Smile Gaps and Topology in the Andreev Spectrum of Superconducting Junctions

Wolfgang Belzig[1], Johannes Reutlinger[1], Tomohiro Yokoyama[2,3], Leonid Glazman[4], and Yuli V. Nazarov[2]

(1) Universität Konstanz
SFB 767

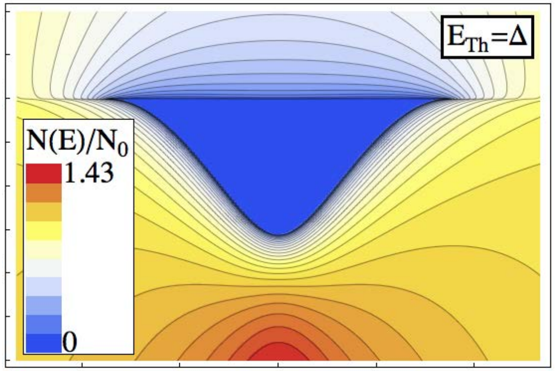
(2) TU Delft
Delft University of Technology

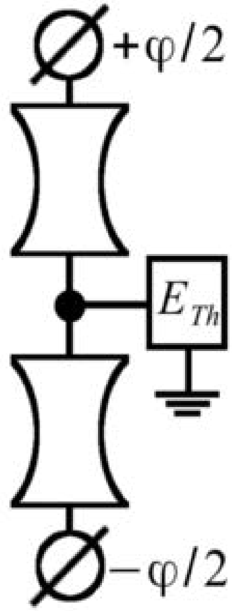
(4) Yale

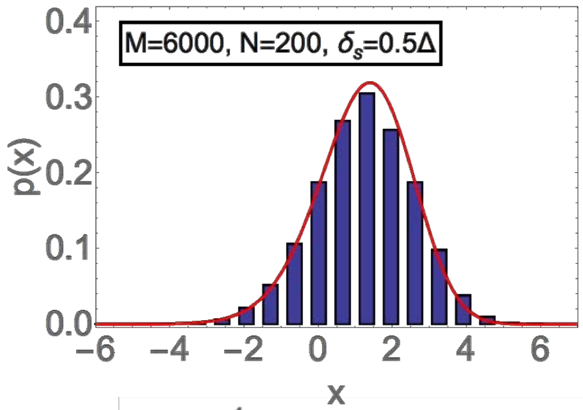
(3) THE UNIVERSITY OF TOKYO

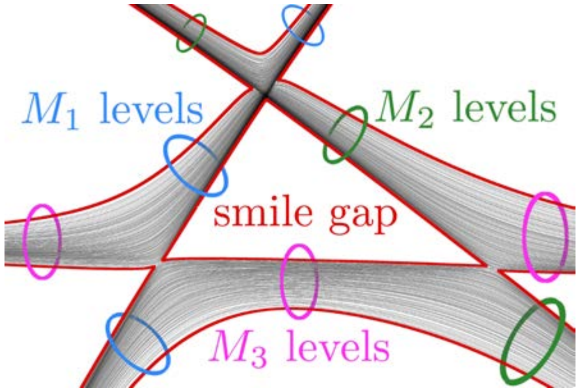
Content

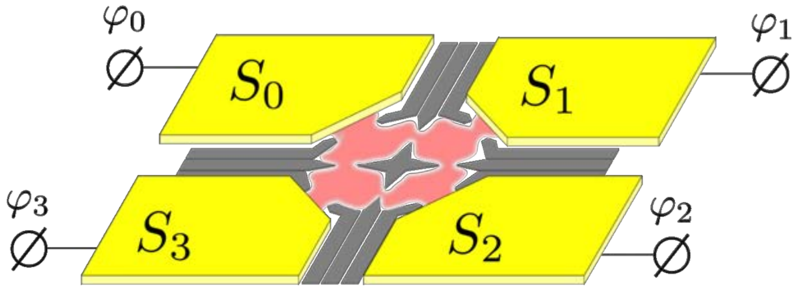
1. Introduction: proximity effect and local density of states
2. The minigap and the smile gap in a ballistic cavity Josephson junction
3. Minigaps and smile gaps in a 4-terminal Josephson junction ring and topological protection of the spectral properties
4. ~~Universality of level statistics at the smile gap edge~~

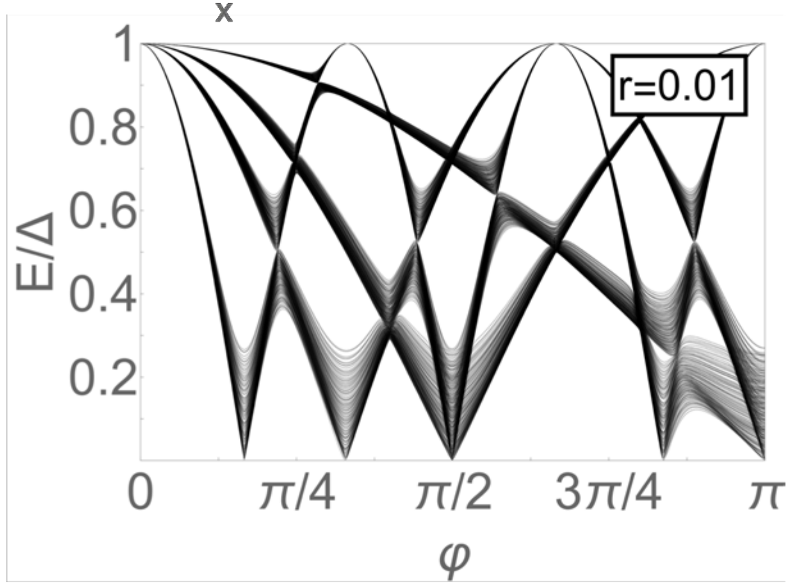












1) Introduction, History, Experiments

Tunnel model

$$\Omega_N \approx \Gamma_N = \hbar/\tau_N$$

This appears to be a general result.

[McMillan, Phys. Rev. 175, 573 (1968)]

Quasiclassical model (diffusive)

[Golubov, Kupriyanov, J. Low Temp. Phys. **70**, 83 (1988)]

The local density of states

- Continuous transition from a superconducting to a normal density of states
- Distant-dependent gap feature

S. Gueron, H. Pothier, N. Birge, D. Esteve, and M. Devoret, Phys. Rev. Lett. **77**, 3025 (1996).

The Minigap

Mesoscopic energy scale:
Thouless energy!

$$E_{Th} = \frac{\hbar D}{L^2}$$

e.g. Belzig, Bruder, Schön, Phys. Rev. B (1996)

Further works (small collection):

- Influence on transport [Volkov, Zaitsev, Klapwijk, Physica C **210**, 21 (1993)]
- Reentrant conductivity [Nazarov, Stoof, PRL **76**, 823 (1996)]
- Phase dependence [Zhou, Charlat, Spivak, Pannetier, JLTP **110**, 841 (1998)]
- Mesoscopic fluctuations [Ostrovsky, Skvortsov, Feigel'man, PRL **87**, 027002 (2001)]
- Quantum fluctuation origin [Micklitz, Altland PRL **103**, 1 (2009)]

Minigap beyond the dirty limit

Review: Beenakker, (2005)

Elastic scattering rate $\hbar/\tau_{el}$

Thouless Energy $\hbar/\boldsymbol{\tau}_{diff}$

Pilgram, Belzig, Bruder, PRB (2000)

Lodder, Nazarov PRB (1998)

