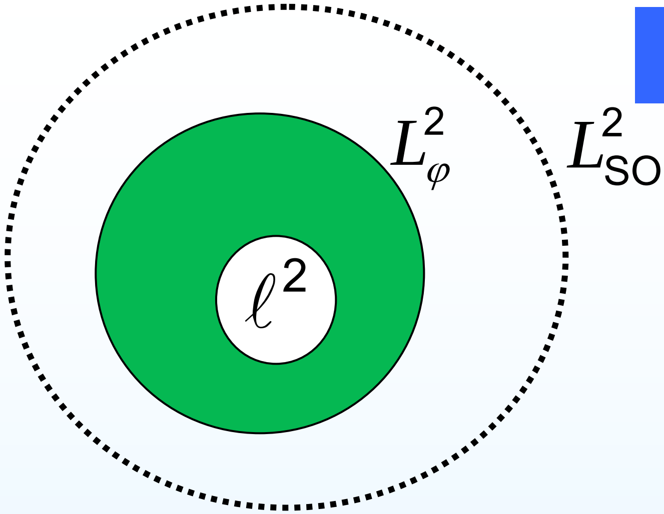
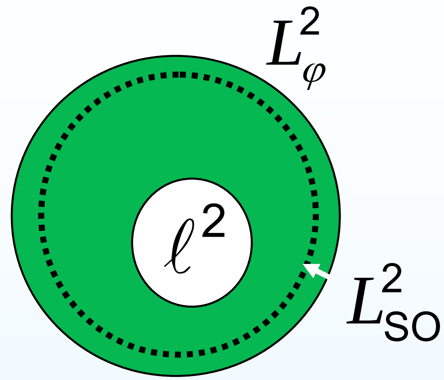
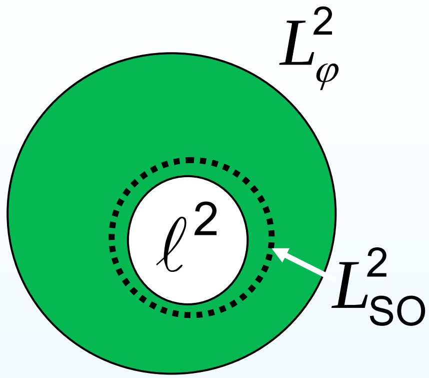
WL and WAL magnetoresistance



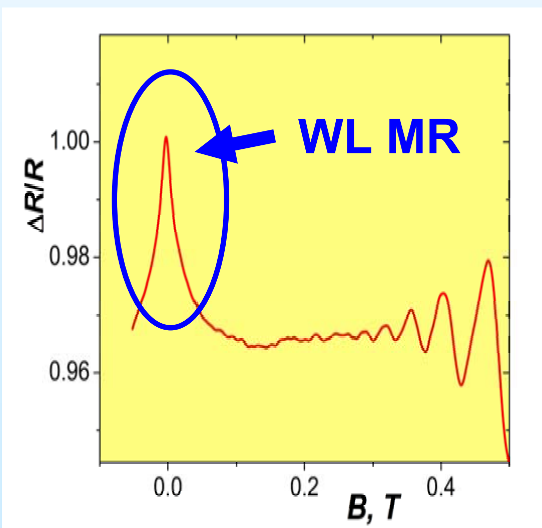
Only WL



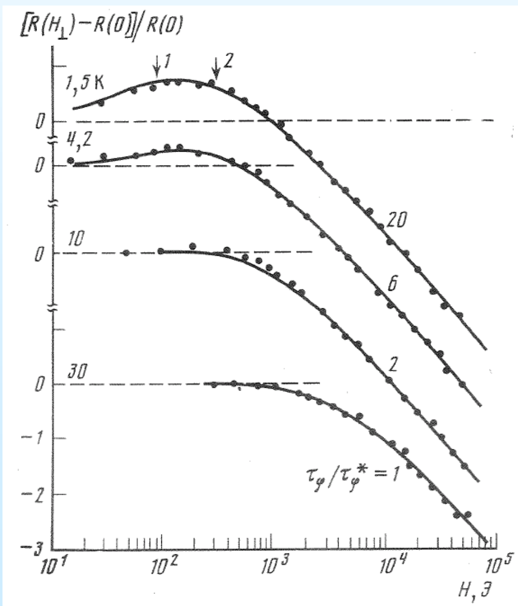
WL+WAL



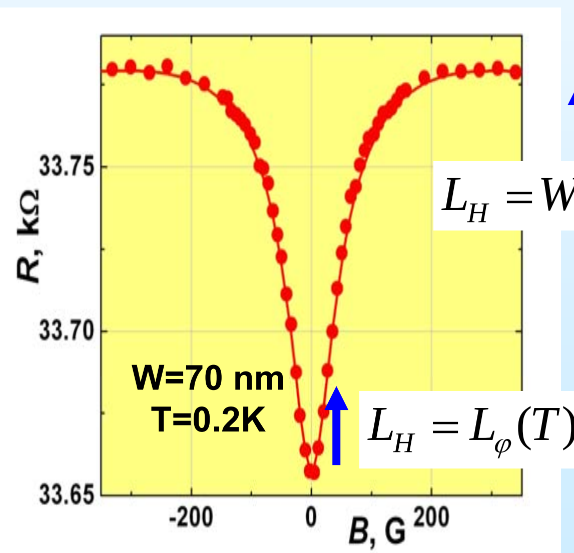
Only WAL



Si MOSFET



2D Ag film



1D Ag wire

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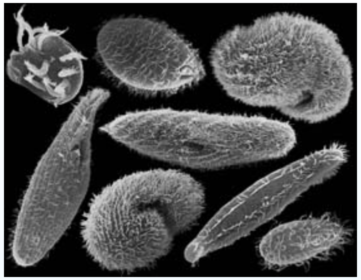
Non-locality of Quantum Corrections

2D and 3D: due to the non-locality of QC, the disorder is averaged over a large length (in comparison with the defect size) $\Rightarrow$ "***universality***" of QC.



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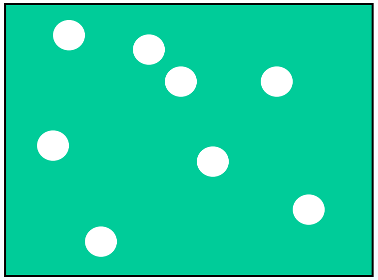
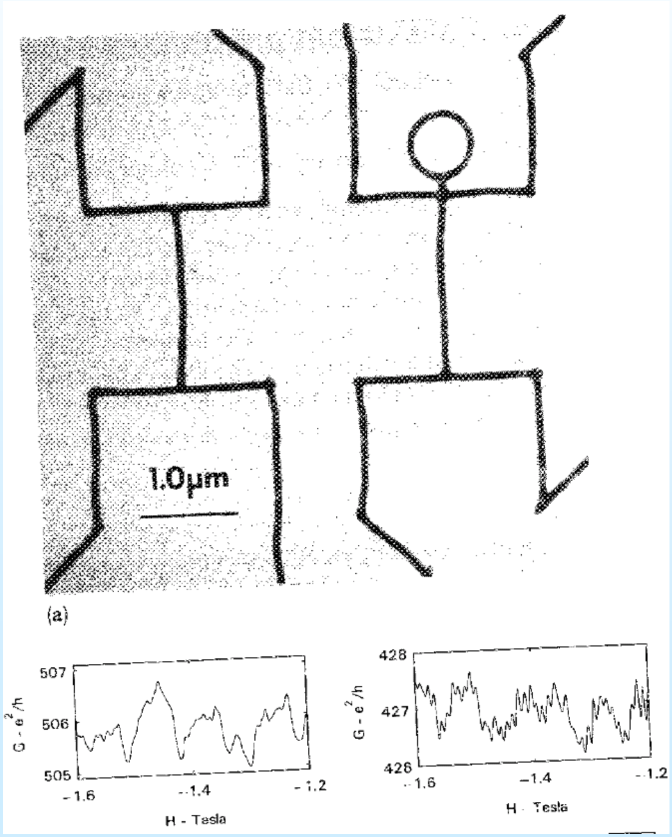
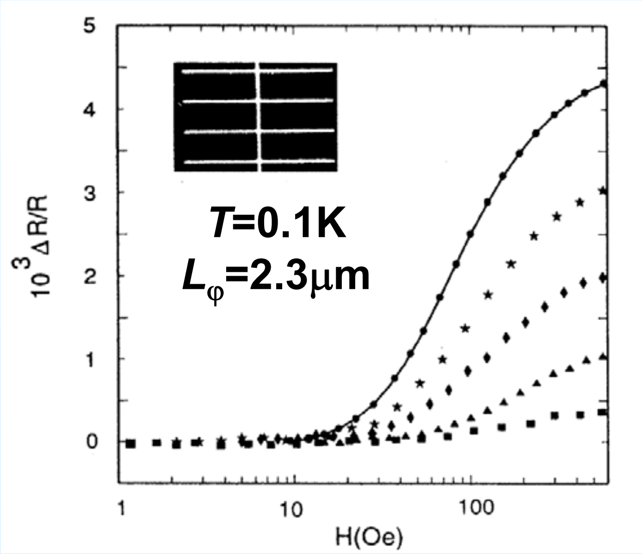


$|\leftarrow$ L_φ $\rightarrow|$



Umbach *et al.*, '87

0D and 1D: Because of non-locality, QC can be strongly influenced by the phase coherent regions extending beyond the classical current paths.



