Quantum Geometry of Josephson Matter

(and how to probe it)

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[arXiv:1810.11277]

The University of Konstanz has been successful in the German Excellence Initiative since 2007.

Outline

- Topology in solid state physics
- Geometry of quantum states
- Andreev bound states and topology in multiterminal Josephson junctions
- Microscopic model of a multiterminal Josephson junction.
- Weyl nodes and topologically nontrivial phases
- Microwave spectroscopy of Andreev bound states
- Accessing the quantum geometry by polarized μw-spectroscopy in multiterminal Josephson junctions

Q1: Construction of topological Josephson matter?

Q2: How to access the topology with microwaves?

Topology in solid state physics

[M. Z. Hasan, C. L. Kane, RMP (2010)]

- Intrinsic geometrical properties of quantum states in periodic crystals.
- Quantized Hall conductance:
  2DEG placed in a strong magnetic field

  [von Klitzing et al., *PRL* (1980).]

$$\sigma_{xy} = N\frac{e^2}{h}$$

Kubo formula: $N = \sum_{m \in occ} n_m$

with $n_m = \frac{1}{2\pi}\int \mathrm{d}^2\boldsymbol{k}\, F_m(\boldsymbol{k})$

n_m: Chern number of the m−th **occupied** band
$F_m(\boldsymbol{k})$: Berry curvature [E.g. Thouless, PRL 1982]

- Chern number: robust topological **invariant** (does **not** change for **smooth** deformations)
- Topological insulators, topological superconductors, topological quantum computing, etcetc.

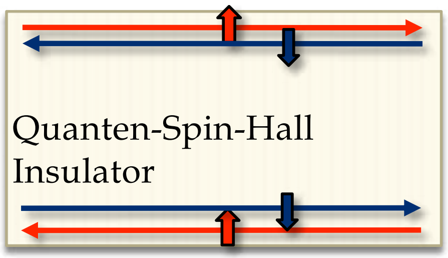
3D Strong Topological Insulators (3D-STI)

Topological Insulators are bulk insulators, but possess metallic surface states, which are topologically protected by the band gap

2D Topological Insulators

Broken time-reversal symmetry:

Time-reversal invariant



Time-reversed pairs of boundary states

Experimental Realizations of a 3D-STI: Bi_2Se_3, Bi_2Te_3

Dispersion of Bi_2Se_3

ARPES-Diagram

[Hsieh et al, *Nature*, **2009**, *460*, 1101]

Spin-Orbit Coupling!

- SSB: Surface-state band
- BVB: Bulk valence band

[Chen et al, *Science*, **2009**, *325*, 178]

At Γ-Point: Dirac-Fermions in 2D

$$\mathcal{H}_{\text{surface}} = \hbar v_F \boldsymbol{\sigma} \cdot (\hat{\boldsymbol{e}}_z \times \boldsymbol{k}) = \hbar v_F (k_y \sigma_x - k_x \sigma_y)$$

SARPES= spin- and angle-resolved PES

Electronic Structure of the Topological Insulator $Bi_2 Se_3$ Using Angle-Resolved Photoemission Spectroscopy: Evidence for a Nearly Full Surface Spin Polarization

Z.-H. Pan, E. Vescovo, A. V. Fedorov, D. Gardner, Y. S. Lee, S. Chu, G. D. Gu, and T. Valla
Phys. Rev. Lett. **106**, 257004 – Published 22 June 2011

Spin-resolved (circulary polarized) light absorption probes the topology!

Quantum geometry

- Quantum states $|\psi(\boldsymbol{\phi})\rangle$ of Hamiltonian $H(\boldsymbol{\phi})$ depend on set of parameters $\boldsymbol{\phi}$
- Berry 1984: Phase acquired during an adiabatic evolution of states can have observable consequences. [Berry, *Proc. R. Soc. London, Ser. A* **392**, 45–57 (1984)]
- Question: **adiabiatic evolution** of states on the **complex manifold of quantum states** while changing parameters $\{\phi_j\}$ **smoothly**? What is the **invariant "distance"** $\mathrm{d}s$ between nearby state $|\psi(\boldsymbol{\phi})\rangle$ and $|\psi(\boldsymbol{\phi}+\mathrm{d}\boldsymbol{\phi})\rangle$?
- Distance: $\mathrm{d}s^2 = 1 - |\langle\psi(\boldsymbol{\phi})|\psi(\boldsymbol{\phi}+d\boldsymbol{\phi})\rangle|^2 = \sum_{jk}\chi_{jk}\,\mathrm{d}\phi_j\,\mathrm{d}\phi_k$
- Quantum geometric tensor (hermitian):

$$\chi_{jk} = \left\langle\partial_{\phi_j}\psi\middle|(1-|\psi\rangle\langle\psi|)\middle|\partial_{\phi_k}\psi\right\rangle = g_{jk} - \frac{i}{2}F_{jk}$$

Symmetric part: Fubini-Study **quantum metric** (measures distance between physical states)

Antisymmetric part: **Berry curvature** (contains information about phase)

- The geometrical phase φ_{geo} acquired along a path P is obtained from

$$\boldsymbol{A} = i\langle\psi_0|\partial_{\boldsymbol{\phi}}|\psi_0\rangle \quad \text{(Berry connection)}$$

- Gauge-invariant field strength $F_{jk} = \partial_j A_k - \partial_k A_j$, the so-called Berry curvature, from which we obtain the **first Chern number**:

$$C = \frac{1}{2\pi}\oiint_S F_{jk}\,\mathrm{d}\lambda_j\,\mathrm{d}\lambda_k \in \mathbb{Z}$$

[M. Kolodrubetz et al., *Phys. Rep.* **697**, 1-87 (2017)]

3D example: Two-level system

- Spin-½ $\boldsymbol{\sigma}$ in a magnetic field $\boldsymbol{d}(\theta, \varphi)$, absolute value $d = |\boldsymbol{d}|$
- Hamiltonian $H = \boldsymbol{d} \cdot \boldsymbol{\sigma}$, energies $\varepsilon_{\pm} = \pm d(\theta, \varphi)$, states $|+\rangle = \begin{pmatrix} \cos\theta/2 \\ e^{i\varphi} \sin\theta/2 \end{pmatrix}$

- Berry connection: $A_\theta = 0, A_\varphi = -\cos\frac{\theta}{2}$
- Berry curvature: $F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = \frac{1}{2}\sin\frac{\theta}{2}$

Vector potential $\boldsymbol{A}$ of a Dirac monopole of strength ½ and total flux of 2π.