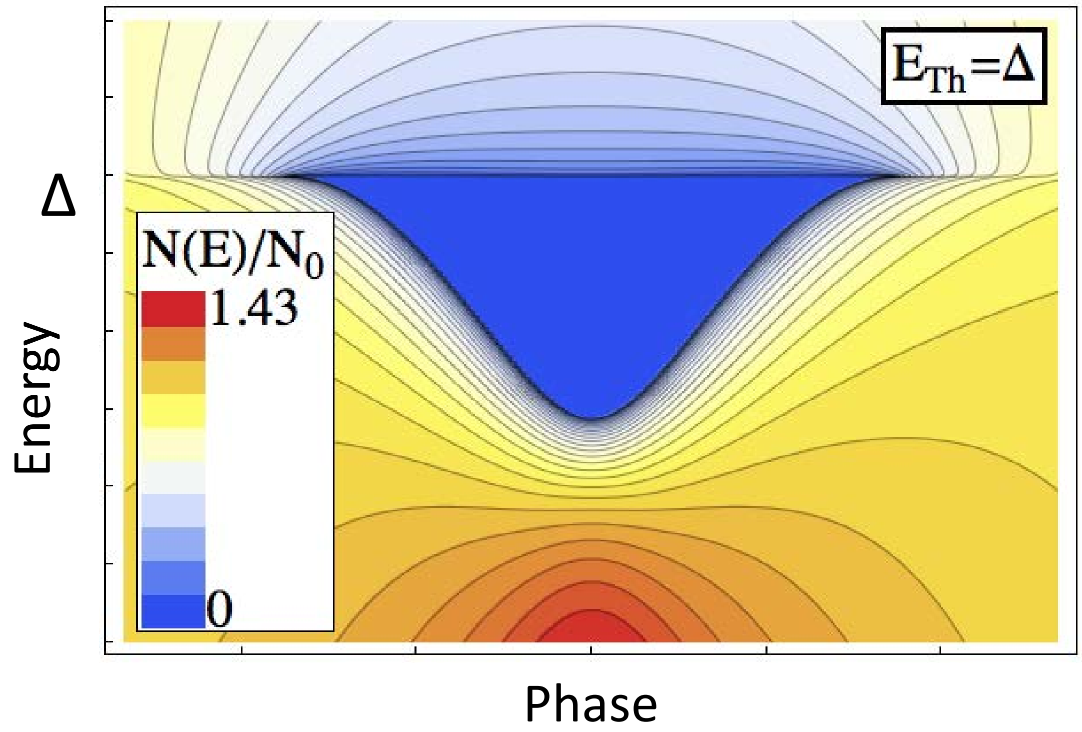
The phase-dependent minigap

Clear observation of the minigap and the phase dependence

H. Le Sueur, P. Joyez, H. Pothier, C. Urbina, and D. Esteve, Phys. Rev. Lett. (2008).

2) Proximity density of states of a cavity at <u>large</u> E_{TH}:

The "smile" gap



J. Reutlinger, L. Glazman, Yu. V. Nazarov, and WB
Phys. Rev. Lett. **112**, 067001 (2014)
Phys. Rev. B **90**, 014521 (2014)

Method: Quasiclassical Greens functions (Quantum circuit method)

Matrix current conservation:

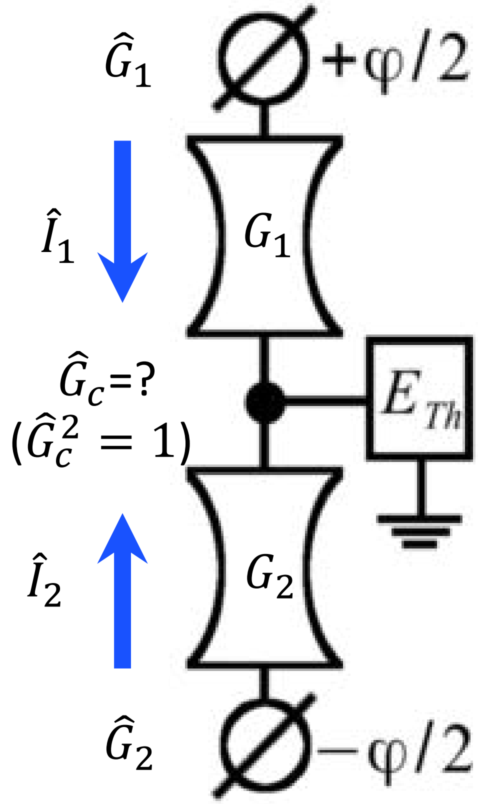
$$\hat{I}_1 + \hat{I}_2 + i(G_1 + G_2)\frac{E}{E_{Th}}\left[\hat{\tau}_3, \hat{G}_c\right] = 0$$

Nonlinear matrix boundary condition ("Nazarovs law", analog Ohms law) for a ballistic connector

$$\hat{I}_i = G_i \frac{\left[\hat{G}_c, \hat{G}_i(\varphi_i)\right]}{1 + \left\{\hat{G}_c, \hat{G}_i(\varphi_i)\right\}/2}$$

Superconducting contacts:

$$\hat{G}_i(\varphi_i) = \frac{1}{\sqrt{E^2 - \Delta^2}}\begin{pmatrix} -iE & \Delta e^{i\varphi_i} \\ e^{-i\varphi_i} & iE \end{pmatrix}$$

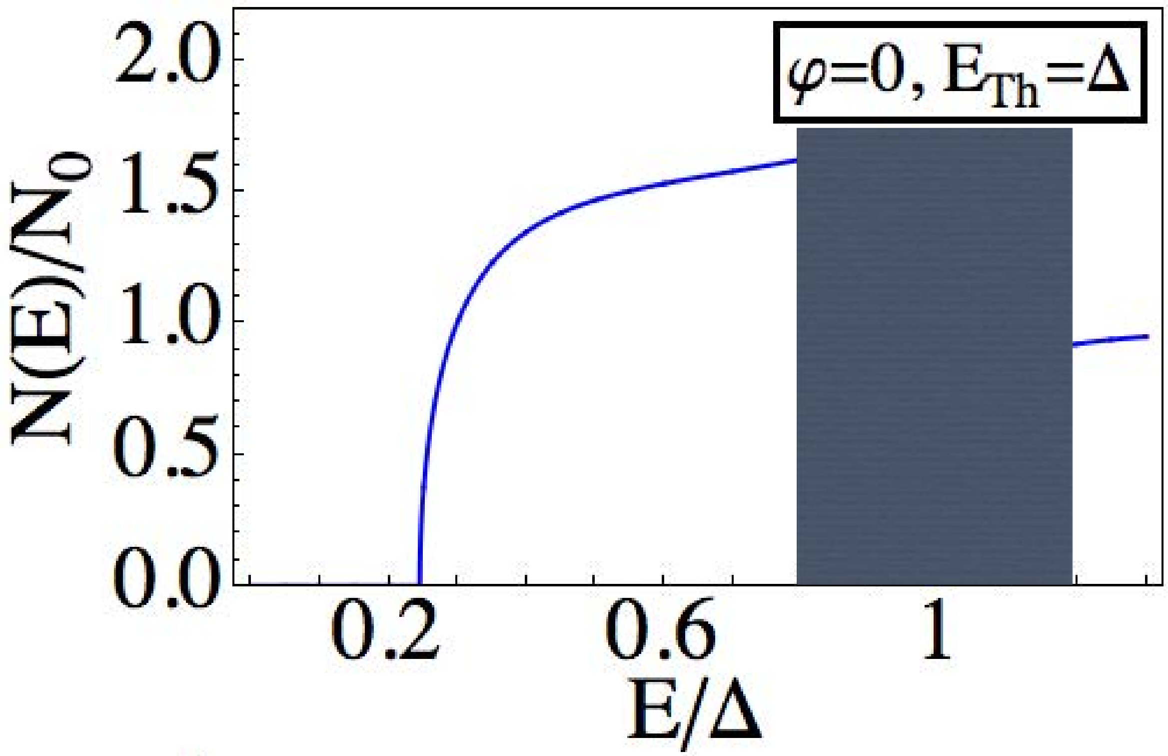


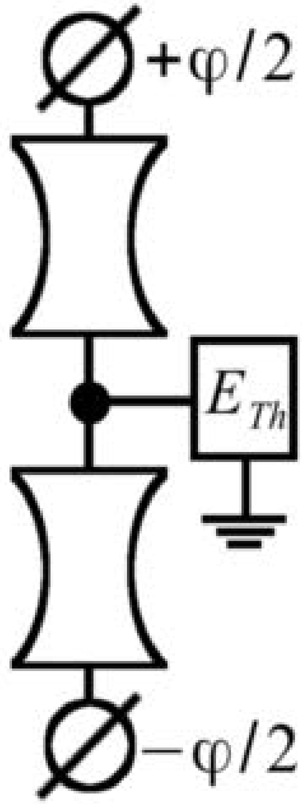
DOS on central node from matrix Greens function

$$\frac{N(E)}{N_0} = \Re\left[\mathrm{Tr}\hat{\tau}_3\hat{G}_c(E)\right]$$

[Nazarov, Superlatt. Microstr. (1999)]