MG *et al.*, '95

∞

3.6μm

1.6μm

0.4μm

0.2μm

distance between side branches

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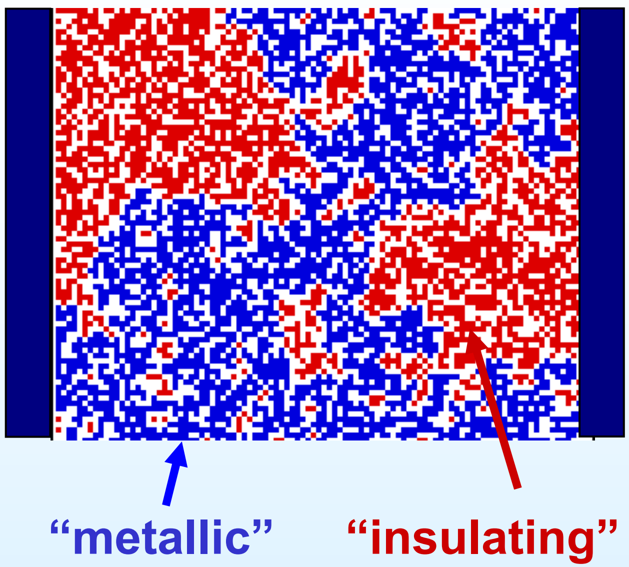
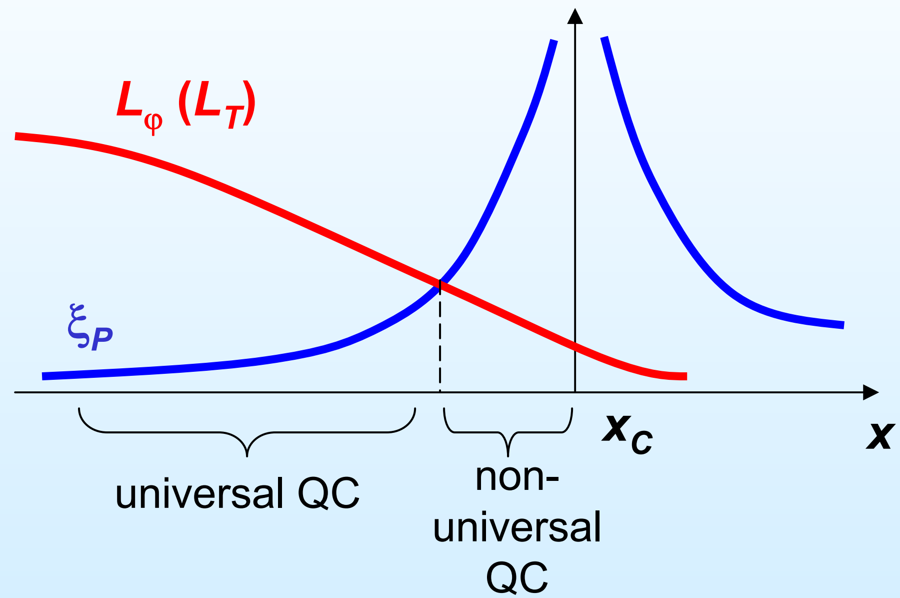
WL in macroscopically non-homogeneous conductors (2D and 3D)



To be specific, we’ll consider WL (though many conclusions apply to the INT corrections as well).

New scale: ***the percolation correlation length*** ξ_P



At $L \gg \xi_P$ the conductivity scales with the conductor dimensions as in a homogeneous system (R_{macro}).

At $L < \xi_P$ - the anomalous diffusion, the diffusion constant $D(L)$ depends on the scale L.

Far from the percolation threshold ($L_\varphi \gg \xi_P$) - “universal” corrections,

$$\Delta\sigma_{WL} = \frac{\Delta\rho_{WL}}{\rho^2_{macro}}$$

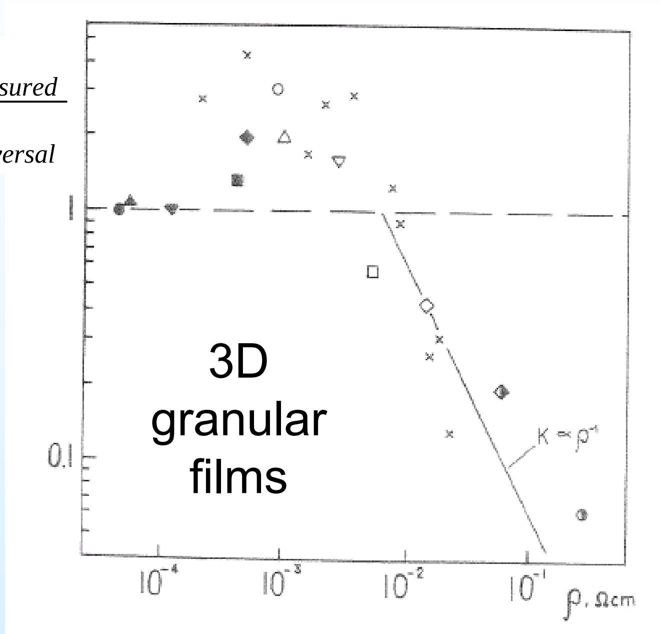
Close to the threshold ($L_\varphi < \xi_P$): an interplay between QC and percolation.

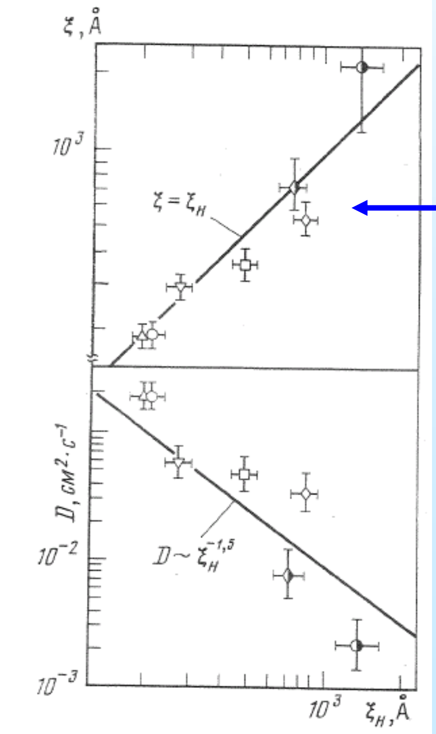
Consequences of macroscopic non-homogeneity

- Near the percolation threshold ($L_\varphi, L_H < \xi_P$) , the quantum corrections are much smaller than that expected for a homogeneous system with the same total resistance.

$$\frac{\Delta\rho_{WL}(T,B)}{\rho^2_{macro}} = "\Delta\sigma_{WL}" << \Delta\sigma_{WL}$$

Aronov, MG, and Zhuravlev ('84) - experiment + theory
Palevski and Deutscher ('86)
Dumpich and Carl ('91)





- using the theory of WL corrections in conductors with percolation, one can extract ξ_P from the WL magnetoresistance

- one should be careful about interpretation of "L_φ" in systems with percolation [$L_\varphi \Rightarrow (L_\varphi \xi_P)^{1/2}$] .

In particular, τ_φ may be insensitive to $R_{\square\,macro}$ and the temperature dependence of "τ_φ" is weakened.

[see also Germanenko *et al*. *PRB* **64**, 165404 ('01)]

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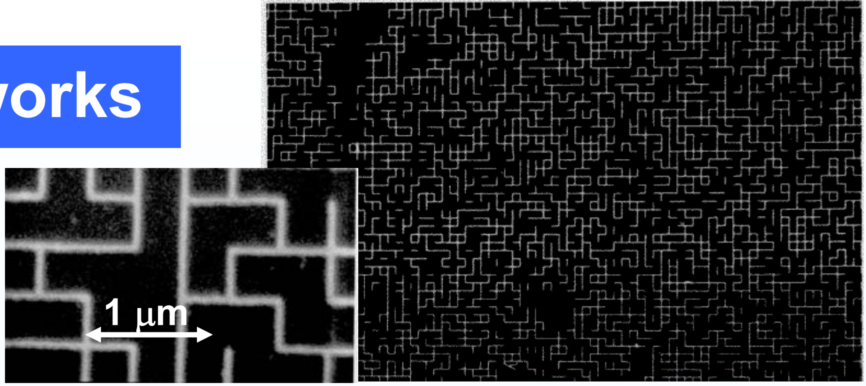
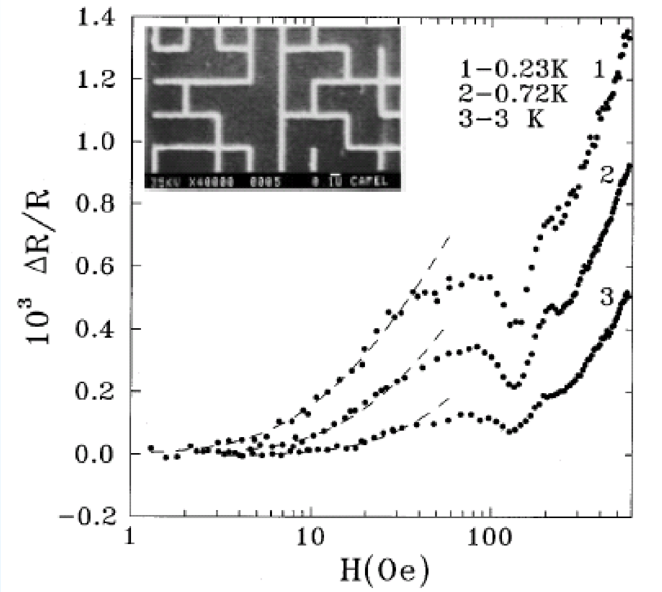
Man-made 2D percolation networks

2D network made of 0.05μm-wide Au wires. The unit cell size – 0.4 μm, # of cells ~ 10^6. In the process of e-beam writing, the probability of skipping a bond varied from 0 to 0.5





MG *et al.* PRL **74**, 446 (‘95)

Sample	x	ξ (μm)	$R_\square(\xi)$ (Ω)	$R_\square^{\mathrm{MR}}$	$R_\square^{\mathrm{TD}}$
a	0	0.4	14	14	20
b	0.3	1.4	50	38	~50
c	0.4	3.4	130	47	—
d	0.45	8.6	527	51	—
e	0.47	17	1039	78	—

The percolation network made of 1D wires behaves like a ***homogeneous 2D film*** with respect to the quantum interference effects if $L_\varphi, L_H > \xi_P$.

Close to the percolation threshold, when ξ_P becomes much larger than L_φ, L_H, the QC become ***quasi-1D***.

These 1D QC are affected by the presence of dead ends of the infinite percolation cluster when $L_\varphi > a$ (a – the unit cell size).

тупик ≠ мертвый конец

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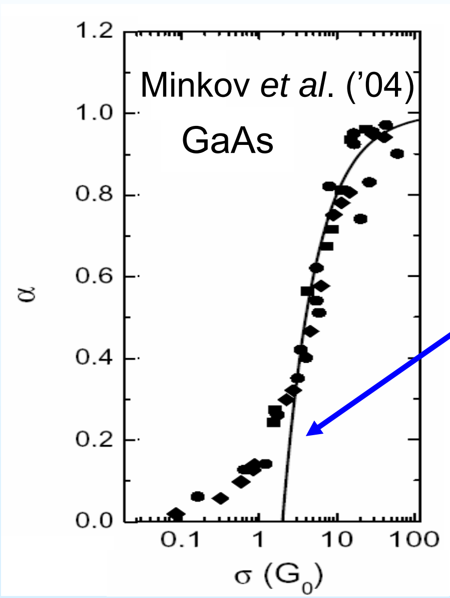
What happens when σ_{2D} approaches e^2/h?