- Quantum geometric tensor: $\chi_{jk} = g_{jk} - \frac{i}{2} F_{jk}$ with

$$(g_{jk}) = \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}, \quad (F_{jk}) = \frac{\sin\theta}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

metric of a sphere — Berry curvature

Chern number:

$$C = \frac{1}{2\pi}\int_0^\pi \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi\, F_{\theta\varphi} = 1$$

Phase space

φ_{geo}

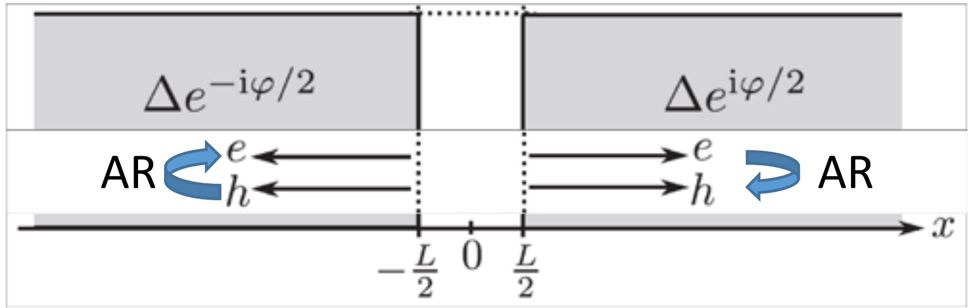
Bloch sphere (M)

[M. Nakahara, *Differentialgeometrie, Topologie und Physik* (Springer Spektrum, Berlin, Heidelberg, ed. 2, 2015).]

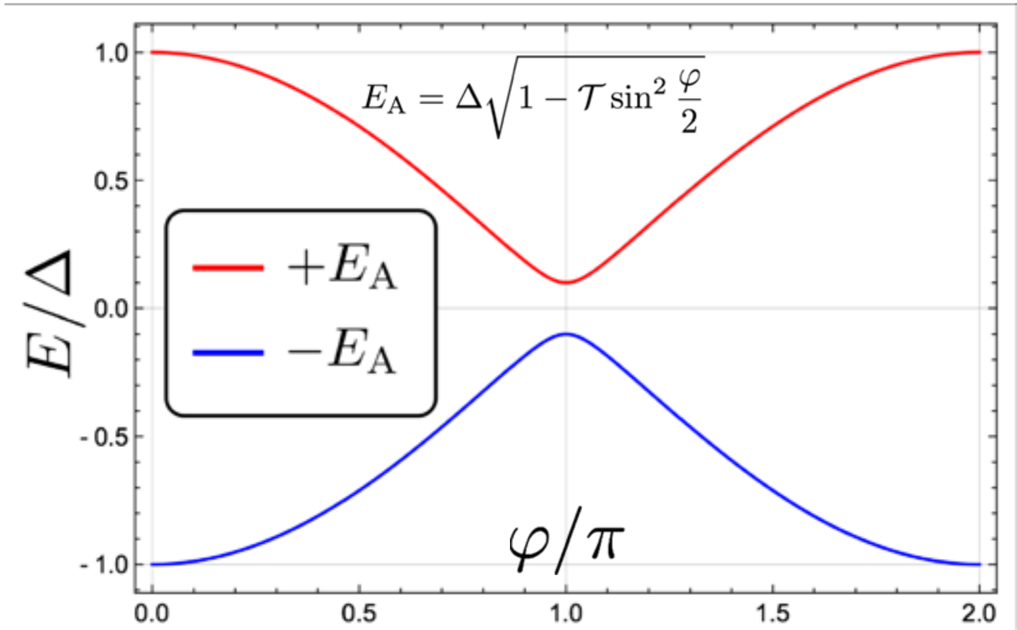
Andreev bound states

ABS= Bogolubov-Quasiparticle
in a Josephson junction

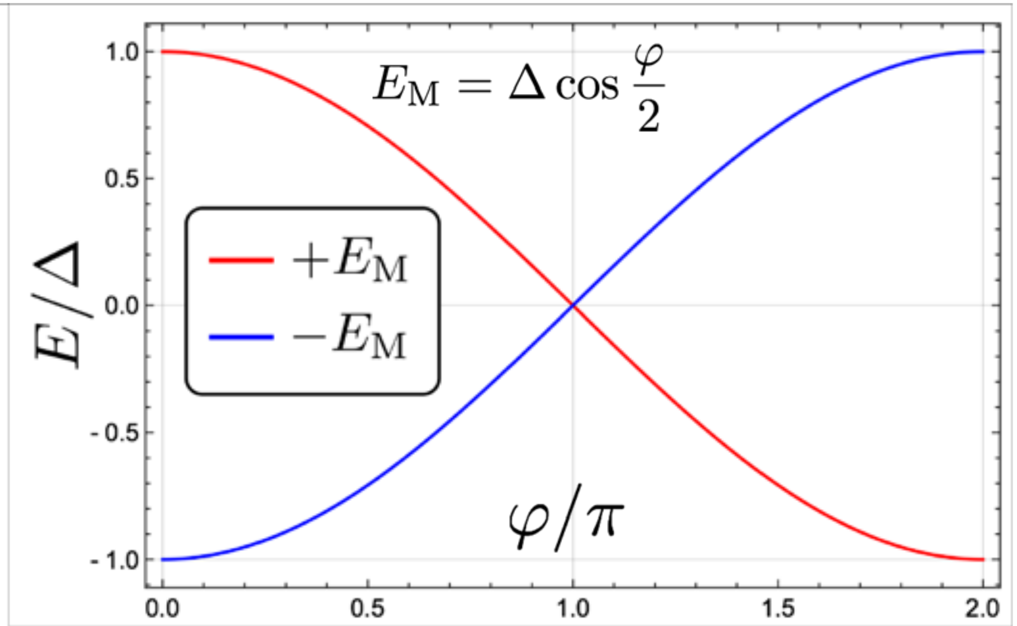
- Electron-hole conversion
- Phase sensitive
- Phase dependent energy
- Current carrying states
  → supercurrent



Conventional:



Topological (Majorana):



Topology in multiterminal Josephson junctions

- Scattering matrix $\hat{S}$ connecting superconductors $0,1,2,\ldots,n-1$
- Andreev bound states (ABS) determined by scattering formalism from
  $$\det\left[1 + e^{-2i\chi(E)}\hat{S}(E)e^{i\hat{\Phi}}\hat{S}^*(-E)e^{-i\hat{\Phi}}\right] = 0$$
- For $n = 4$ ABS live in „3D"-Brillouin zone $\boldsymbol{\phi} = (\phi_1 \quad \phi_2 \quad \phi_3)$:
  $H_{eff} \sim \mathbf{d}(\boldsymbol{\phi})\hat{\boldsymbol{\tau}}$ with $\hat{\boldsymbol{\tau}}$ = Pauli-matrices in Nambu space
- Dispersion <u>might</u> have **zero-crossings → topological** transition $\mathbf{d}(\boldsymbol{\phi}_W) = \mathbf{0}$
  Low energy (for $\boldsymbol{\phi}^W$): **Weyl-Hamiltonian** $H_{eff} \sim \boldsymbol{\Delta\phi}\hat{\boldsymbol{\tau}}$ with $\boldsymbol{\Delta\phi} = \boldsymbol{\phi} - \boldsymbol{\phi}^W$

[Riwar, Meyer, Houzet, Nazarov, *Nat. Commun.* **7**, 11167 (2016),
See also van Heck, Mi, Akhmerov, PRB 2014]

Topology in multiterminal Josephson junctions

- Chern number $C = \sum_{k\sigma} C_k (n_{k\sigma} - \frac{1}{2})$ with occupation $n_{k\sigma}$ and

$$C = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi_1 \int_{-\pi}^{\pi} d\phi_2 \; F_{12}$$

- Quantized transconductance:

$$G^{12} = \frac{dI_1}{dV_2} = -\frac{4e^2}{h} C$$

C

[R.-P. Riwar et al., *Nat. Commun.* **7**, 11167 (2016).]