The Effect of finite Δ for a ballistic cavity

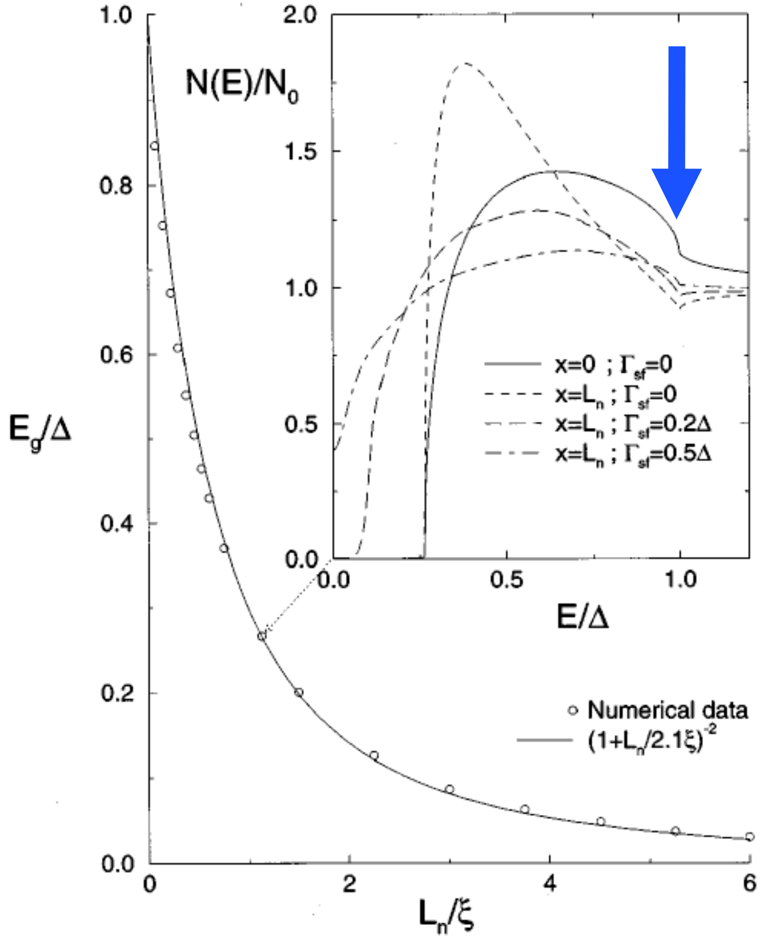




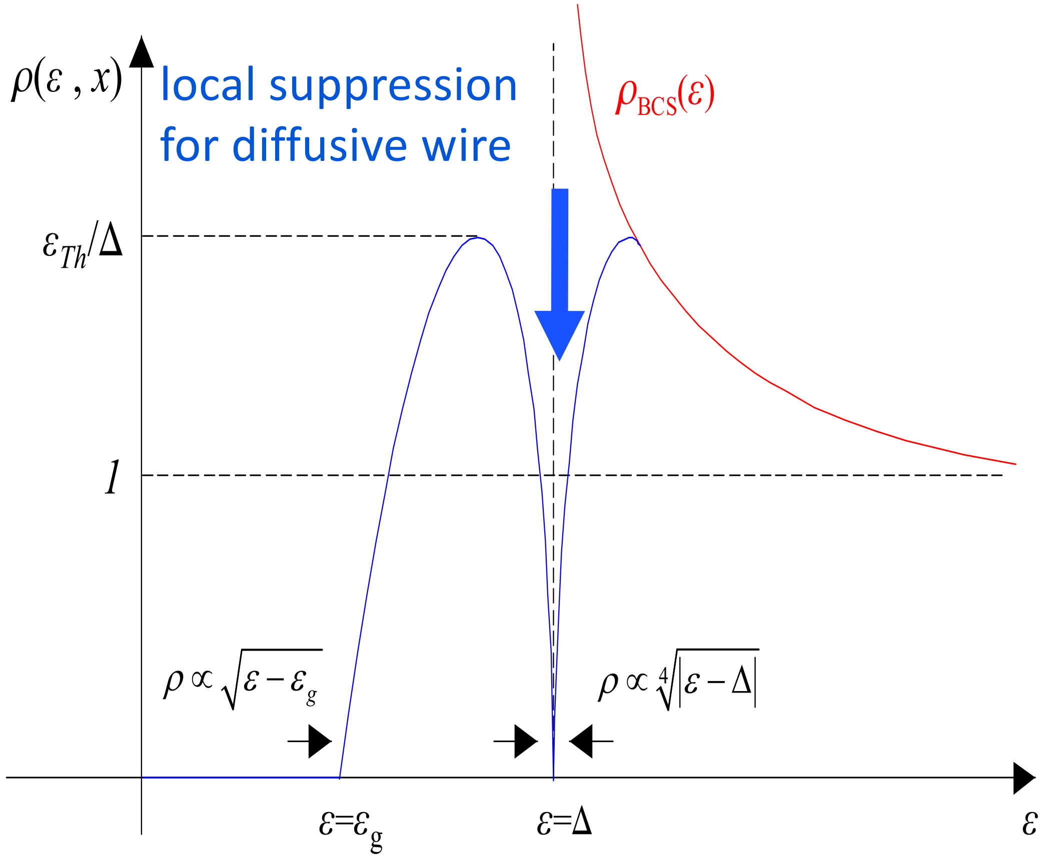
Small secondary gap just below Delta!

J. Reutlinger, L. Glazman, Yu. V. Nazarov, and WB, PRL **112**, 067001 (2014)

Earlier hints?