With diminishing σ, the ***second-loop*** corrections become important (approx. at $R_\square > 5\mathrm{k}\Omega/\square$). Because of these corrections, $\Delta\sigma_{WL}(B)$ is strongly diminished whereas $\Delta\sigma_{WL}(T)$ remains "universal" with pre-factor ~ e^2/h.

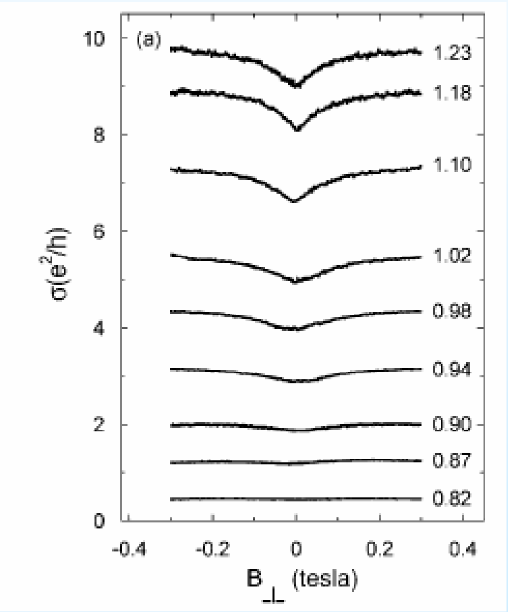
$$\Delta\sigma_{WL}(B) = \alpha \frac{e^2}{2\pi^2 h} f(B)$$



$$\alpha = 1 - \frac{2G_0}{\sigma}$$

$$G_0 = e^2/(2\pi^2\hbar)$$

Gornyi ('04)



Rahimi *et al.* ('03)

Caution: "vanishing" WL MR at $\sigma \Rightarrow e^2/h$ is expected for both ***homogeneous*** and ***macroscopically inhomogeneous*** systems. In the latter case, however, $\Delta\sigma_{WL}(T)$ should also be diminished (similar to the behavior of high-mobility Si MOSFETs) .

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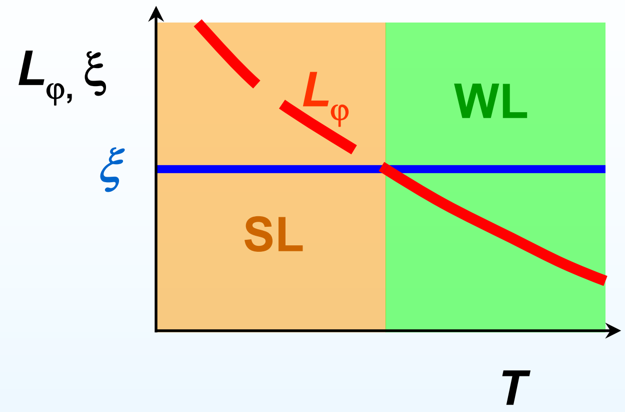
"WL ⇔ SL" crossover in 1D and 2D

What happens when $L_\varphi(T)$ becomes of the order of ξ with cooling?



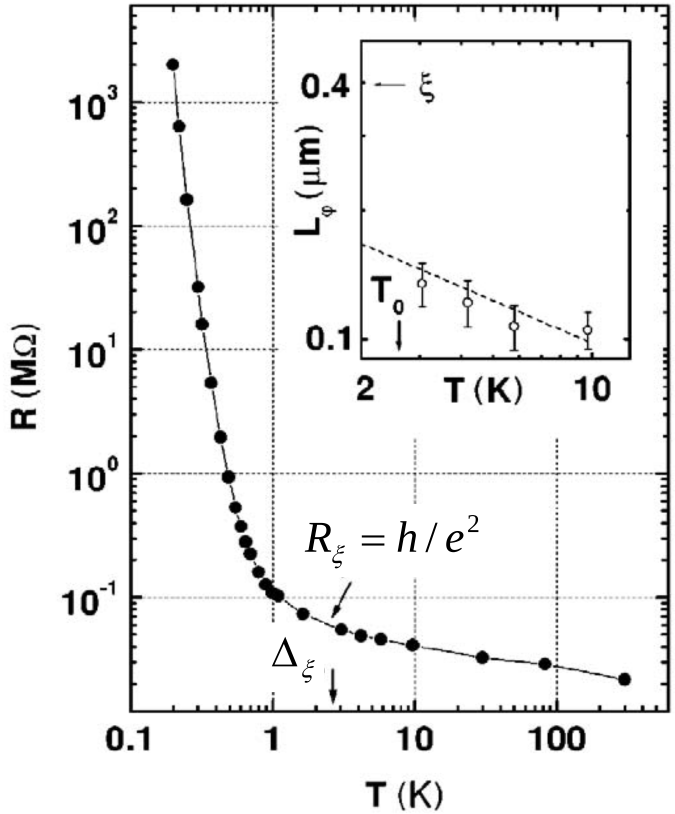
WL: $L_\varphi < \xi$ **SL:** $L_\varphi > \xi$

The QC become of the order of the Drude conductivity:

2D: $$\sigma_0 = \frac{e^2}{h} k_F l \qquad \sigma \approx \frac{e^2}{h}\left(k_F l - \ln\frac{L_\varphi}{l}\right) \Rightarrow 0 \quad at\ L_\varphi \approx \xi_{2D} \qquad \left[\xi_{2D} \approx l \exp(k_F l)\right]$$

1D: $$\sigma_0 = \frac{e^2}{h} k_F l W \qquad \sigma \approx \frac{e^2}{h}\left(k_F l W - L_\varphi\right) \Rightarrow 0 \quad at\ L_\varphi \approx \xi_{1D} \qquad \left[\xi_{1D} \approx l k_F W\right]$$

1D “WL ⇔ SL” crossover

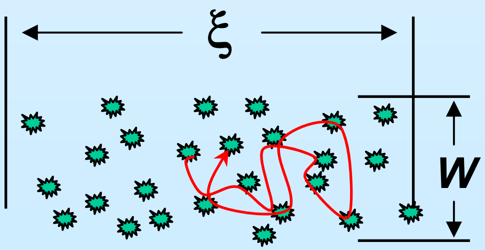


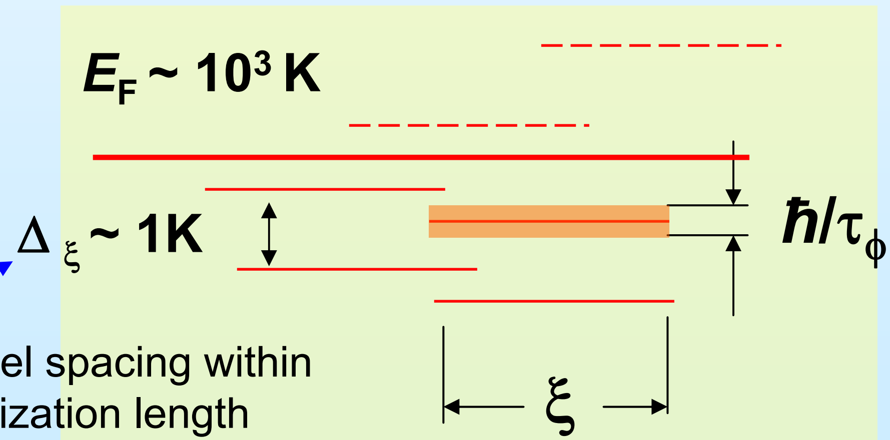
$\xi_{1D} \approx L_\varphi(T_\xi)$, Nyquist dephasing $\Rightarrow T_\xi \sim \dfrac{1}{DW^2(m^*)^2}$

Quasi-1D wires in Si δ-doped GaAs
\# of wires – 470, L = 500 μm, W = 0.05 μm,
$\boldsymbol{N}_{1d}$ = 7, ξ = 0.4 μm, Δ_ξ = 2.1 K

Electrons are localized over **many** impurity states (ξ >> $n^{-1/2}$,l), and there is a diffusive motion within the localization envelope. The localized states strongly **overlap** in space [e.g., in our Si:GaAs samples, there are 10^3 localized electrons confined within an area of a size of ξ·W].

MG et al., PRL **79**, 725 (‘97)
Khavin, MG et al, PRL **81**, 1066 (‘98).
Khavin, MG, Bogdanov, PRB **58**, 8009 (‘98)





the inter-level spacing within
the localization length
~ $E_F/(nW\xi)$ in quasi-1D

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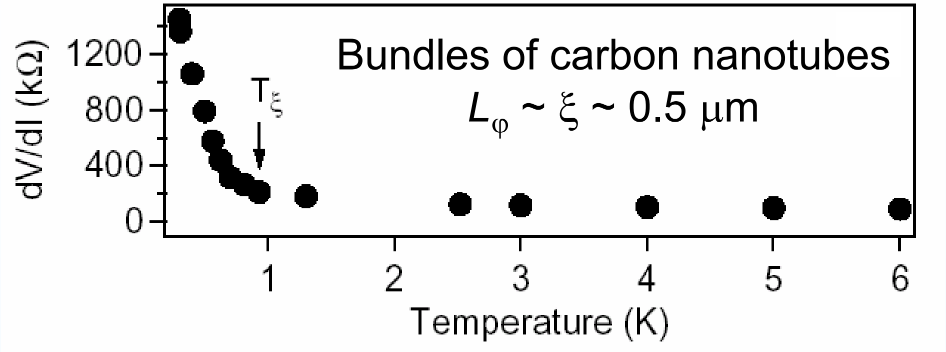
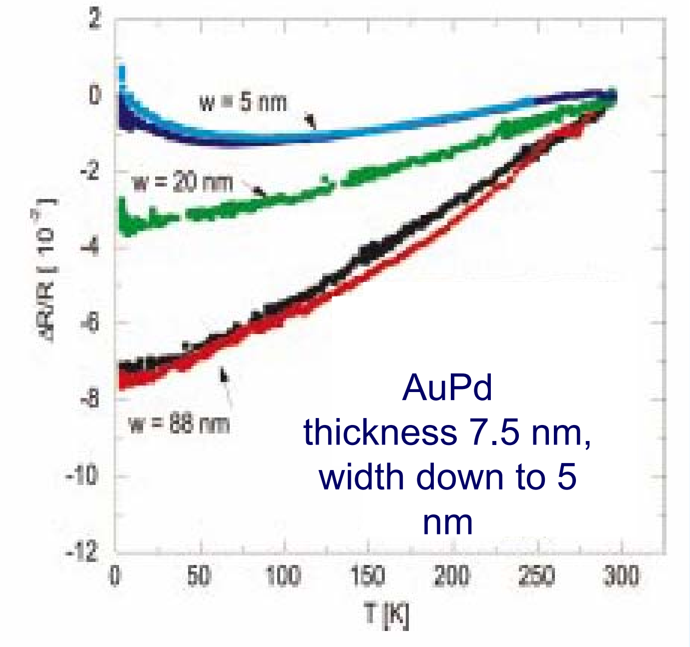
1D "WL ⇔ SL" crossover (cont.)