Topology in multiterminal Josephson junctions

- Random symmetric scattering matrices
- Only about 5% of them show Weyl points.

  Linear response: $C = \frac{h}{8e^2}(G_{12} - G_{21})$

[Meyer&Houzet, PRL 2017,
Also H&M, arxiv:1810.09962]

[Eriksson et al., *PRB* **95**, 075417 (2017)]

Problem:

- Quantization of the transconductance for low voltages $\mathrm{V} \sim 10^{-2}\Delta/e$ (Al: $\Delta \approx 180\mu\mathrm{eV} \Rightarrow V \approx 1.8\mu\mathrm{V}$), masked by supercurrent peak
- Difficult to construct (…random scattering matrix)

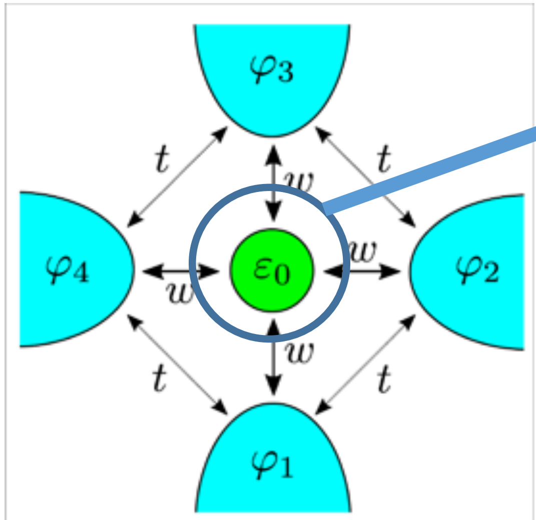
Q1: Construction of topological Josephson matter?

- Construction scheme?
- Materials?
- Realistic parameters?

Microscopic model of multiterminal junctions

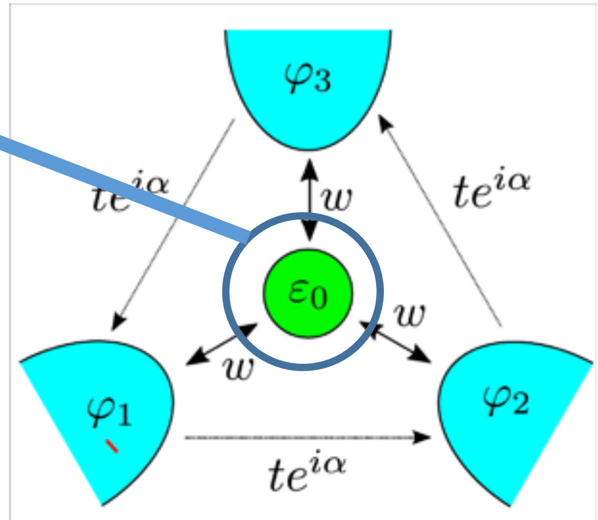
- **Question**: What are the minimal ingredients to obtain nontrivial topological Andreev quantum states?

- Two proposed microscopic models of multiterminal Josephson junctions:



$$H_{\mathrm{D}} = \boldsymbol{d}(\boldsymbol{\phi}) \cdot \boldsymbol{\tau}$$

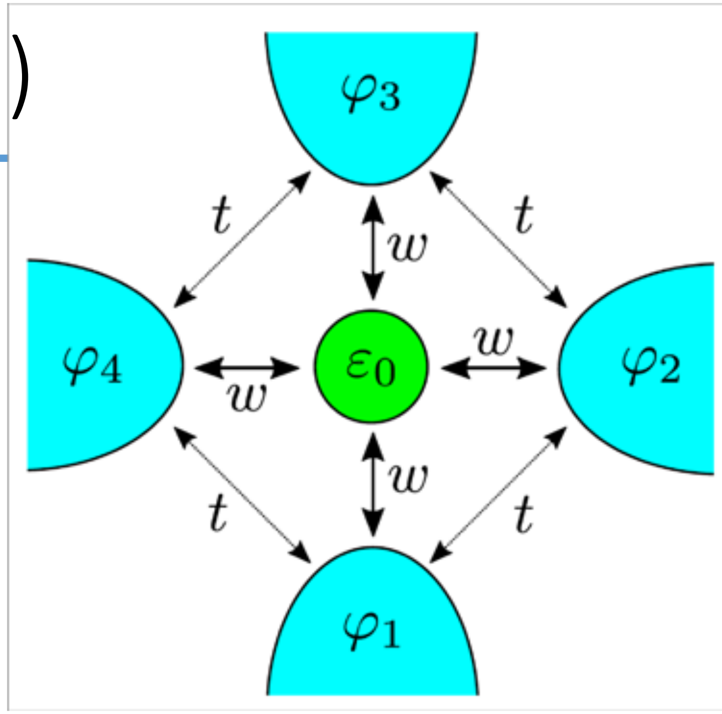
$$\boldsymbol{\phi} = (\varphi_1, \varphi_2, \varphi_3)$$



Hamiltonian of the coupled dot can be mapped to a pseudo-spin in an effective magnetic field!

[Klees, Rastelli, Cuevas, WB, arXiv:1810.11277]

4-terminal Josephson junction (I)



- Total Hamiltonian:

$$H = H_{\mathrm{D}} + \sum_{j=1}^{4} \left(H_{\mathrm{S}}^{(j)} + H_{\mathrm{S-D}}^{(j)} + H_{\mathrm{S-S}}^{(j,j+1)} \right)$$