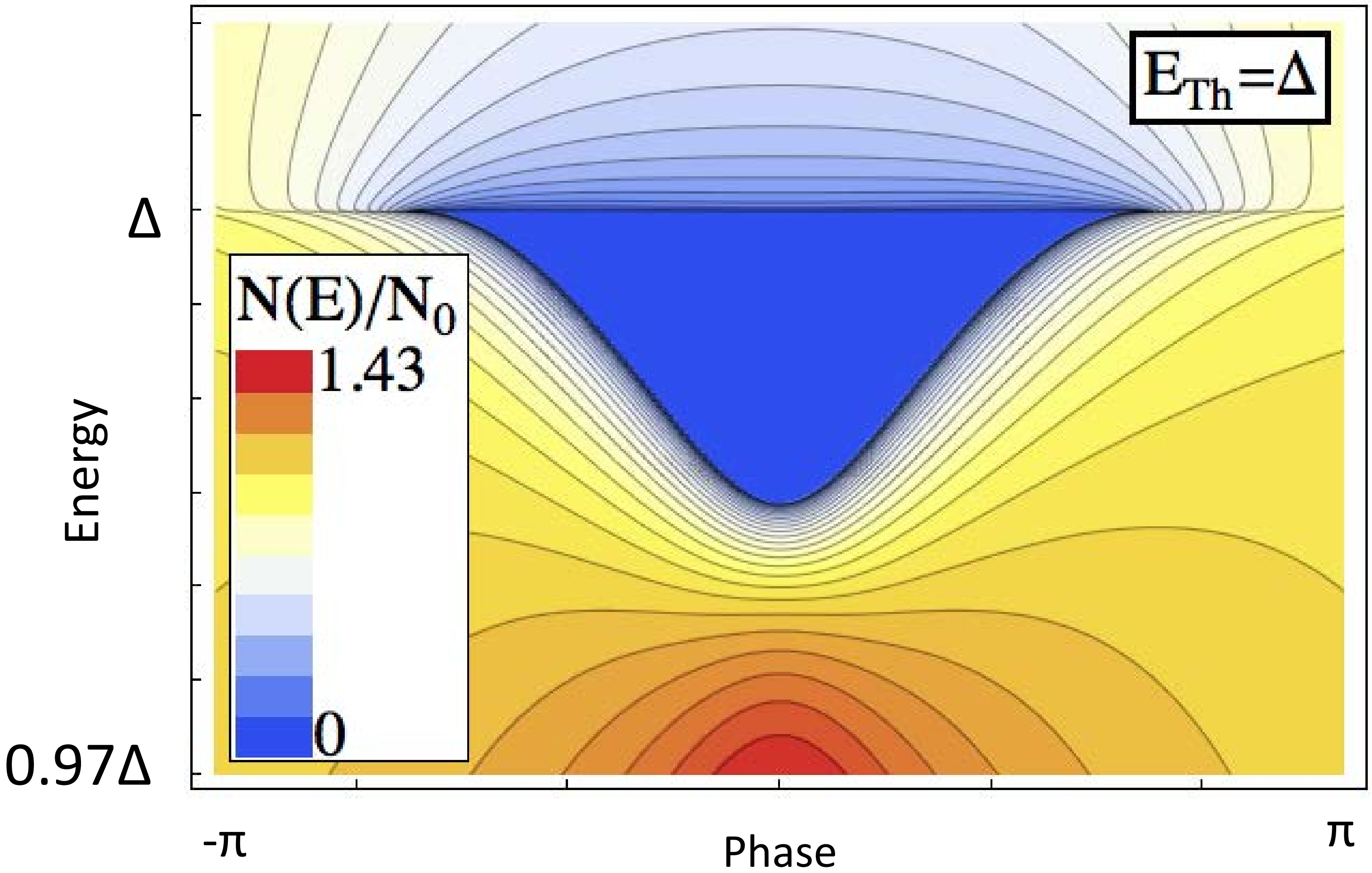
Belzig, Bruder, Schön, PRB (1996)



Levchenko, PRB (2008)

Further hints: Golubov, Kupryanov; Bezuglyi; Kuipers, Richter;....

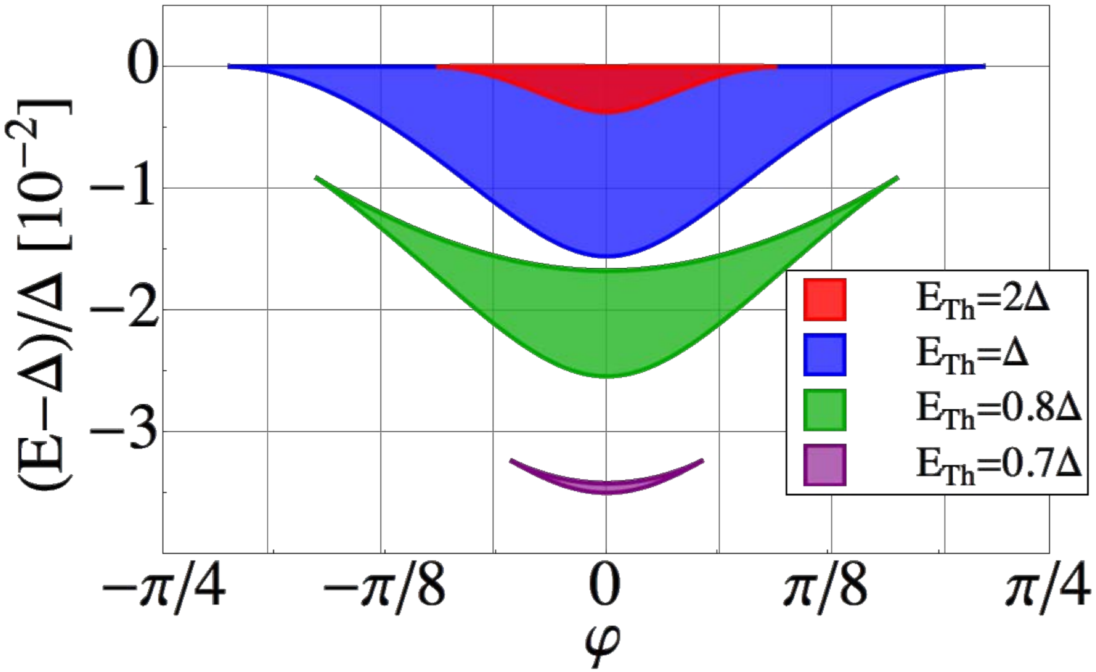
Phase-dependence of the secondary gap:



- Secondary gap vanishes at a critical phase with a “smile”

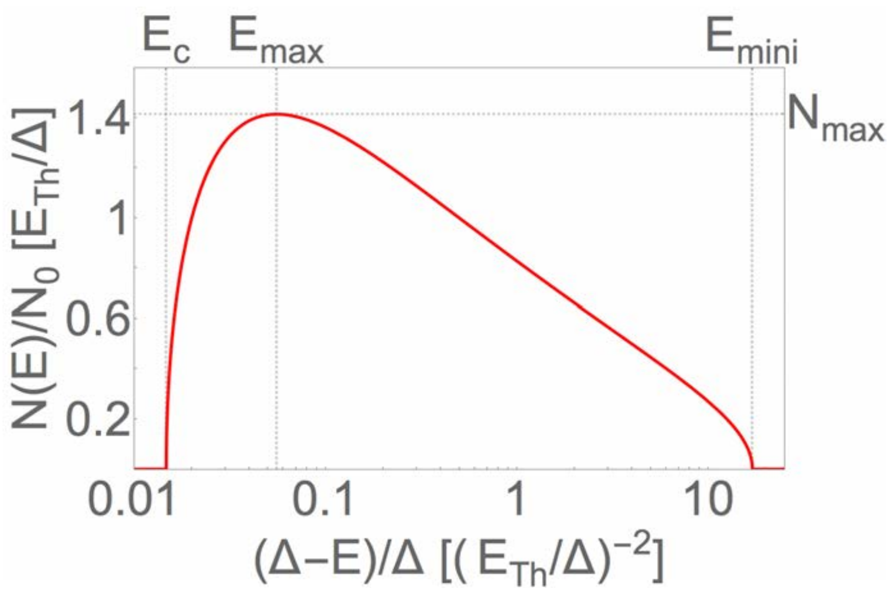
J. Reutlinger, L. Glazman, Yu. V. Nazarov, and WB, Phys. Rev. Lett. **112**, 067001 (2014)