Shea, Martel, Avouris, *PRL* **84**, 4441 ('00)



Natelson *et al., SSC* **115**, 269 ('00)
Natelson *et al., PRL* **86**, 1821 ('01)

<u>At the crossover :</u>

- $L_\varphi (T_\xi) \sim \xi$
- The conductance of a wire segment of the length $\xi \Rightarrow e^2/h$
- At the crossover temperature T_ξ , all energy scales are ***of the same order of magnitude***:

$$\hbar/\tau_\varphi(T_\xi) \sim \Delta_\xi \sim k_B T_\xi \sim \hbar D/\xi^2$$

level broadening — Inter-level distance — crossover temperature — Thouless time ξ^2/D

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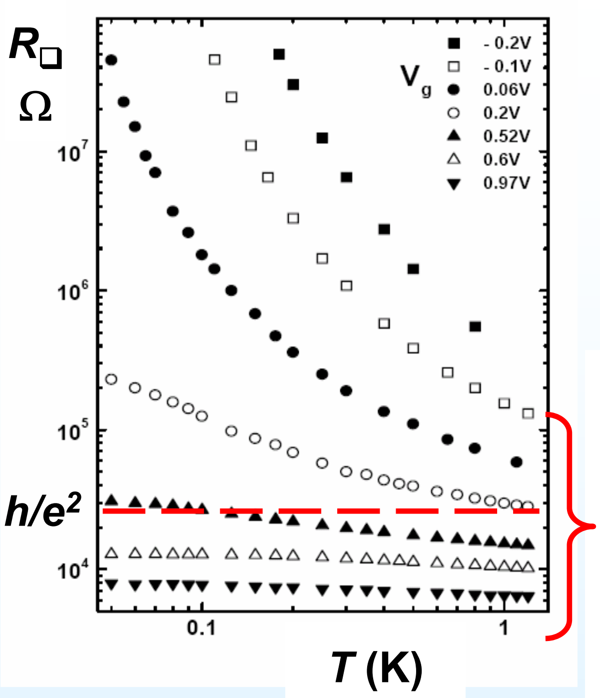
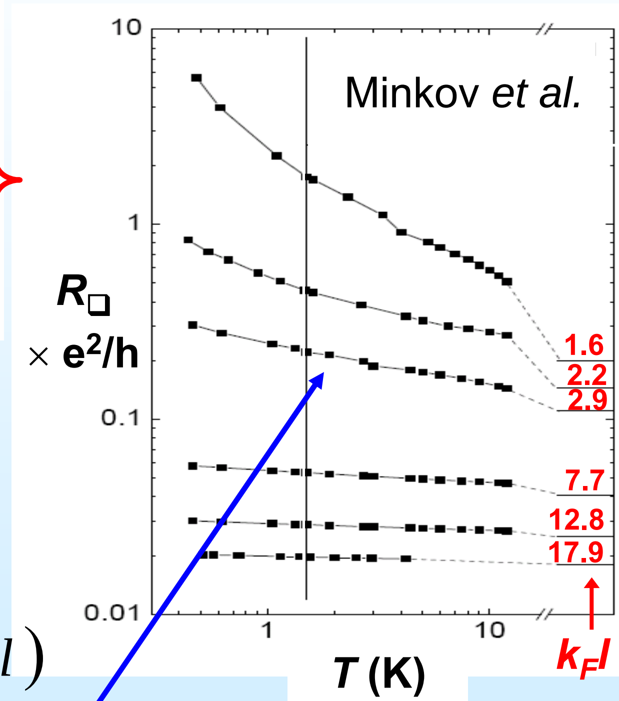
2D “WL ⇔ SL” crossover

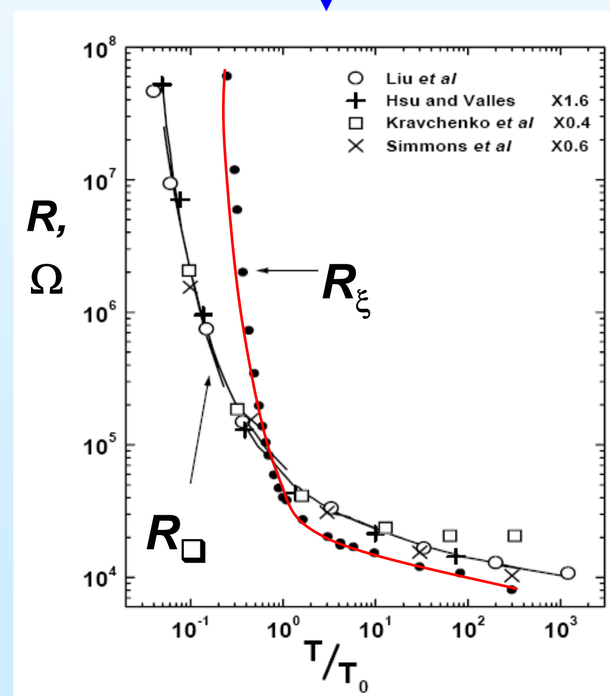
Qualitatively – similar to the 1D “WL-SL” crossover



gated Si:GaAs
MG *et al.*, unpublished



The WL-SL crossover is “sharper” for 1D samples.



$$L_\varphi(T) \Leftrightarrow \xi \approx l \exp\left(k_F l\right)$$

the 2D crossover can be observed only if $k_F l \sim 1$

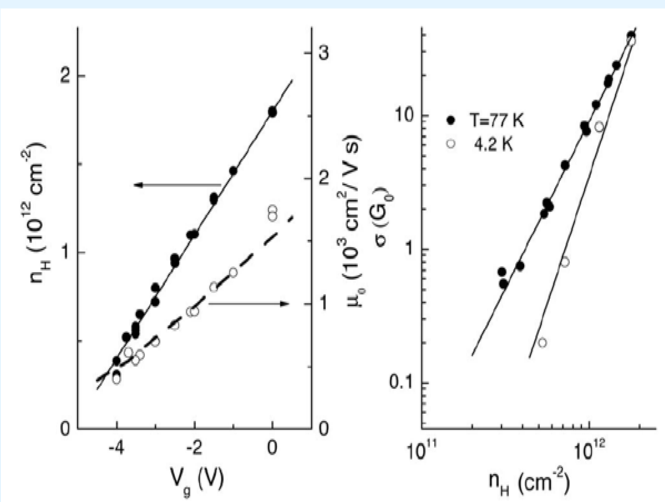
(more room for that in 1D)

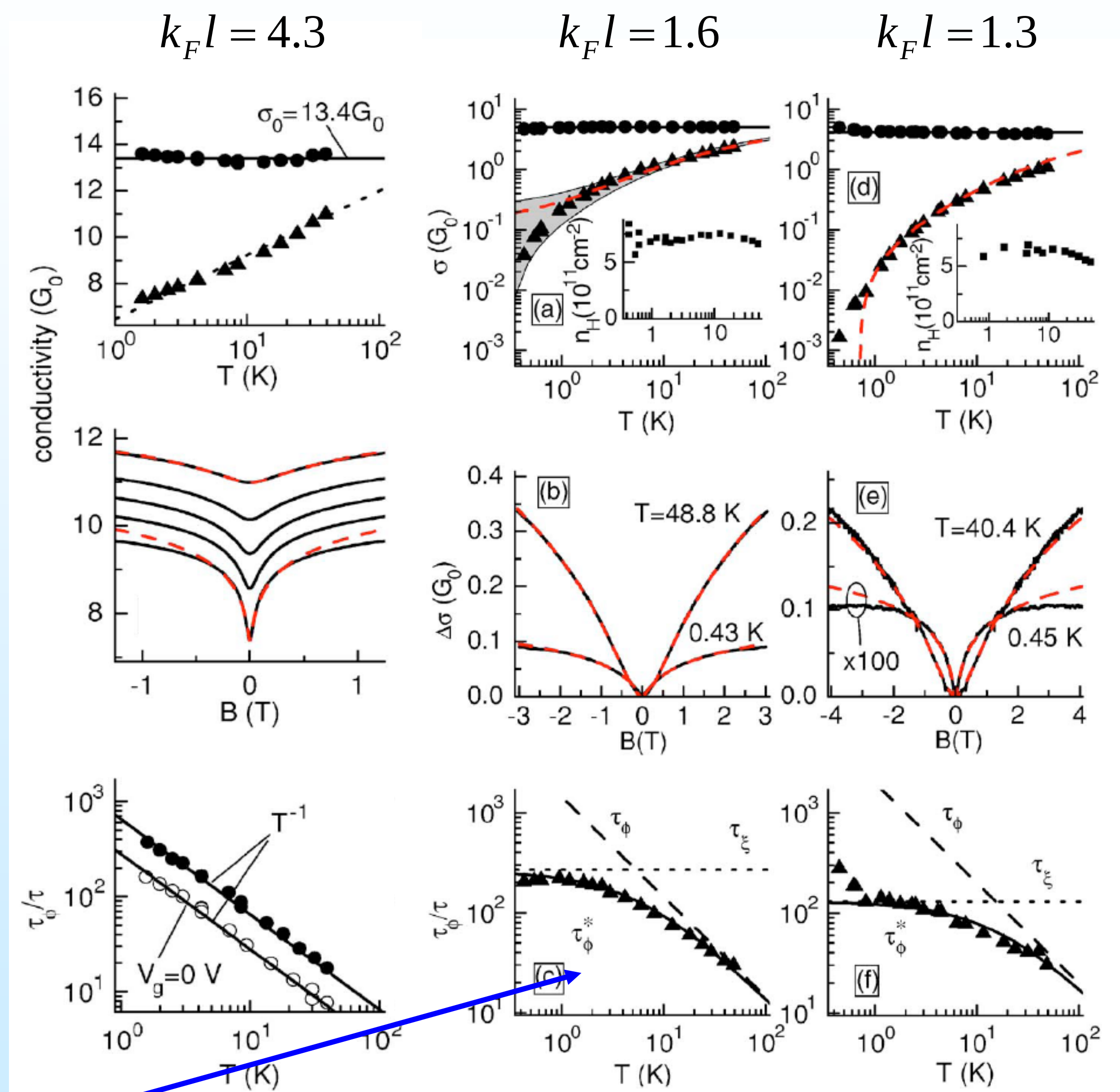
Limits of 2D WL Description

σ ⇔ e^2/h - where should we stop using the WL theory?