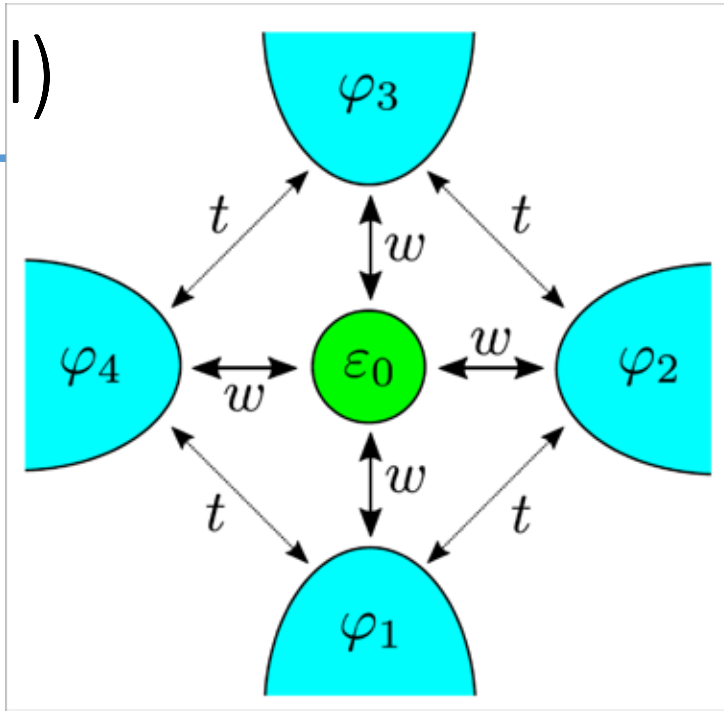
- Dot: $H_{\mathrm{D}} = \varepsilon_0 \sum_\sigma d_\sigma^\dagger d_\sigma$

- Leads: $H_{\mathrm{S}}^{(j)} = \sum_{\boldsymbol{k}\sigma} \xi_k c_{j\boldsymbol{k}\sigma}^\dagger c_{j\boldsymbol{k}\sigma} + \Delta \sum_k (e^{i\varphi_j} c_{j\boldsymbol{k}\uparrow}^\dagger c_{j(-\boldsymbol{k})\downarrow}^\dagger + \mathrm{H.c.})$

- Lead-dot: $H_{\mathrm{S-D}}^{(j)} = w \sum_{\boldsymbol{k}\sigma} (c_{j\boldsymbol{k}\sigma}^\dagger d_\sigma + \mathrm{H.c.})$

- Lead-Lead: $H_{\mathrm{S-S}}^{(j,j+1)} = t \sum_{\boldsymbol{k}\sigma} (c_{j\boldsymbol{k}\sigma}^\dagger c_{(j+1)\boldsymbol{k}\sigma} + \mathrm{H.c.})$

4-terminal Josephson junction (II)



- Limit of large superconducting gap: $\Delta \to \infty$
- Weak coupling between the leads: $t \ll w$
- Greens function calculation
- Effective low-energy Hamiltonian: $H_{\mathrm{D}} = \boldsymbol{d} \cdot \boldsymbol{\tau}$

Pseudo-spin: Pauli matrices in particle-hole space $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$

Effective magnetic field controlled by superconducting phases:

$$\boldsymbol{d}(\phi) = \begin{pmatrix} \Gamma \sum_j \cos\varphi_j \\ -\Gamma \sum_j \sin\varphi_j \\ \varepsilon_0 - 2\, t_0\, \Gamma \sum_j \cos(\varphi_j - \varphi_{j+1}) \end{pmatrix}$$

Definitions:

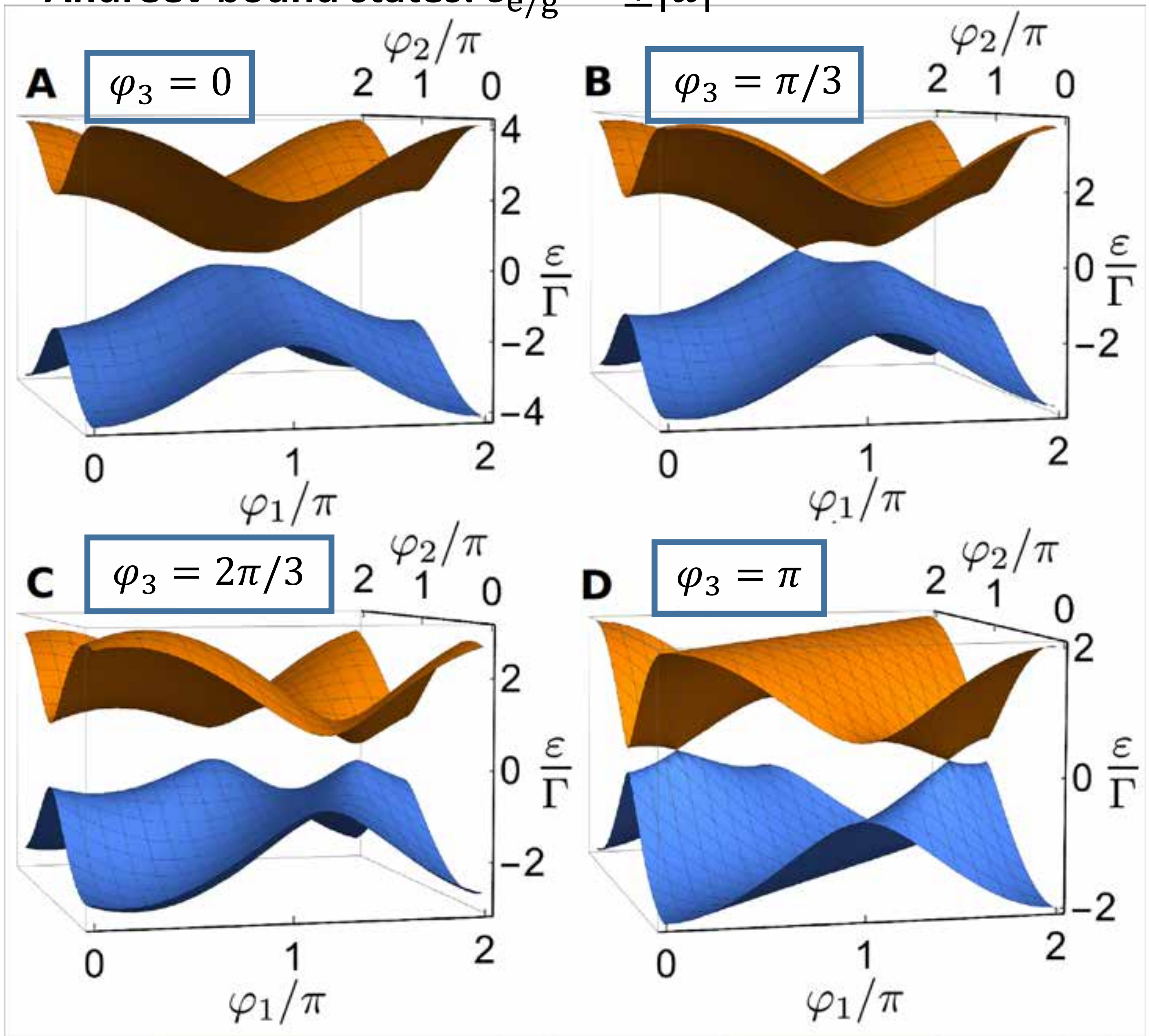
$\Gamma = \pi\, N_0\, w^2$

$t_0 = \pi\, N_0\, t$

[Klees, Rastelli, Cuevas, WB, arXiv:1810.11277]