Further properties of the smile gap



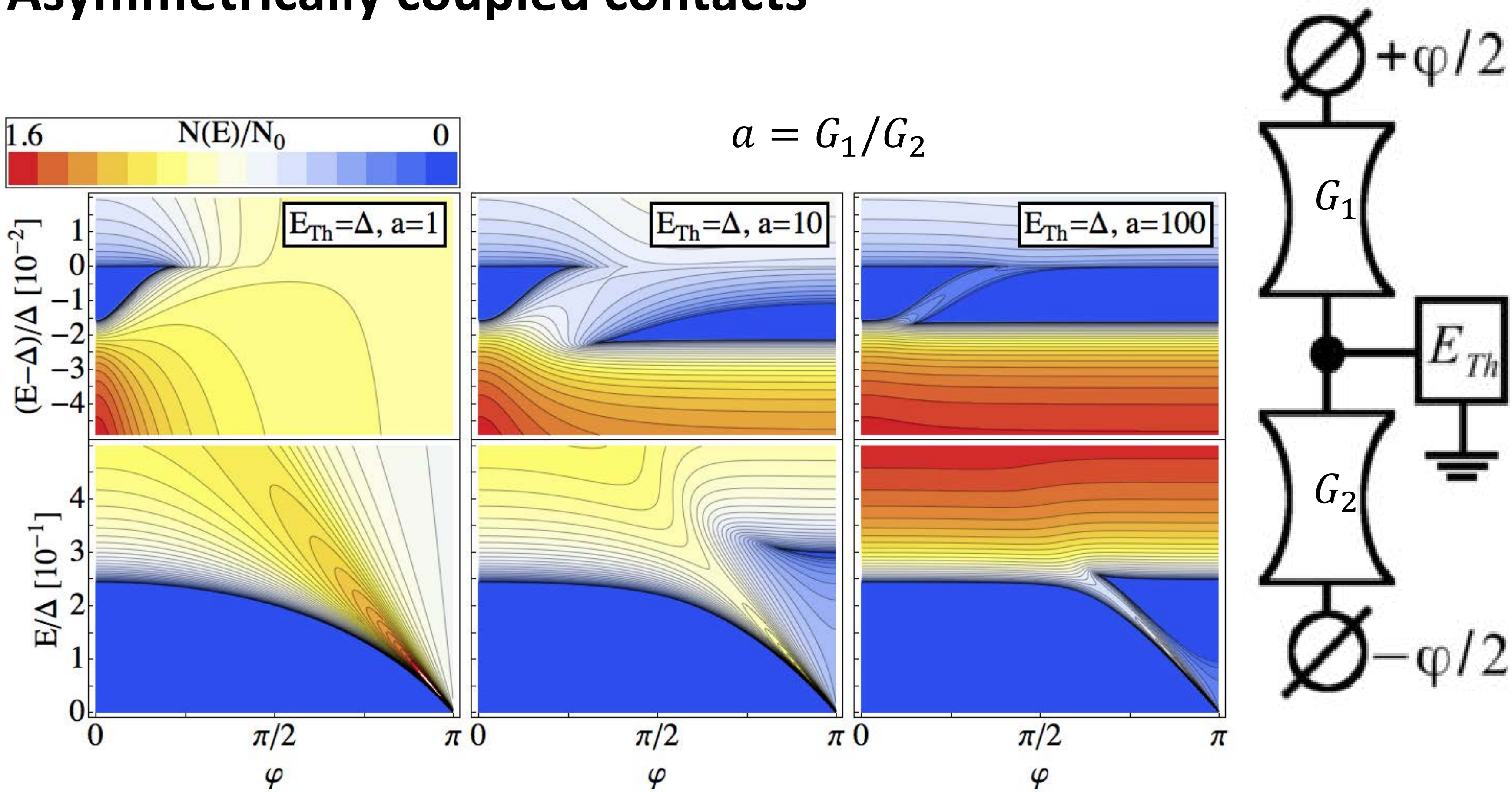
- Decreases with increasing E_{Th}
- Closes at a critical phase
- Detaches from Δ for $E_{Th} < \Delta$
- Vanishes for $E_{Th} < 0.68\ \Delta$

Universality for $E_{Th} \gg \Delta$



- Full subgap DOS scales with Δ/E_{Th}
- Size of the smile gap $\sim \Delta^3/E_{Th}^2$
- Generalization to arbitrary contacts: smile gap is related to a **gap in transmission eigenvalue distribution**!

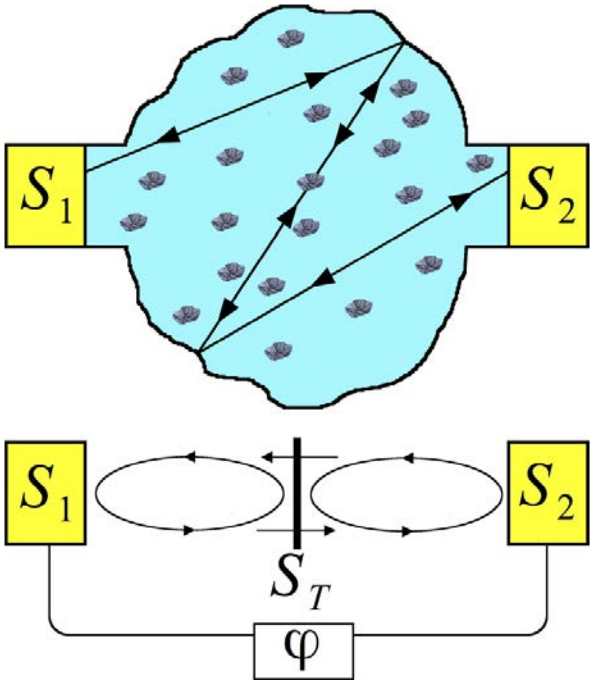
Asymmetrically coupled contacts



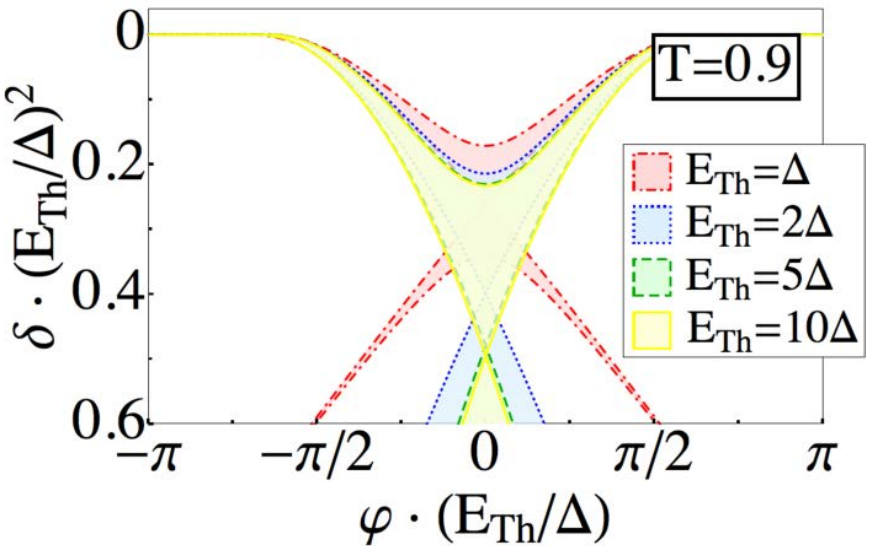
- The secondary gap ist robust against asymmetries
- Further gaps appear around phase difference π
- Phase dependent energy bands develop
- More gaps develop (e.g. centered at φ=π)

J. Reutlinger, L. Glazman, Yu. V. Nazarov, and WB, Phys. Rev. B **90**, 014521 (2014)

Simple physical picture:



Adding a scatterer with variable position

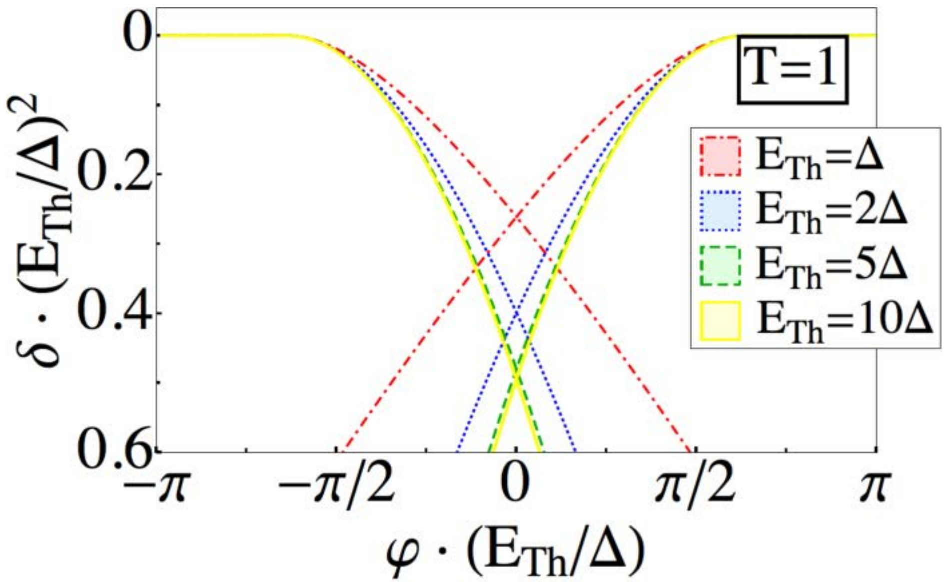


Condition for Andreev bound states

$$2E/E_{\rm Th} - 2\arccos(E/\Delta) \pm \varphi = 2\pi n$$

"Not so short" junction $E_{Th} \gg \Delta$

$$E(\varphi) = \Delta\cos(\Delta/E_{\rm Th} \pm \varphi/2)$$
$$\approx \Delta[1 - (\Delta/E_{\rm Th} \pm \varphi/2)^2/2]$$