Minkov et al. '07:

δ-doped GaAs structures, n~1.5-2.5x10^{12}cm^{-2}





$$\frac{1}{\tau_\varphi^*} = \frac{1}{\tau_\varphi} + \frac{\xi^2}{D}$$

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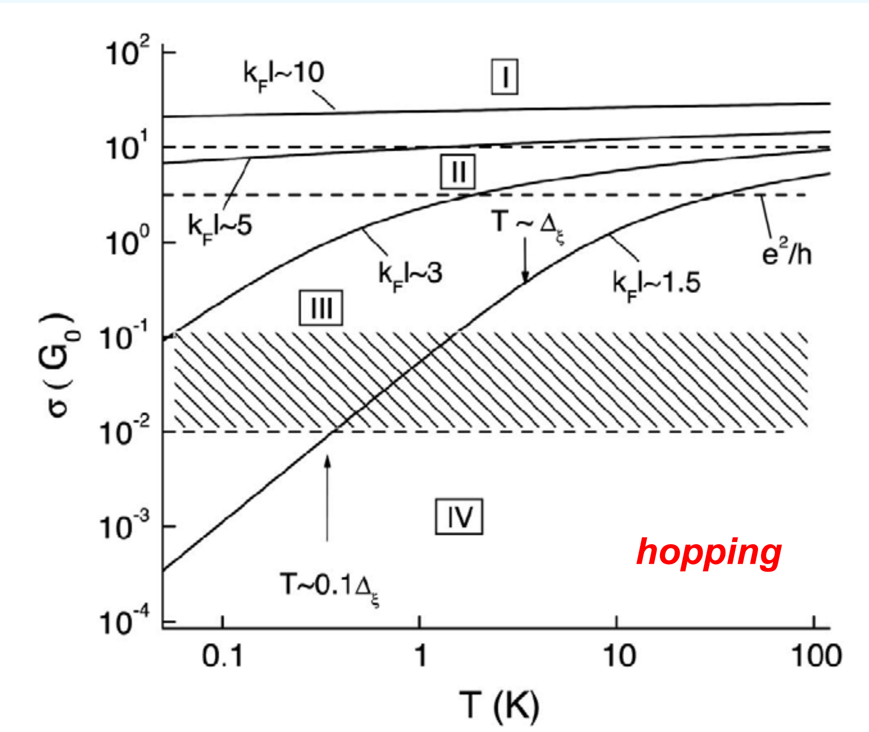
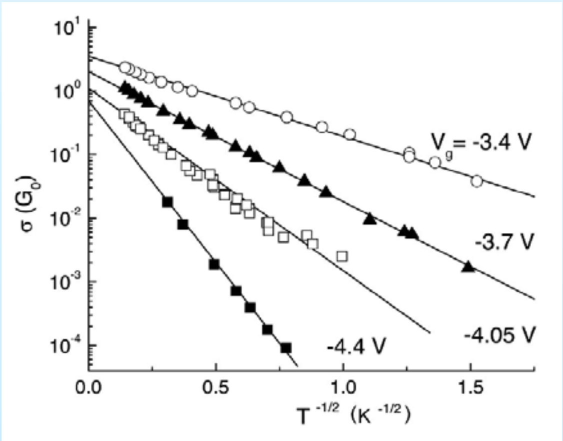
The Limits of 2D WL Description (cont.)

Area	$\sigma(T,B=0)$	$\Delta\sigma(B)$
I	$\sigma_0+G_0\ln(\tau/\tau_\phi)+K_{ee}G_0\ln T\tau$	$\alpha G_0\mathcal{H}(\tau/\tau_\phi,B/B_{tr})$,[a] $\alpha=1$
II	$\sigma_0+G_0\ln(\tau/\tau_\phi)+K_{ee}G_0\ln T\tau,\ K_{ee}\to 0$ at $k_Fl\searrow$	$\alpha G_0\mathcal{H}(\tau/\tau_\phi,B/B_{tr}),\ \alpha=1-2G_0/\sigma$
III	$\sigma_0+G_0\ln(\tau/\tau_\phi+\tau/\tau_\xi),\ \tau_\xi\sim\tau\exp(\pi k_Fl)$	$\alpha G_0\mathcal{H}(\tau/\tau_\phi+\tau/\tau_\xi,B/B_{tr}),\ \alpha\to 0$ at $\sigma\searrow$
IV	$\sigma_3\exp(-\varepsilon_3/T),\ \varepsilon_3\simeq(0.1-0.3)\Delta_\xi$	Determined by $\xi(B)$

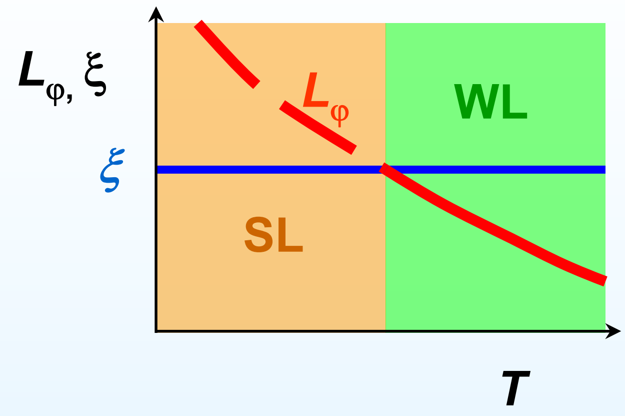


Minkov et al. '07: "…down to the very low conductivity value, $\sigma\sim10^{-2}e^2/h$, the experimental results are described by the same expressions which are valid in the WL regime. One should only replace the dephasing rate τ_φ^{-1} by the quantity $\tau_\varphi^{-1}+\tau_\xi^{-1}$."

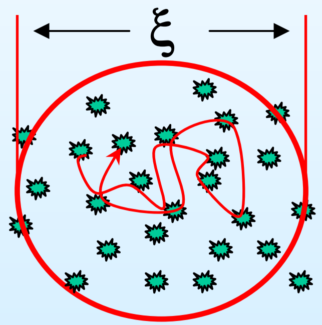


Hopping in Systems with Large Localization Length

Low-T *WL-SL* crossover - a unique opportunity to explore electron hopping with a large ξ, up to a few μm in 1D and ~0.5μm in 2D.



Hopping with large ξ in 1D and 2D systems – a very interesting transport regime, which is qualitatively different in many respects from the conventional hopping with small ξ in lightly doped semiconductors



$$\lambda_F < \ell < \xi < L_\varphi(T)$$

- the electron motion is still diffusive within the localization length

- Signatures of hopping with large ξ: orbital magnetoresistance, hot-electron effects in strong electric fields.
- Several experimental facts ("universality" of $R(T)$, hot-electron effects in SL regime) suggest that this hopping might be driven by ***electron-electron*** interactions rather than ***electron-phonon*** interactions

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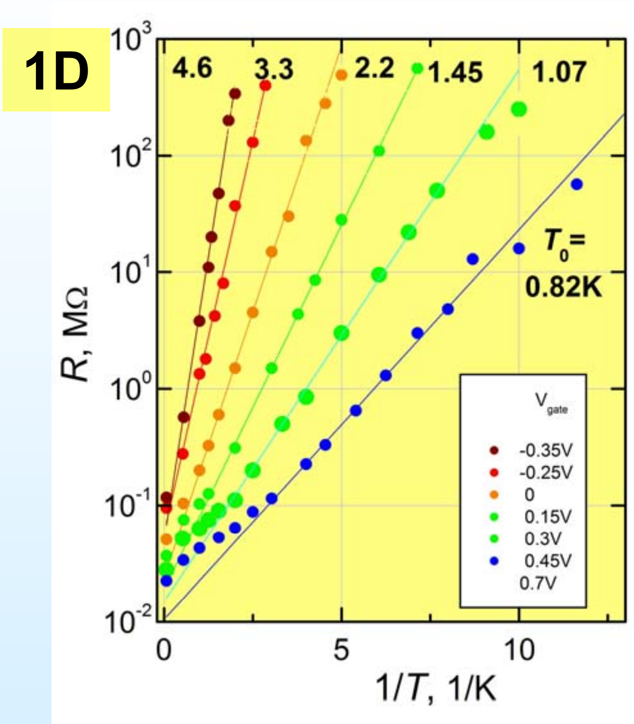
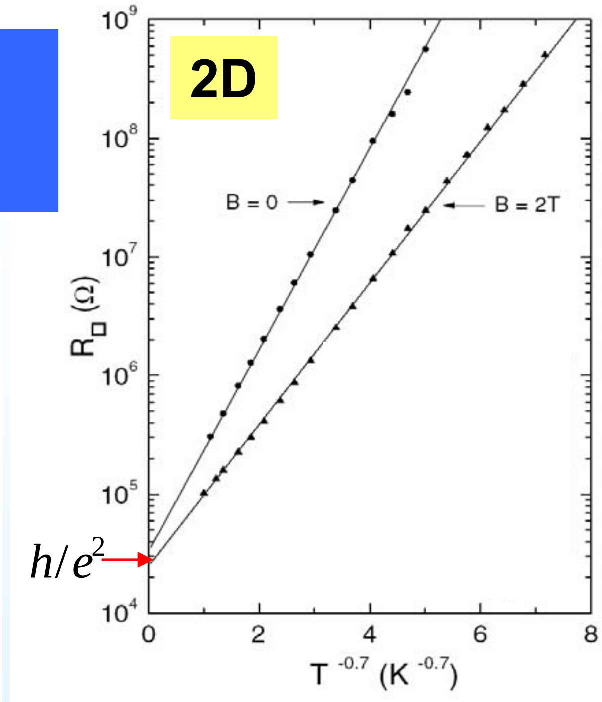
$R(T)$ in Hopping Regime



Gated ***quasi-1D wires*** in Si δ-doped GaAs

Khavin, MG, Bogdanov, PRB **58**, 8009 ('98)

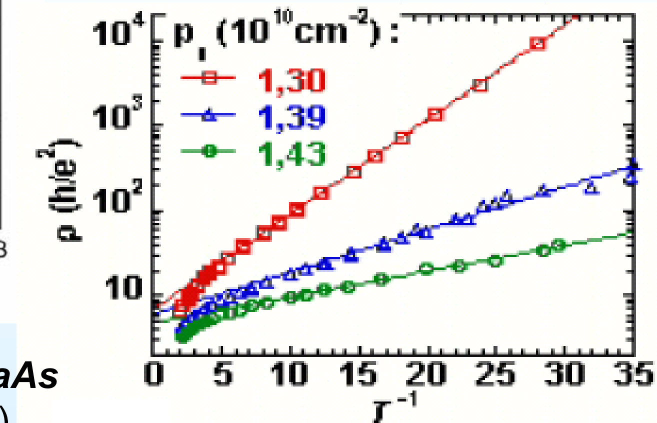


2D Si:GaAs

MG et al,

PRL **85**, 1718 ('00)

Similar dependences – in ultra-thin metal films (e.g., Goldman *et al*.)