4-terminal Josephson junction (III)

Andreev bound states: $\varepsilon_{\mathrm{e/g}} = \pm|\boldsymbol{d}|$



Weyl points ($d = 0$):

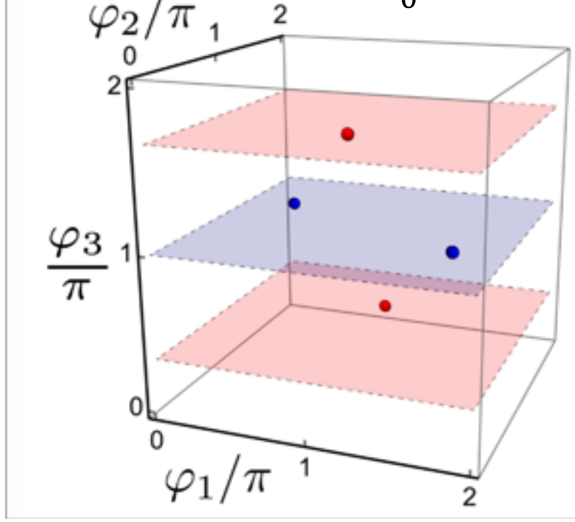
$\boldsymbol{\varphi}_{\mathrm{W}}^{(1)} = (-\delta, \pi - \delta, \pi)$

$\boldsymbol{\varphi}_{\mathrm{W}}^{(2)} = (\delta, \delta - \pi, \pi)$

$\boldsymbol{\varphi}_{\mathrm{W}}^{(3)} = (\pi, \pi - \delta, -\delta)$

$\boldsymbol{\varphi}_{\mathrm{W}}^{(4)} = (\pi, \delta - \pi, \delta)$

$\delta = \arccos(1 + \frac{\epsilon_0}{4t_0\Gamma})$



Parameters: $t_0 = 0.1$

$\frac{\varepsilon_0}{\Gamma} = -0.2, \varphi_4 = 0$

Klees et al., unpublished

4-terminal Josephson junction (III)

Ground state geometry: Normalized effective field $\boldsymbol{n} = \boldsymbol{d}/d$

Metric tensor: $$g_{jk} = \frac{1}{4}(\partial_j \boldsymbol{n}) \cdot (\partial_k \boldsymbol{n})$$

Berry curvature: $$F_{jk} = \frac{1}{2}\, n \cdot [(\partial_j \boldsymbol{n}) \times (\partial_k \boldsymbol{n})]$$

Chern number: $$C(\varphi_3) = \frac{1}{2\pi} \iint_0^{2\pi} \mathrm{d}\varphi_1\, \mathrm{d}\varphi_2\, F_{12}$$

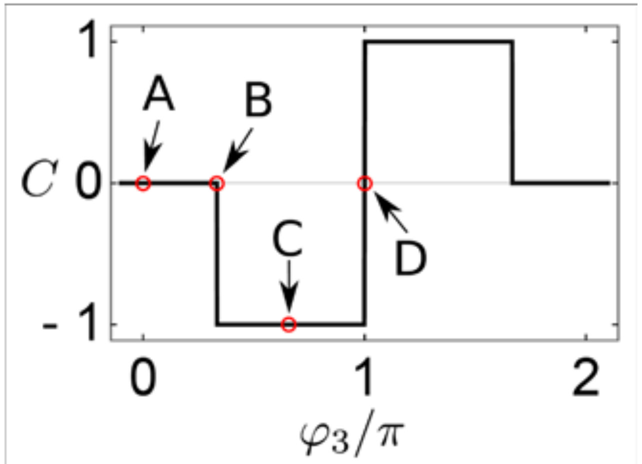
Weyl points:

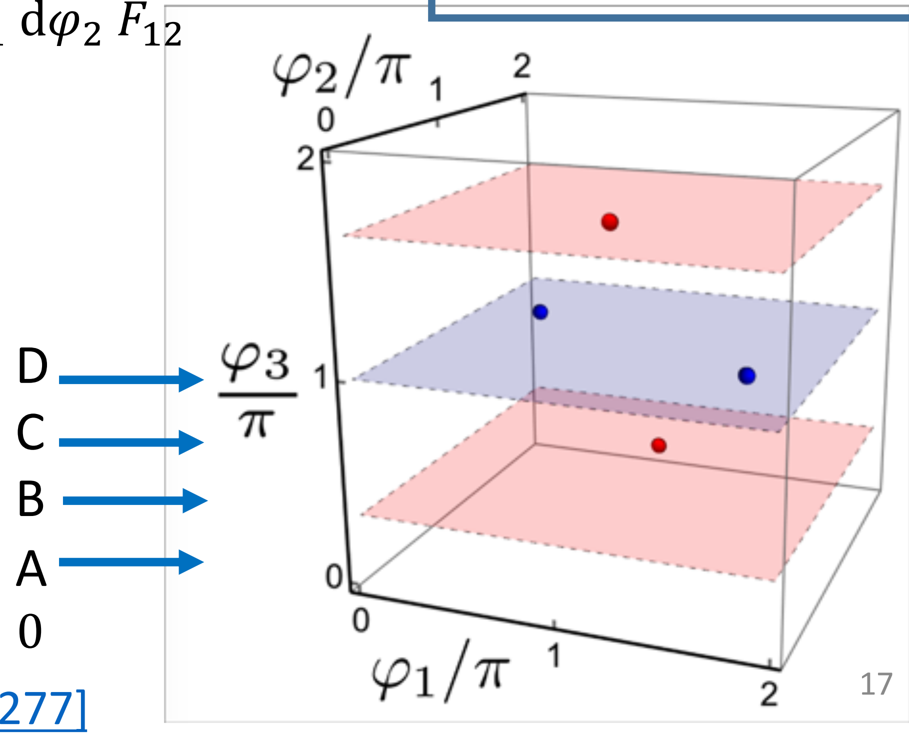
$\boldsymbol{\varphi}_{\mathrm{W}}^{(1)} = (-\delta, \pi - \delta, \pi)$

$\boldsymbol{\varphi}_{\mathrm{W}}^{(2)} = (\delta, \delta - \pi, \pi)$

$\boldsymbol{\varphi}_{\mathrm{W}}^{(3)} = (\pi, \pi - \delta, -\delta)$

$\boldsymbol{\varphi}_{\mathrm{W}}^{(4)} = (\pi, \delta - \pi, \delta)$





Parameters: $t_0 = 0.1, \frac{\varepsilon_0}{\Gamma} = -0.2, \varphi_4 = 0$

[Klees, Rastelli, Cuevas, WB, arXiv:1810.11277]