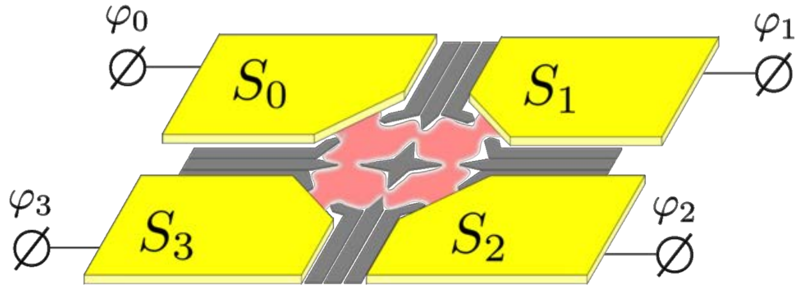
3) Topology and smile gaps in a 4-Terminal Josephson junction

T. Yokoyama, J. Reutlinger, W. Belzig, and Y. V. Nazarov

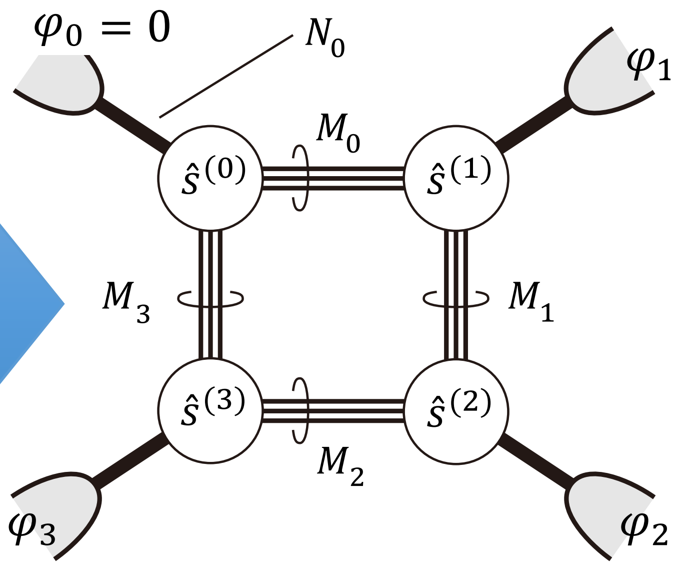
Phys. Rev. B 95, 1915 (2017)

„Smile“-Gaps in Multi-Terminal Josephson-contacts

➢ <u>4-Terminal-Josephson-Ring</u>



Scattering matrix description



M : # in internal modes

N : # of external modes

$\hat{s}^{(i)}$: random scattering matrix (COE)

$$\vec{\varphi} = (\varphi_0, \varphi_1, \varphi_2, \varphi_3)^T = (0, \varphi, 3\varphi, 6\varphi)$$

- Elimination of internal modes → effective ring scattering matrix
- Andreev levels determined by [Beenakker, Phys. Rev. Lett. **67**, 3836 (1991)]

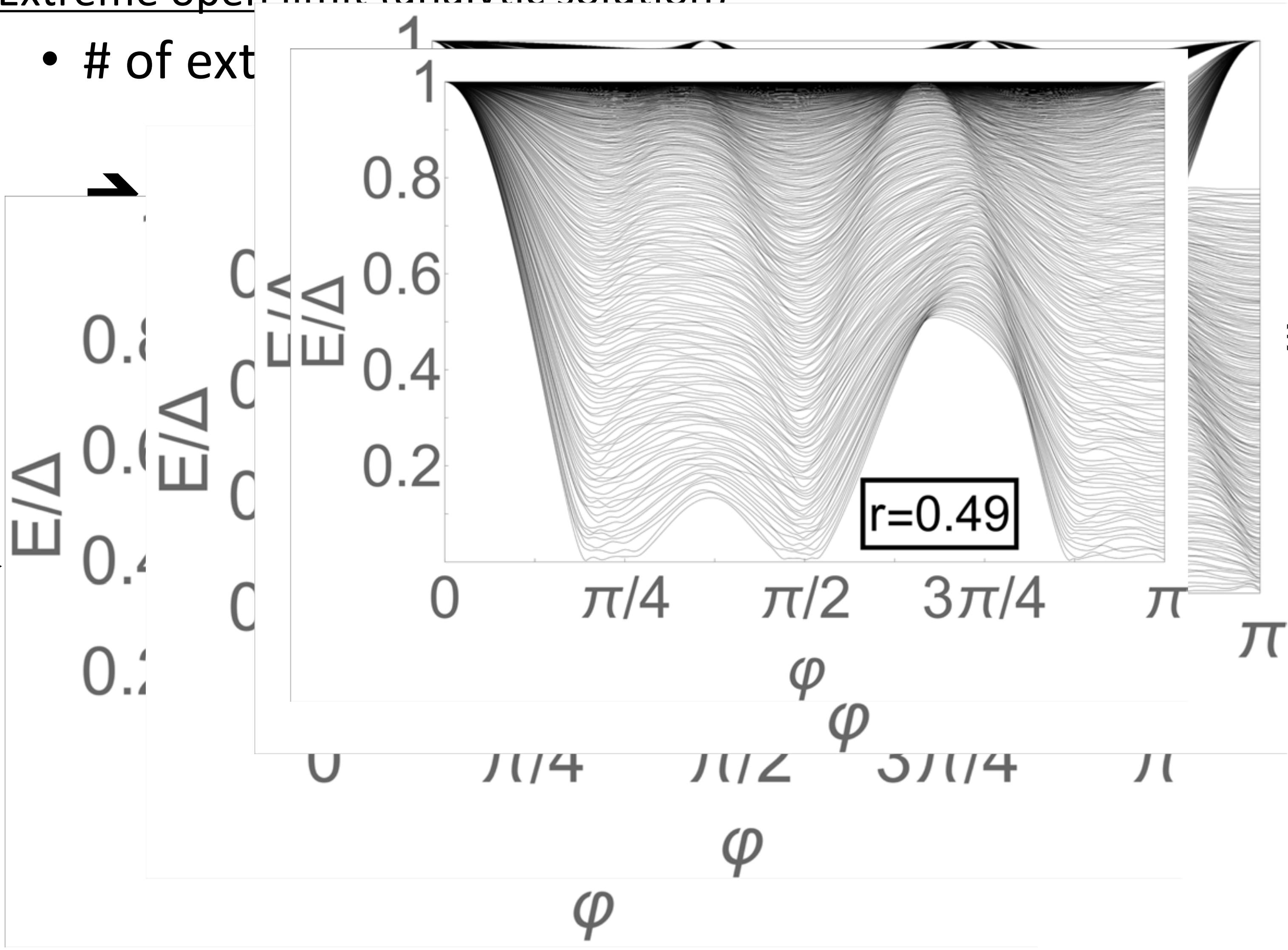
$$\det\left[e^{2i\chi} - e^{i\hat{\varphi}}\hat{S}_h(E)e^{-i\hat{\varphi}}\hat{S}_e(E)\right] = 0 \qquad \chi = \arccos E/\Delta$$