Holes in GaAs/AlGaAs

Leturcq et al. ('02)

The "Arrhenius" dependences $R(T)$ have been observed for both 1D and 2D conductors in the SL regime.

The activation energy is close to the mean level spacing within the localization envelope, ~(0.2-1) Δ_ξ.

Puzzle: for different 2D systems, the pre-factor is "universal" – not expected if hopping is phonon-assisted.

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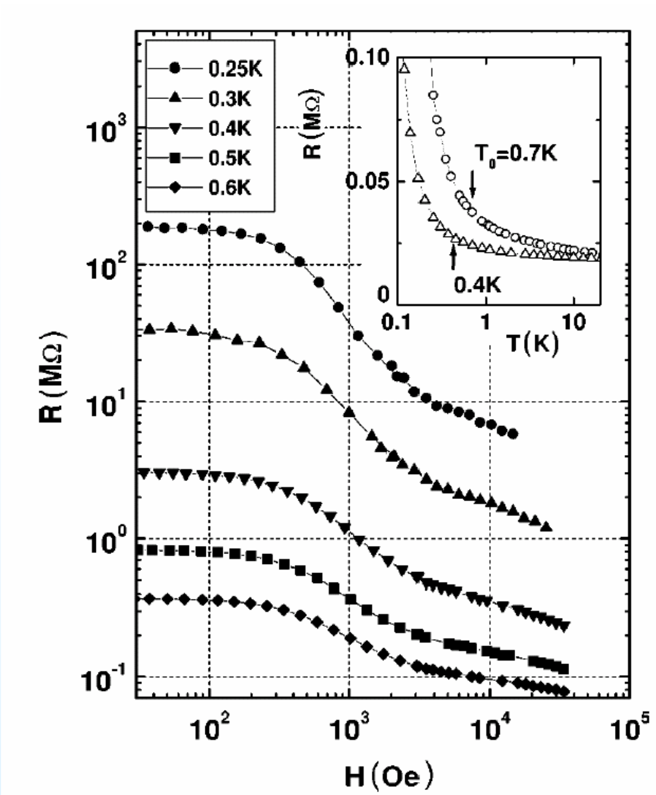
MR in strongly-localized 1D systems

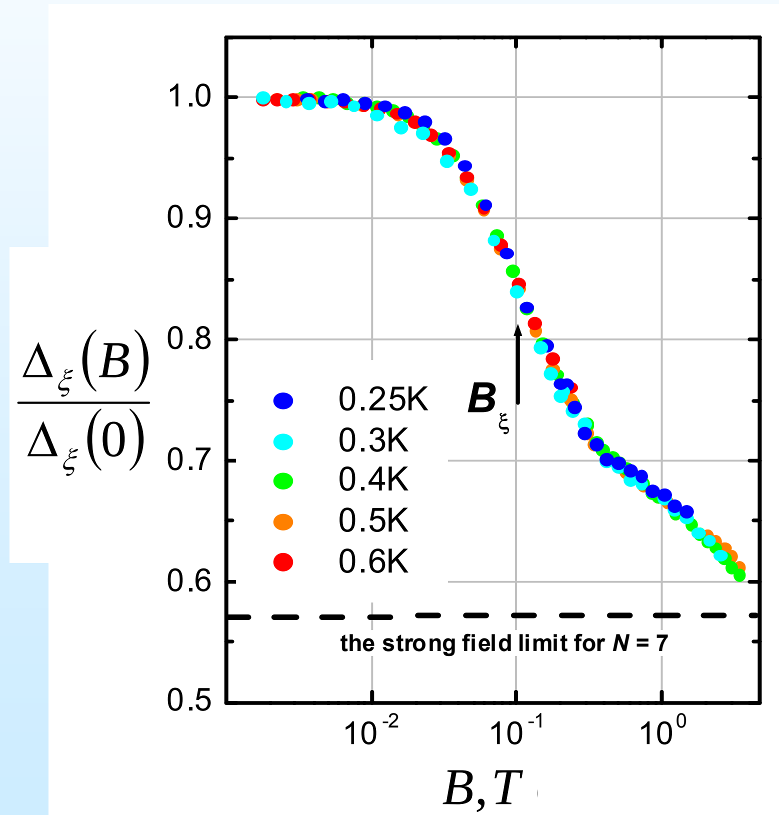


$$R(T,B) = R_0 \exp\left[\frac{\Delta_\xi(B)}{T}\right] \qquad \Delta_\xi = (\nu_{1D}\xi)^{-1}$$

the pre-factor R_0 does not depend on B

- negative
- exponentially strong
- orbital: vanishes in $B_{||}$

Khavin, MG, Bogdanov, PRB **58**, 8009 (1998)



$$\xi_{1D}^{O} = N_{1D} \cdot l$$

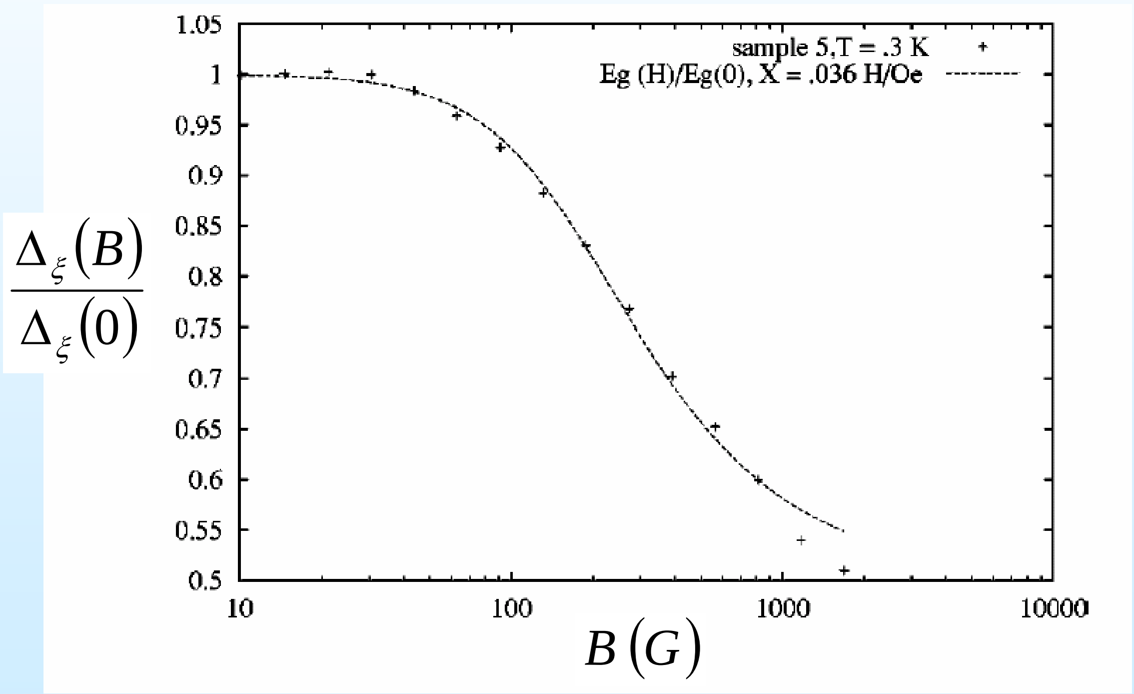
$$\xi_{1D}^{U} = 2N_{1D} \cdot l$$

$$B >> \frac{\Phi_0}{\xi W}$$

<u>Doubling of</u> $\xi \Rightarrow$ <u>Cutting</u> $\Delta_\xi = (\nu_{1D}\xi)^{-1}$ <u>in half</u>

What does the theory say?

- Kolesnikov and Efetov, *PRL* **83**, 3689 (‘99)
- Schomerus and Beenakker, *PRL* **84**, 3927 (‘00)
- Kettemann, *PR* B **62**, R13282 (‘00)
- Kettemann and Mazzarello, *PR* B **65**, 085318 (‘02)



S. Kettemann,
PRB **62**, R13282 (2000)

- direct measurements of ξ in 1D (and 2D) conductors with a large ξ

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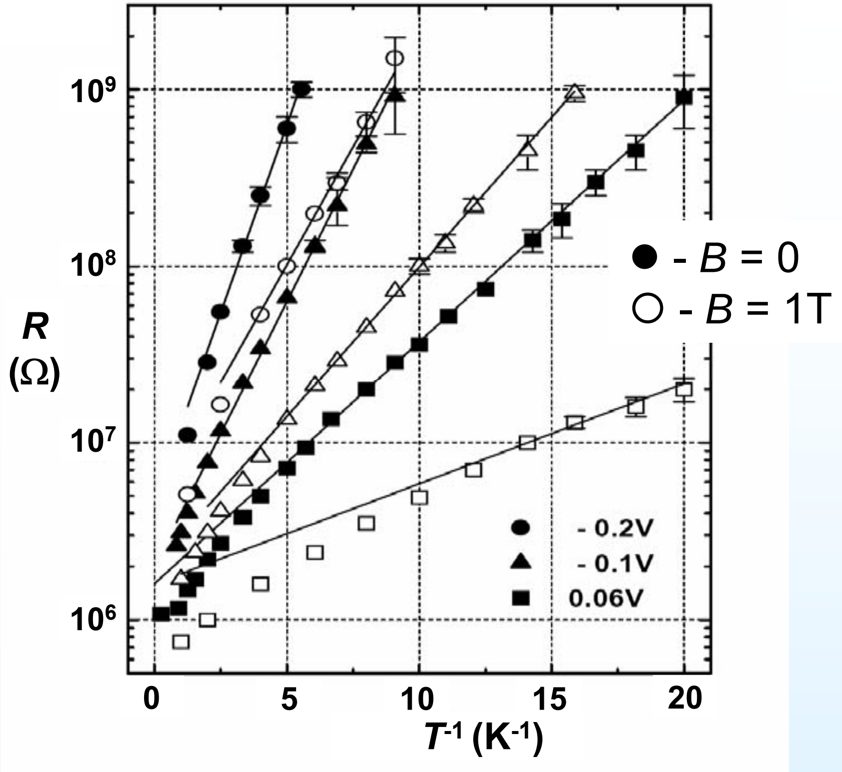
MR in the 2D SL regime