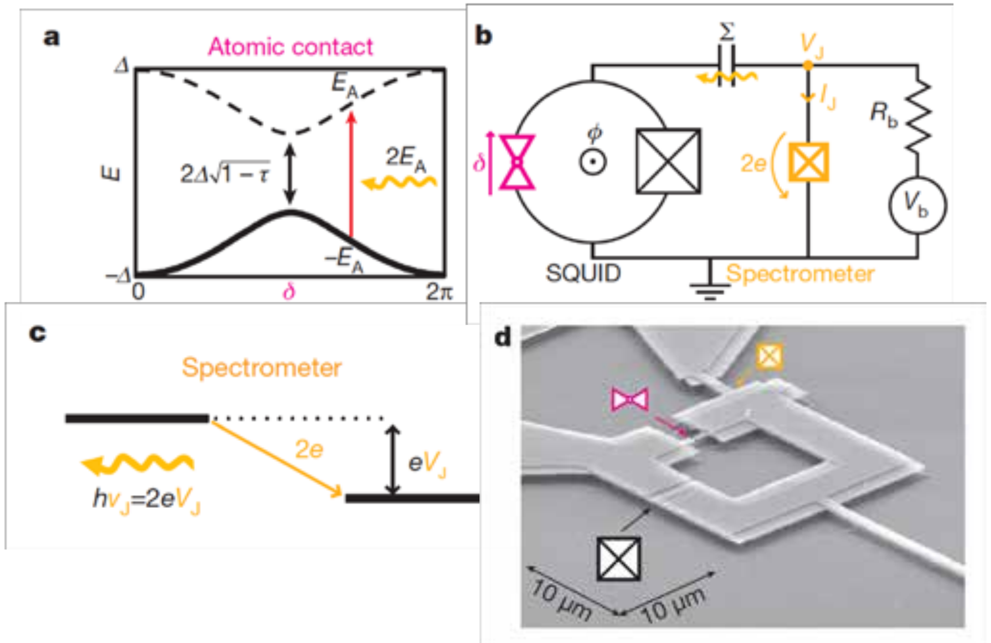
Microwave spectroscopy of ABS

- How to observe Andreev bound states in Josephson junctions without contacts? → RF-SQUID
- Advantage: voltage noise and (some) other noise sources can be avoided.

Microwave spectroscopy of conventional ABS

Setup in two-terminal junction:

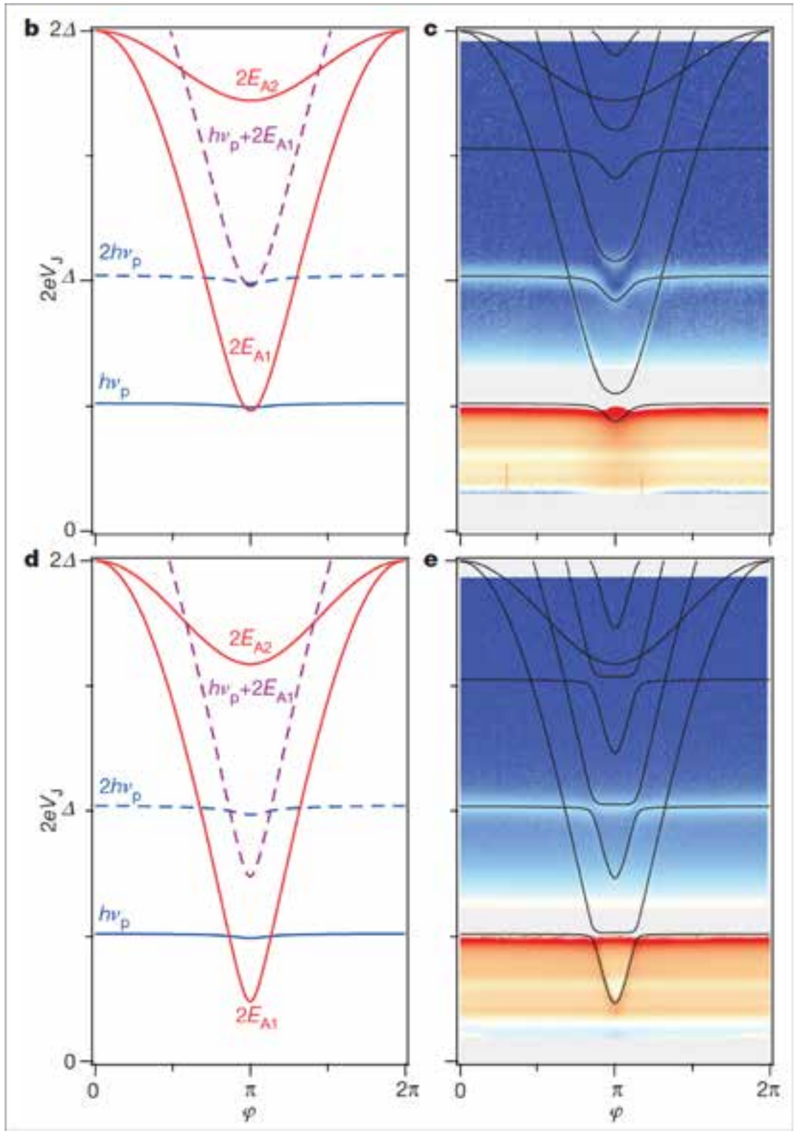
$$E_{\mathrm{A}} = \Delta\sqrt{1 - \tau\, sin^2(\delta/2)}$$



[L. Bretheau et al., *Nature* **499**, 312–315 (2013).]

Microwave absorbtion for ABS energies!

Microwave absorption spectra:



Q2: How to access the topology with microwaves?

1. Microwave excitation leads to transitions, but frequencies are not sensitive to quantum geometry (still information about zero crossings, see e.g. Varynen, Rastelli, WB, Glazman, PRB 2016 or quasiparticle dynamics, see Klees, Rastelli, WB PRB 2017)

2. **Key idea:** Circularly or linearly polarized light is topologically equivalent to the variation of two parameters with a controlled phase difference!

General protocol to access quantum geometry

- Hamiltonian $H(\boldsymbol{\phi})$ which depends on set of parameters $\boldsymbol{\phi} = \{\phi_j\}$
- Recall: Quantum geometric tensor $\chi_{jk} = g_{jk} - \frac{i}{2} F_{jk}$ with $g_{jj} \sim \left| \left\langle \psi \middle| \partial_{\phi_j} \psi \right\rangle \right|^2$
- **Drive of one parameter:** $\phi_j \rightarrow \phi_j + (2E/\hbar\omega)\cos(\omega t)$, $\mathrm{E}/\hbar\omega \ll 1$
- Fermi's Golden Rule:

$$R_{jj}(\omega) = \frac{2\pi}{\hbar} \frac{E^2}{(\hbar\omega)^2} \left| \left\langle f \middle| \frac{\partial H}{\partial \phi_j} \middle| i \right\rangle \right|^2 \delta(E_f - E_i - \hbar\omega)$$

Equivalence to quantum geometry by: $\left\langle f \middle| \frac{\partial H}{\partial \phi_j} \middle| i \right\rangle = (E_i - E_f) \left\langle f \middle| \partial_{\phi_j} i \right\rangle$

$$\Rightarrow \bar{R}_{jj} = \int d\omega \, R_{jj}(\omega) = \frac{2\pi \, E^2}{\hbar^2} g_{jj}$$

Intensity ≙ Diagonal elements of the quantum metric!