[Multi-terminal JJ: Cuevas, Pothier, PRL (2007); Freyn, Doucot, Feinberg, and Mélin, PRL (2011); van Heck, Mi, Akhmerov, PRB (2014); Riwar, Houzet, Meyer, Nazarov, Nat. Comm. (2016)]

- Extreme open limit (analytic solution)
  - # of ext

ized

e



Topological protection of minigaps (at E=0)

Quasiclassical Greens functions at E=0:

$$\hat{G}_i(E=0) = \begin{pmatrix} 0 & e^{i\eta_i} \\ e^{-i\eta_i} & 0 \end{pmatrix}$$

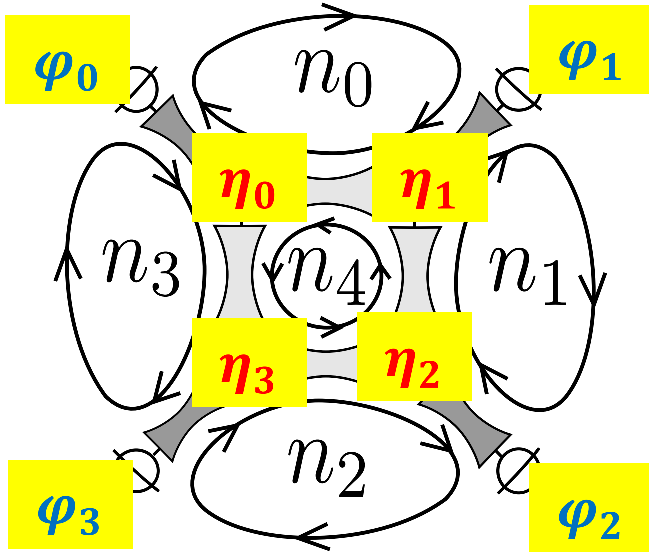
Phase differences around possible loops

$$P(\alpha) = \{\alpha/2\pi + 1/2\} - \pi \quad (\alpha \to [-\pi, \pi])$$

$$2\pi n_0 = P(\eta_0 - \varphi_0) + P(\eta_1 - \eta_0) + P(\varphi_1 - \eta_1) + \varphi_0 - \varphi_1$$

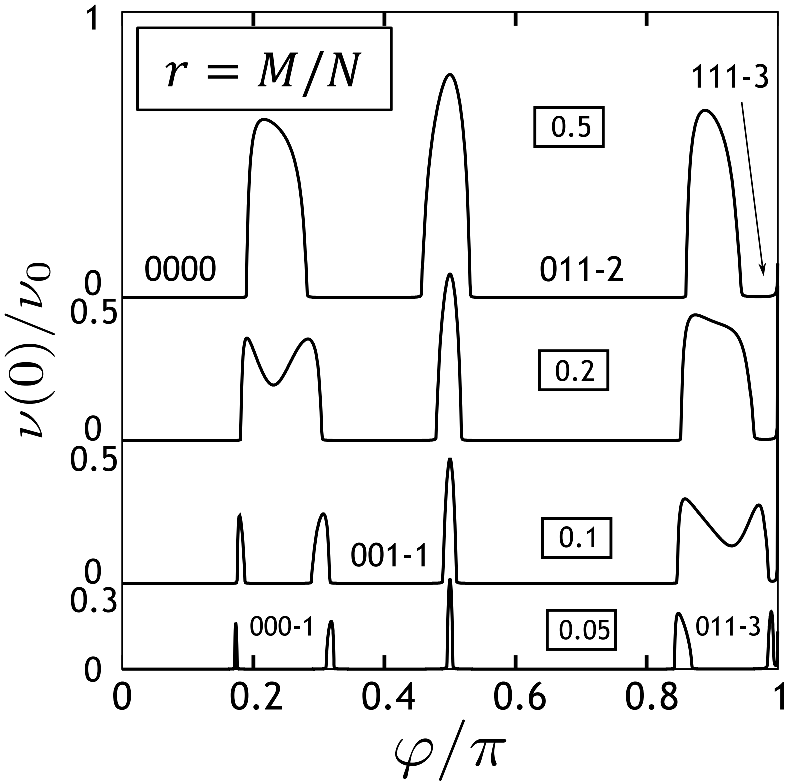
$n_{1,2,3}$ from cyclic permutations

$$n_4 = \Sigma_{i=0}^{3} n_i \in 0, \pm 1$$



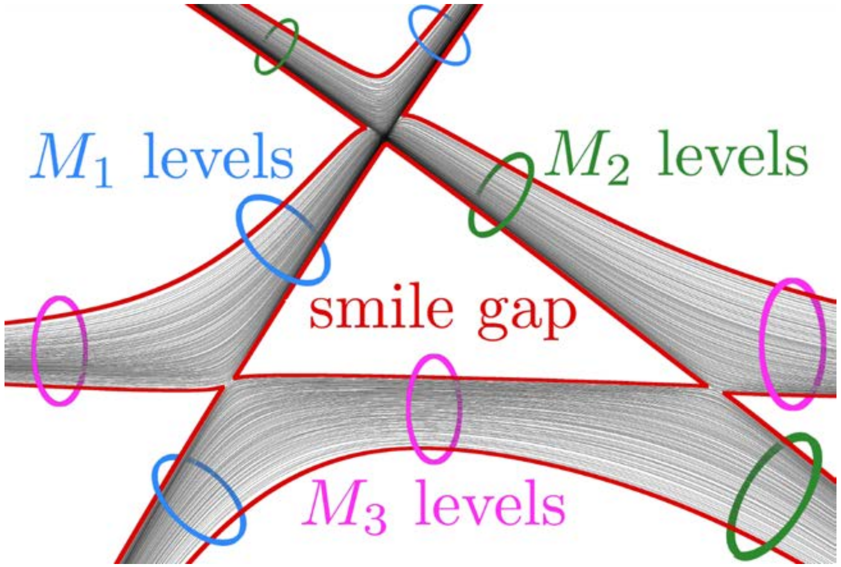
Quasiclassical calculation (circuit theory)

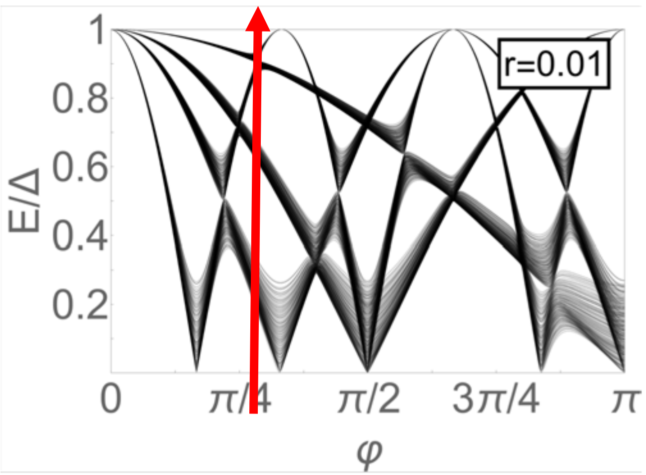
- Different topological regions characterized by a set $n_0 n_1 n_2 n_3$
- Separated by 'gapless' states
- Gaps most pronounced for 'open' case ($N \gg M$)