The activation energy is decreased in the magnetic field ⊥ to the plane of a 2D sample:

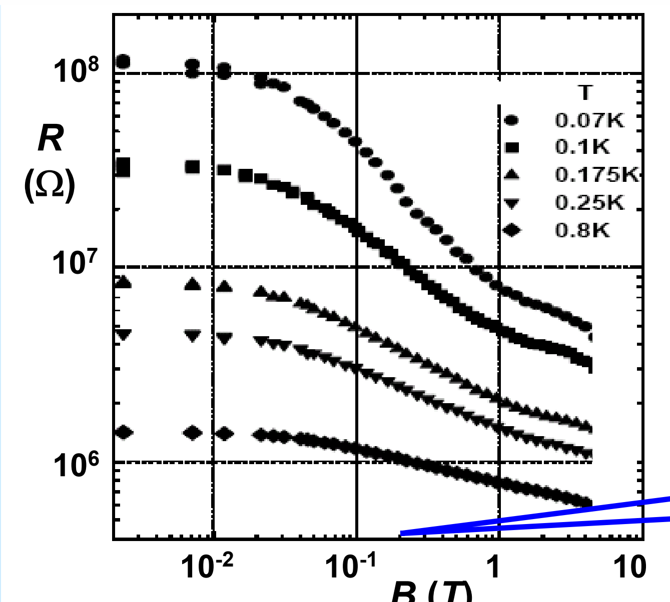
$$R(T,B) = R_0 \exp\left[\frac{\Delta_\xi(B)}{T}\right] \qquad \Delta_\xi = \left(\nu_{2D}\xi^2\right)^{-1}$$

$R_0 \approx h/e^2$ - does not depend on B

In 2D, one might expect more radical drop of the activation energy in ***B***:

$$\xi_{\mathrm{O}}^{2\mathrm{D}} = l \cdot \exp\left(\frac{\pi}{2}k_F l\right) \longrightarrow \xi_{\mathrm{U}}^{2\mathrm{D}} = l \cdot \exp\left[\left(\frac{\pi}{2}k_F l\right)^2\right]$$

However, $\boldsymbol{k_F l}$ ~1 for the 2D conductors which demonstrate the SL-WL crossover at low T (= large ξ).



$\xi \sim 0.1\,\mu\mathrm{m}\ \left(\sim 10l\right)$

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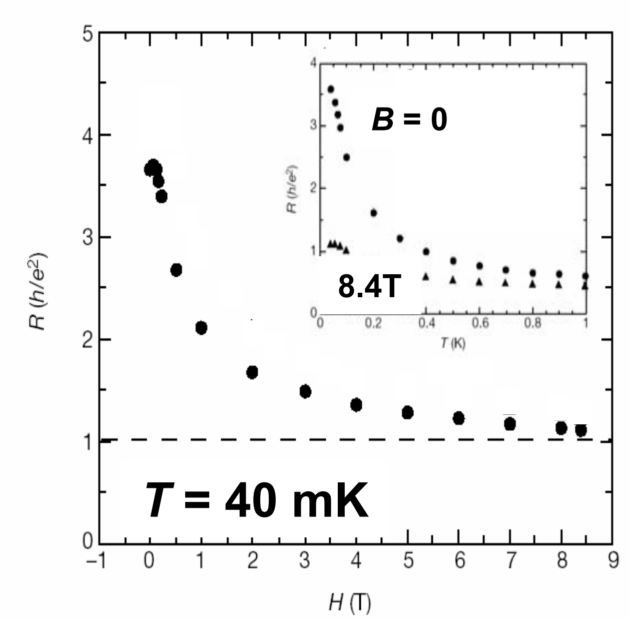
Examples of strong orbital MR in 2D



1.7 nm thick Be film
MR: large, anisotropic, negative in strong fields

Butko and Adams, *Nature* **409**, 161 ('01)

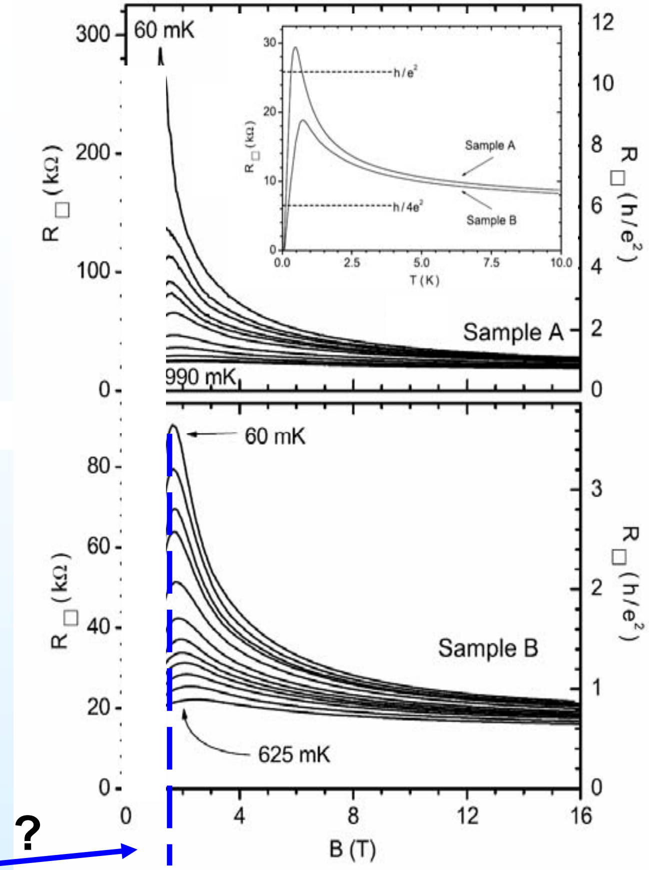
Interesting parallel:
strong-field MR of 2D systems close to the disorder-driven Superconductor-Insulator Transition

Baturina *et al*., TiN films

Shahar *et al*., InO_x films

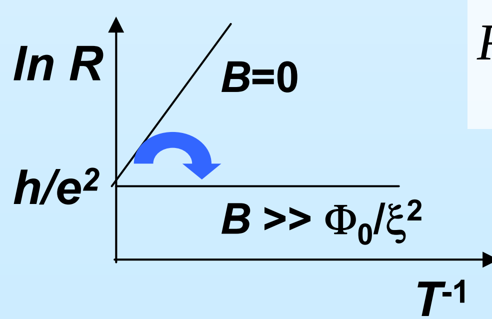
$$\left(\frac{\Phi_0}{B}\right)^{0.5} \sim \xi \sim \xi_{SC}$$

?



Baturina *et al*.,'06

"Quantum metallicity" – strong reduction of the activation energy due to the growth of ξ in $B>\Phi_0/\xi^2$?



$$R(T,B) = \frac{h}{e^2}\exp\left[\frac{\Delta_\xi(B)}{k_B T}\right]$$

$\Delta_\xi(B)$ – mean level spacing within ξ^2 (B)

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Aharonov-Bohm Effect in Hopping

Poyarkov *et al*. '86: disordered $PbTeO_x$ with persistent low-T photoconductivity