[see also Ozawa et al., *Phys. Rev. B* **97**, 201117(R) (2018).]

General protocol to access quantum geometry

- Hamiltonian $H(\boldsymbol{\phi})$ depending on set of parameters $\boldsymbol{\phi} = \{\phi_j\}$
- Recall: Quantum geometric tensor of the ground state $\chi_{jk} = g_{jk} - \frac{i}{2} F_{jk}$
- **Drive of two parameters:** with a relative phase γ

$$\phi_j \to \phi_j + (2E/\hbar\omega)\cos(\omega t),$$
$$\phi_k \to \phi_k + (2E/\hbar\omega)\cos(\omega t - \gamma)$$

- Fermi's Golden Rule (generalized)

$$R_{jk}^{(\gamma)}(\omega) = \frac{2\pi}{\hbar}\frac{E^2}{(\hbar\omega)^2}\left|\left\langle f\left|\frac{\partial H}{\partial \phi_j} + e^{i\gamma}\frac{\partial H}{\partial \phi_k}\right|i\right\rangle\right|^2 \delta(E_f - E_i - \hbar\omega)$$

Compare different „polarizations“:

$$\Rightarrow \bar{R}_{jk}^{(0)} - \bar{R}_{jk}^{(\pi)} = \frac{8\pi\, E^2}{\hbar^2} g_{jk}$$

off-diagonal metric

$$\bar{R}_{jk}^{(\pi/2)} - \bar{R}_{jk}^{(-\pi/2)} = \frac{4\pi\, E^2}{\hbar^2} F_{jk}$$

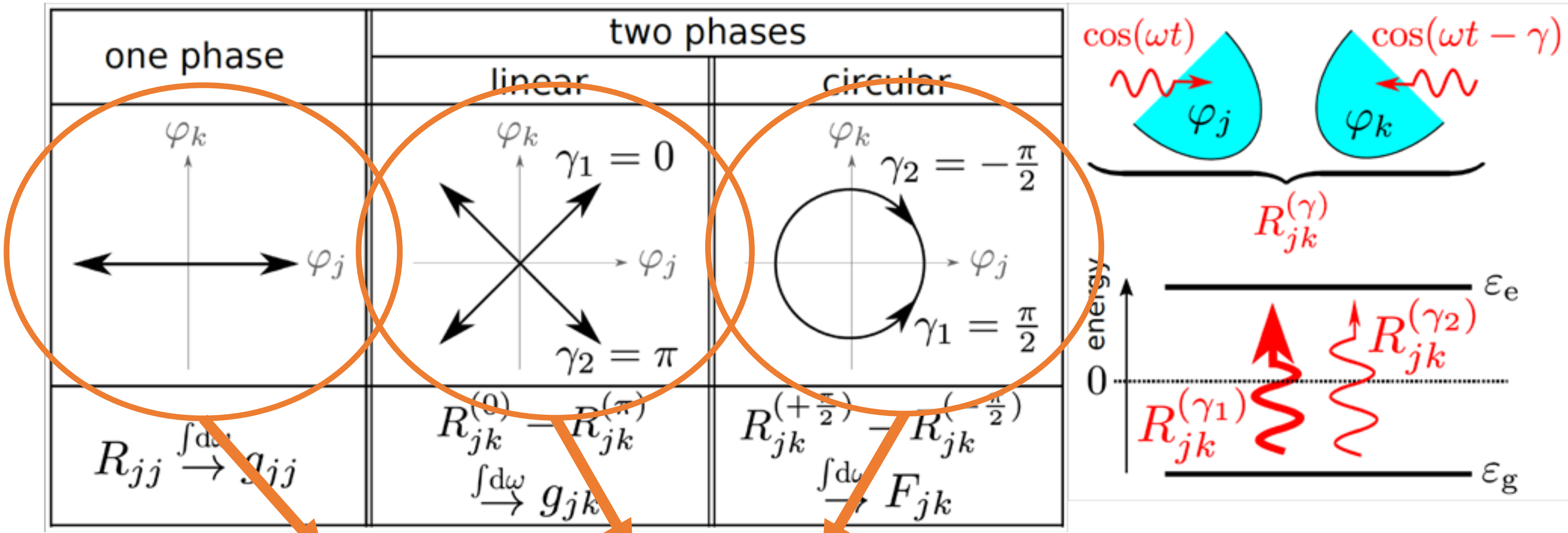
Berry curvature

[Klees, Rastelli, Cuevas, WB, arXiv:1810.11277]

[see also Ozawa et al., *Phys. Rev. B* **97**, 201117(R) (2018).]

Application to multiterminal Josehpson junctions

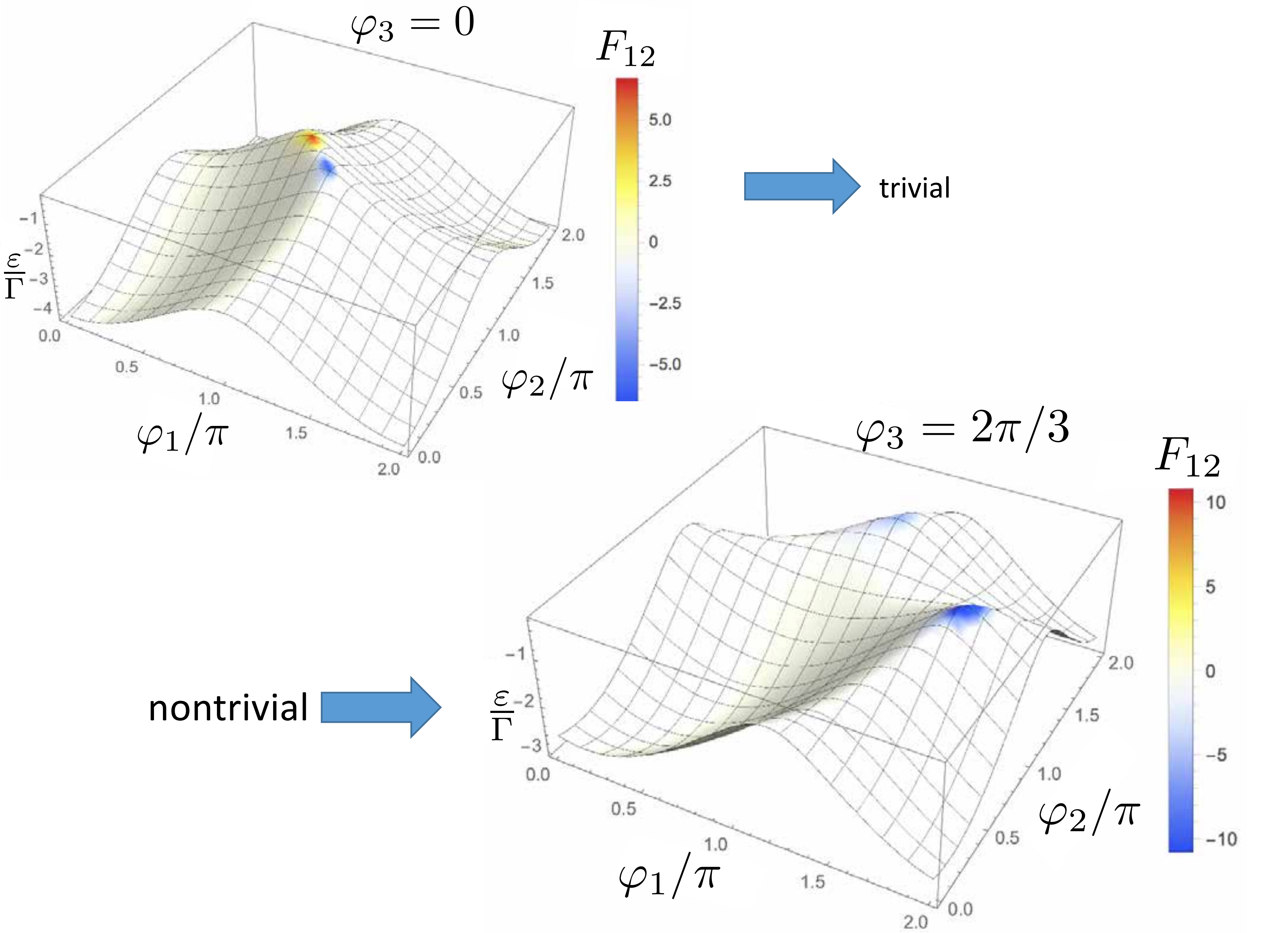
- Modulate **one or two** superconducting phases:



$$ds^2 = \sum_j g_{jj}\, d\varphi_j^2 + \sum_{\substack{j,k \\ j\neq k}} \left(g_{jk} - \frac{i}{2} F_{jk}\right) d\varphi_j\, d\varphi_k$$

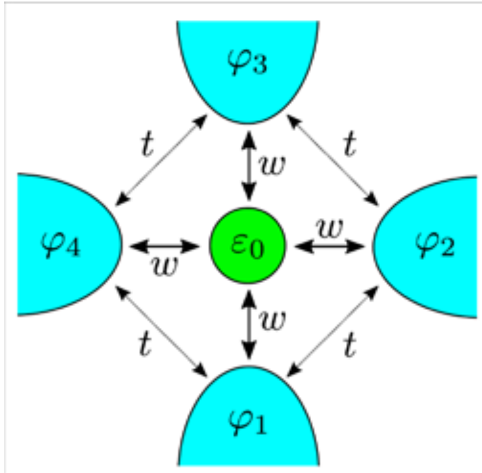
- Measurement of the ground state **quantum geometry** of the Andreev bound states in multiterminal Josephson junctions by polarized microwave spectroscopy.
- **Intensity** of the lines in absorption spectra is related to quantum geometry of the Andreev bound states.

[Klees, Rastelli, Cuevas, WB, arXiv:1810.11277]

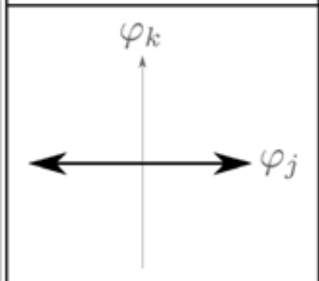
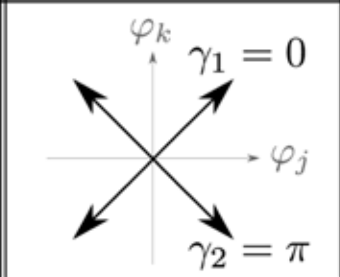
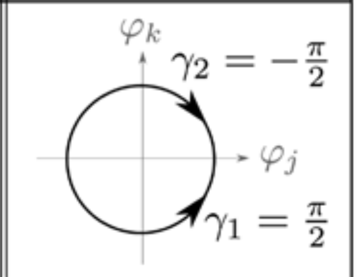


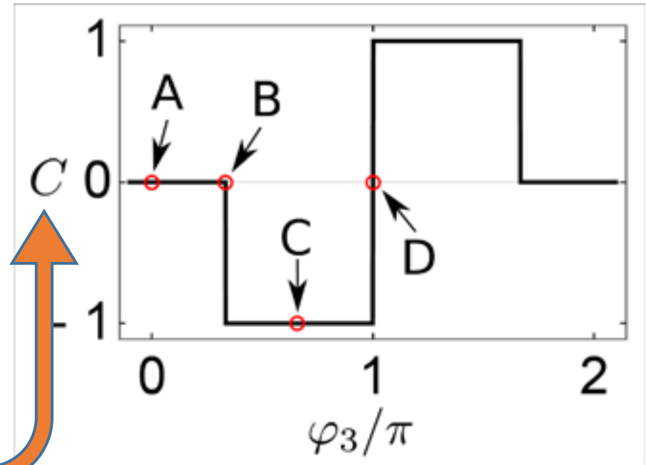
$\varphi_3 = 0$
F_{12}
5.0
2.5
0
−2.5
−5.0
trivial
$\frac{\varepsilon}{\Gamma}$
−1
−2
−3
−4
0.0
0.5
1.0
1.5
2.0
φ_1/π
φ_2/π
$\varphi_3 = 2\pi/3$
F_{12}
10
5
0
−5
−10
nontrivial
$\frac{\varepsilon}{\Gamma}$
−1
−2
−3
φ_1/π
φ_2/π

Summary



- Topology in multi-terminal Josephson matter
- 4-terminal junction with multiple paths shows Weyl-nodes and non-trivial topology
- Generalized microwave spectroscopy allows access to the full quantum geometric tensor (including quantum metric, Berry curvature and Chern number)

one phase	two phases	
	linear	circular
		
$R_{jj} \overset{\int d\omega}{\rightarrow} g_{jj}$	$R_{jk}^{(0)} - R_{jk}^{(\pi)} \overset{\int d\omega}{\rightarrow} g_{jk}$	$R_{jk}^{(+\frac{\pi}{2})} - R_{jk}^{(-\frac{\pi}{2})} \overset{\int d\omega}{\rightarrow} F_{jk}$



Collaborators: **Raffael Klees**, Gianluca Rastelli and Juan Carlos Cuevas

[Klees, Rastelli, Cuevas, WB, arXiv:1810.11277]

Summary I+II

- 4-terminal Josephson junction with multiple paths shows **Weyl-nodes** and non-trivial topology.
- **Generalized microwave spectroscopy** allows access to the full **quantum geometric tensor** (including quantum metric, Berry curvature and Chern number)
- [Klees, Rastelli, Cuevas, WB, arXiv:1810.11277]