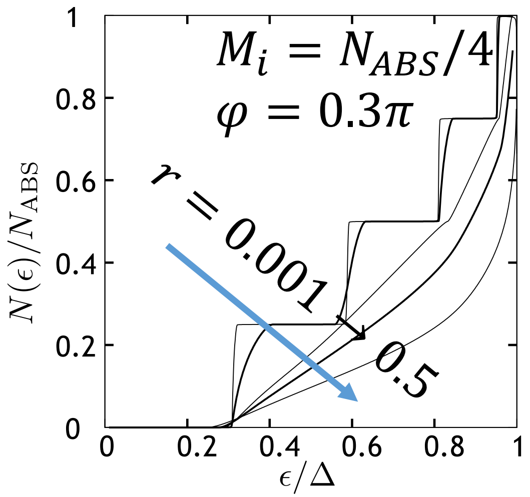


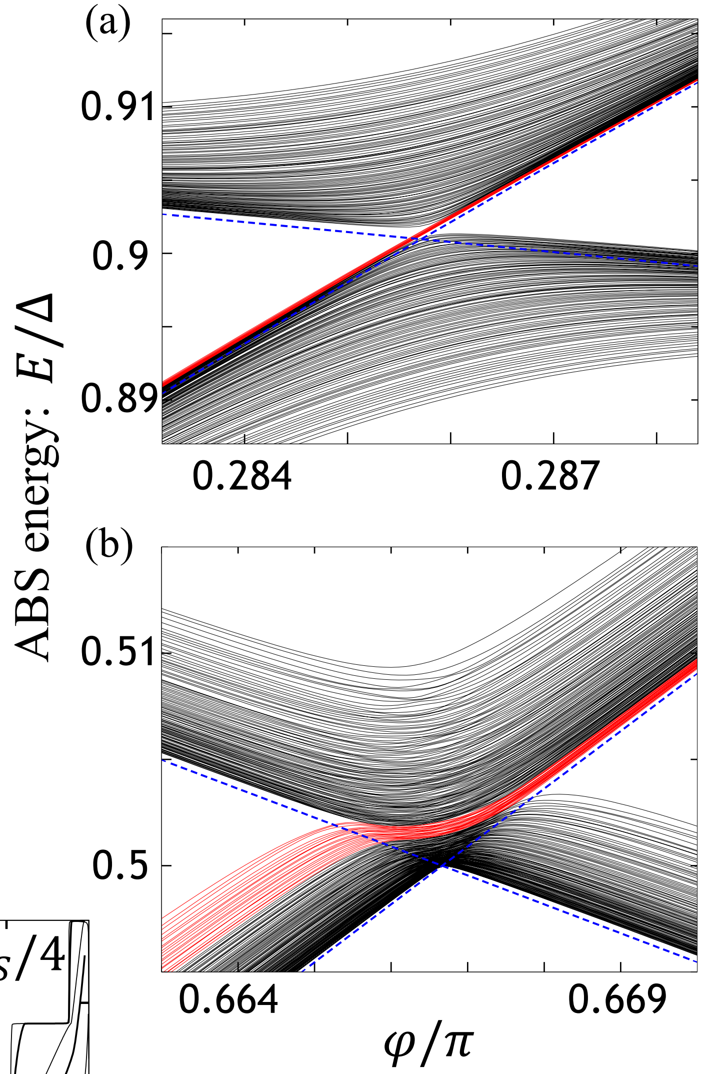
Topological protection of the smile gap

of levels in a bunch is determined by # of open transport channels in the QPCs
→ topological numbers $\{M_1, M_2, M_3, M_4\}$







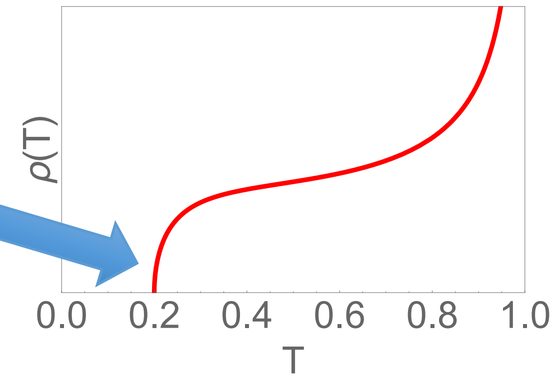


4-Terminal-Josephson ring

➢ Stray level to break topological protection

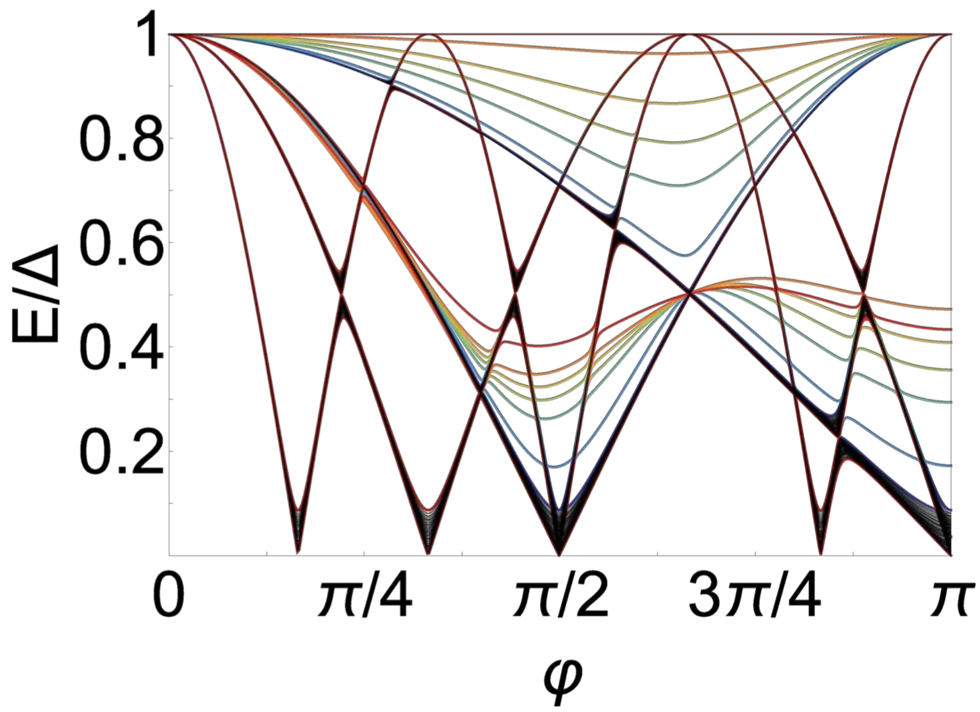
- For $M/N < 1/2$ the scattering matrices are characterized by transmission eigenvalue distributions with a minimal transmission eigenvalue T_c.

$$T_c = \frac{(N-2M)^2}{(N+2M)^2}$$



➔ Introducing one extra transmission eigenvalues T_{ext} in the interval $[0, T_c]$ leads to stray level

T_{ext} between 0 (red) and T_c (blue) at $S^{(1)}$ (numerical)



- Smile gaps are protected by the gap in the transmission eigenvalue distribution
- Smile gaps for ballistic quantum point contacts

Summary

- Proximity density of states still has surprises beyond minigap
- Secondary gap feature at E<Δ in a cavity between superconductors with $E_{Th} > \Delta$ [1]
- Phase-dependent closing in a "Smile"-shape → smile gap
- Robust against asymmetries, weak spatial dependence and weak backscattering [2]
- The level number in the DOS between minigap and smile gap is set by the number of open transport channels [3]
- Fluctuations of the smile gap are universal
- Multi-terminal Josephson junctions provide a rich structure of multiple gaps (smile and mini) [4]
- Gaps are protected by different levels of topology

[1] J. Reutlinger, L. Glazman, Yu. V. Nazarov, W.B. Phys. Rev. Lett. **112**, 067001 (2014)

[2] J. Reutlinger, L. Glazman, Yu. V. Nazarov, W.B. Phys. Rev. B **90**, 014521 (2014)

[3] J. Reutlinger, L. Glazman, Yu. V. Nazarov, W.B. (unpublished)

[4] T. Yokoyama, J. Reutlinger, W. B., and Y. V. Nazarov, arXiv:1609.05455

Summary in pictures

[1] J. Reutlinger, L. Glazman, Yu. V. Nazarov, W.B. Phys. Rev. Lett. **112**, 067001 (2014)
[2] J. Reutlinger, L. Glazman, Yu. V. Nazarov, W.B. Phys. Rev. B **90**, 014521 (2014)
[3] J. Reutlinger, L. Glazman, Yu. V. Nazarov, W.B. (unpublished)
[4] T. Yokoyama, J. Reutlinger, W. B., and Y. V. Nazarov, arXiv:1609.05455