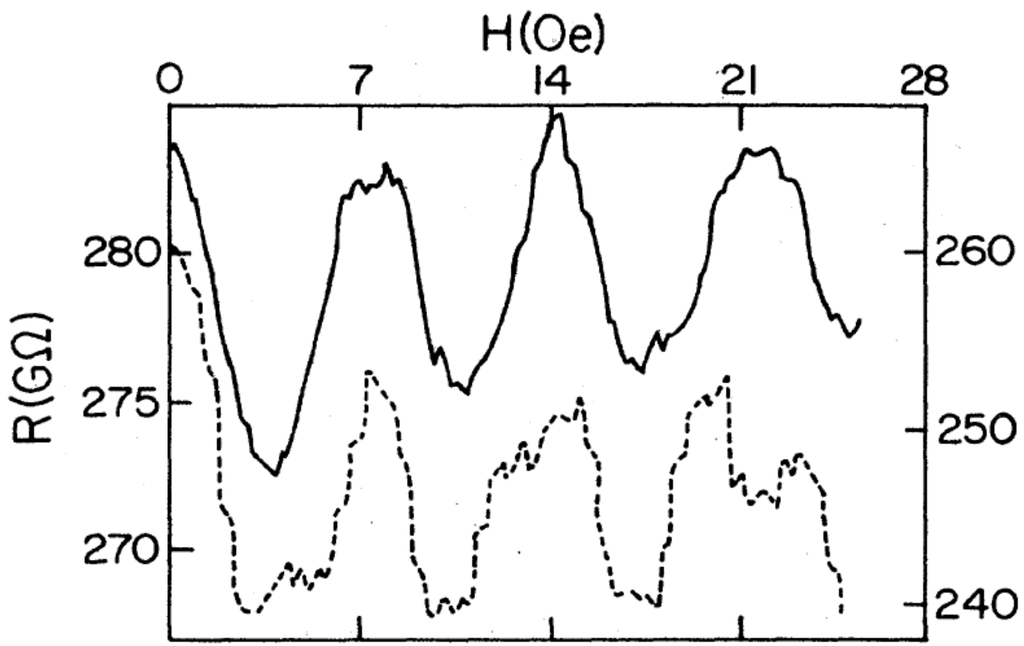
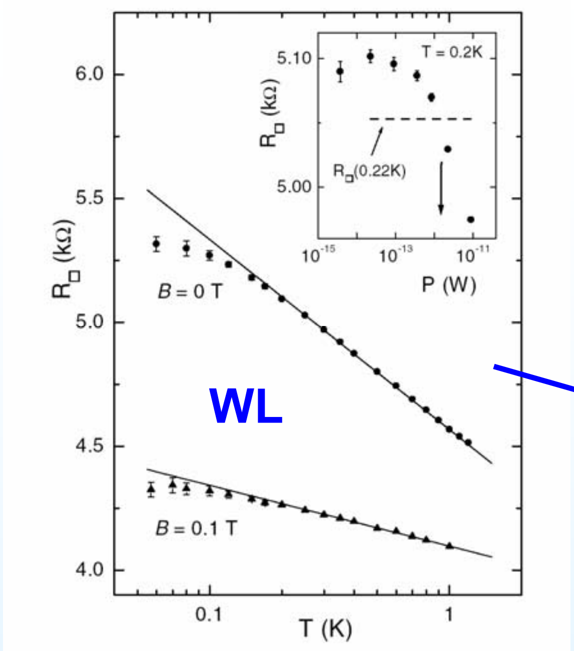
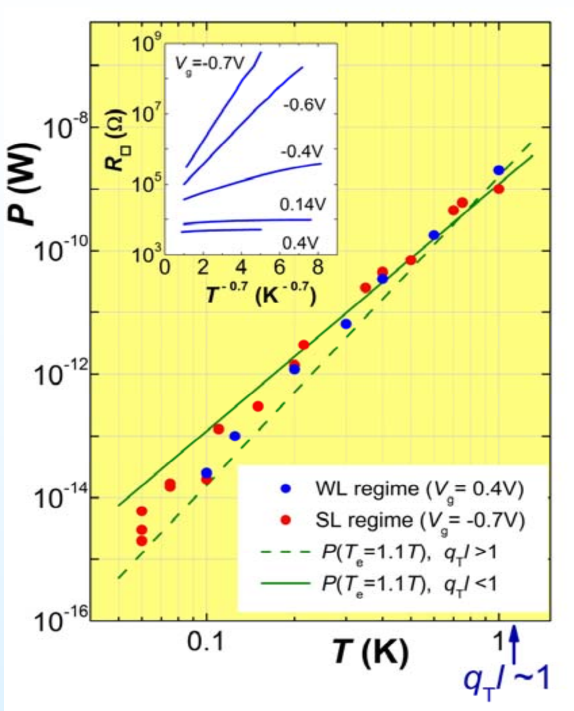


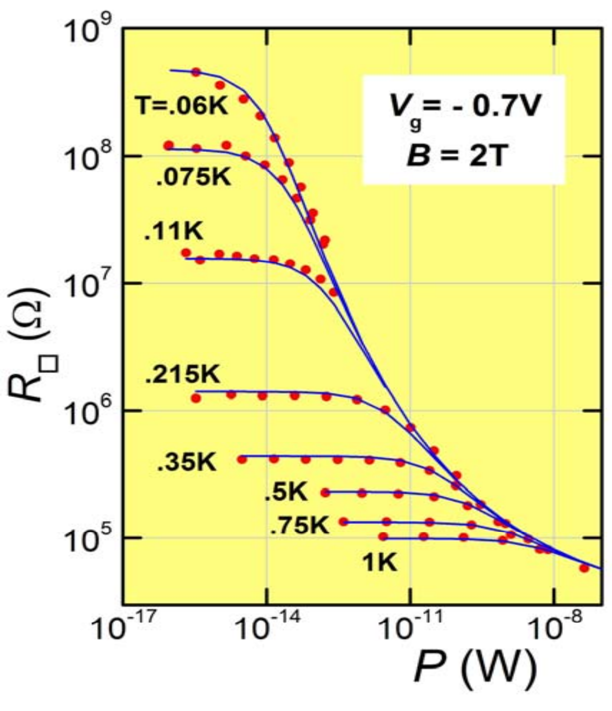
FIG. 30. $R(H)$ dependence at $T=1.12$ K for the same sample as in Fig. 29 (average of 24 runs). The $R(G\Omega)$ scale for the single-run curve (dotted line) is shown on the right (Poyarkov *et al.*, 1986).

Never reproduced!

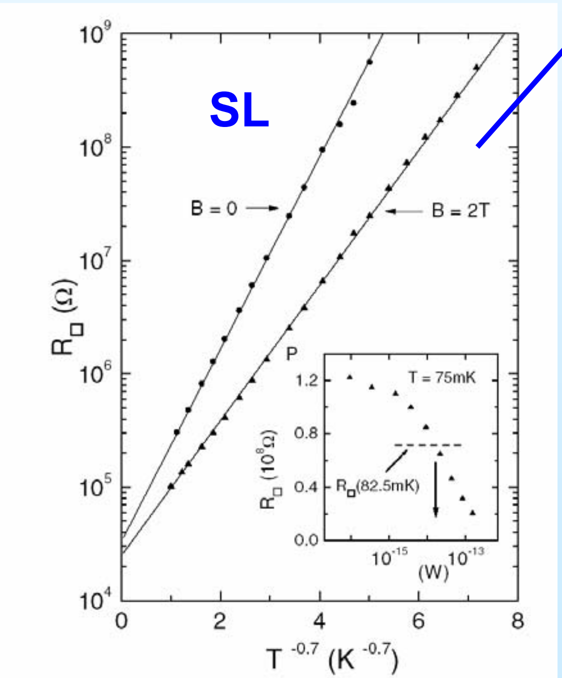
Non-Linear Effects in Hopping Regime







MG et al., PRL **85**, 1718 (2000)



- Non-linearity of the hopping conductivity of 2D systems with a large ξ is due to hot-electron effects rather than the field effects; the electric field, which is sufficient to overheat electrons, does not depend on ξ.

- The thermal conductivity between electrons and phonons, $\boldsymbol{G_{e\text{-}ph} = C_e/\tau_{e\text{-}ph}}$, is the same on both sides of the 2D WL-SL crossover (this might be expected, since the el. motion is still diffusive at $L<\xi$, and $q_T\,\xi \sim 1$ even at T=50 mK).

Non-Linear Effects in Conductors with a large ξ

In the systems with a large ξ, ***electron heating is expected to dominate over the field effects.*** Let's compare two values of the power ***P*** dissipated in a conductor:

P_{NL}, the Joule heat power at which the ***E***-induced non-linearity develops:

$$eE\xi \approx k_B T$$

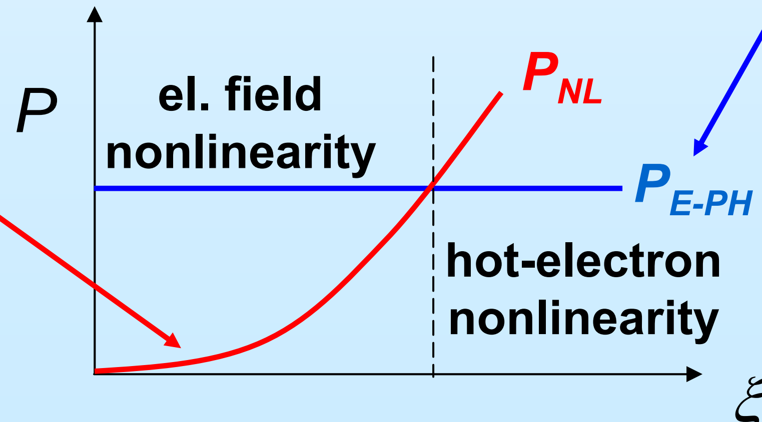
$$E \approx \frac{k_B T}{e\xi}$$

$$P_{NL} = \frac{(EL)^2}{R} = \frac{\left(\frac{k_B TL}{e\xi}\right)^2}{h/e^2 \exp\left(\frac{1}{k_B T \nu_{2D} \xi^2}\right)} \propto \frac{1}{\xi^2 \exp\left(\frac{1}{\xi^2}\right)}$$

- decreases exponentially with decreasing ξ for exp(…)>> 1

$P_{E\text{-}PH}$, the power that can be removed from an electron system due to electron-phonon coupling without significant overheating of electrons

- does not depend on ξ



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What have we learned from the hot-electron experiments:

2D systems: the $\rho(E)$ dependence can be attributed to carrier heating all along the crossover from the diffusive to the VRH regimes [n-GaAs – MG *et al*. ('00), SiGe – Leturcq *et al*., ('03)].

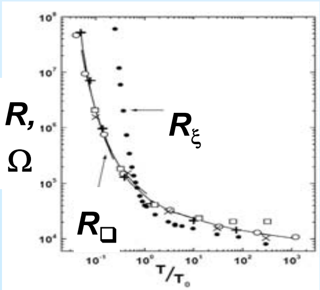
This tendency has been observed for σ as small as 10^{-4} e^2/h. Recall: Minkov *et al*. ('04) considered the hot-electron regime as a signature of "diffusiveness" rather than hopping down to $\sigma \sim 2\times10^{-2}$ e^2/h.

3D Strongly Localized systems: strong evidence for the existence of separate temperatures for the electron and phonon systems analogous to the hot-electron effects in metals [doped Si and Ge - Zhang et al., ('98); doped Ge – Wang et al., ('90,'99); Marnieros et al., ('00); doped Si - Galeazzi et al, '07].

Hopping conductivity depends only on the electron temperature when electrons are removed from equilibrium with phonons. The effective electron temperature is established via interactions among the localized electrons. This suggests that electron-electron interactions play crucial role in hopping in conductors with a large localization length (electron-assisted hopping rather than phonon-assisted one).

In the WL regime: the temperature dependences of $\Delta\sigma_{WL}$ and $\Delta\sigma_{EEI}$ are governed by electron-electron interactions

In the SL regime: "universal" hopping R(T); hot-electron effects when electrons are removed from equilibrium with phonons



Temptation: to develop a unifying (EEI-based) approach that would describe R(T) over a wide range of resistances (WL+SL).

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Summary

- Single-particle effects in disordered conductors: quantum interference leads to WL corrections to the Drude conductivity in the regime of weak disorder/high *T* ($L_\varphi<\xi$) or SL in the regime of strong disorder/low *T* ($L_\varphi>\xi$)
- Due to non-locality, the WL corrections are very universal; WL magnetoresistance provides unique opportunity to study inelastic and spin scattering in disordered conductors.
- Realization of the low-T WL-SL crossover provides an access to the regime of (unconventional) hopping with a large localization length.