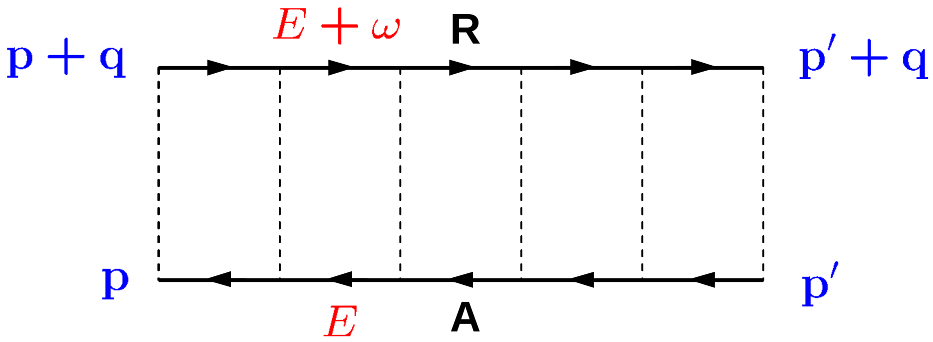
Лекция 2

- Высшие поправки в слабой локализации
- типология σ-моделей
- случаи, допускающие точное решение σ-модели
- пертурбативный анализ σ-моделей
- вывод келдышевской σ-модели для зависящих от времени случайных матриц

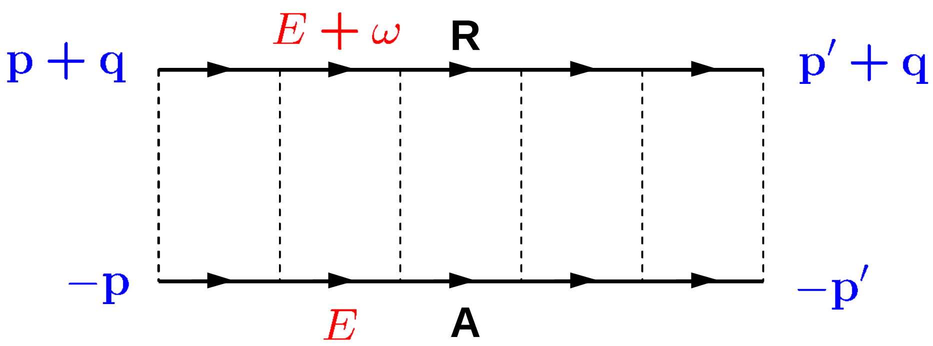
Diffusons & cooperons

Diffuson:

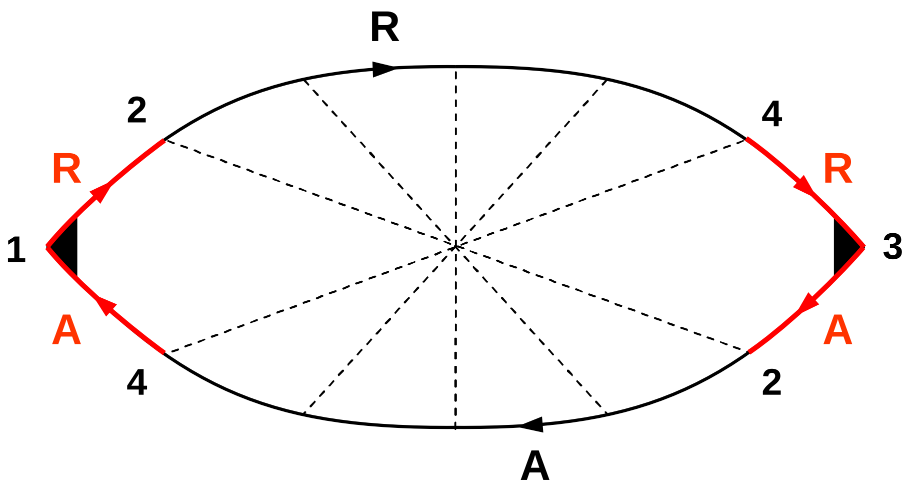


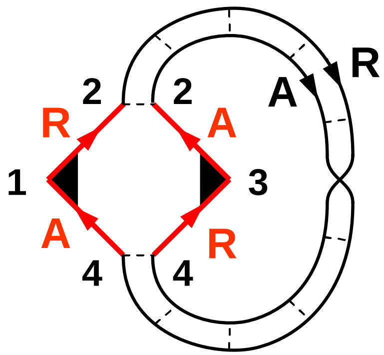
$$\frac{1}{2\pi\nu\tau^2}\,\frac{1}{Dq^2 - i\omega}$$

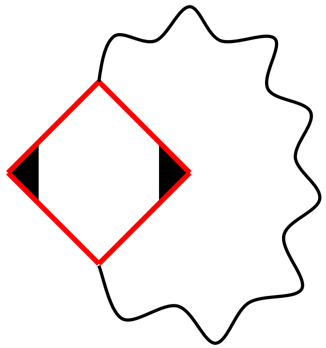
Cooperon:



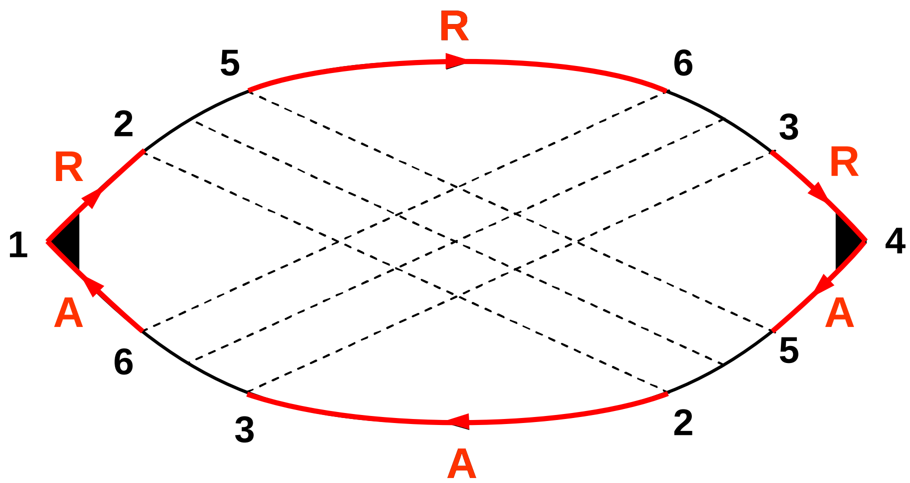
Weak localization (no magnetic field)

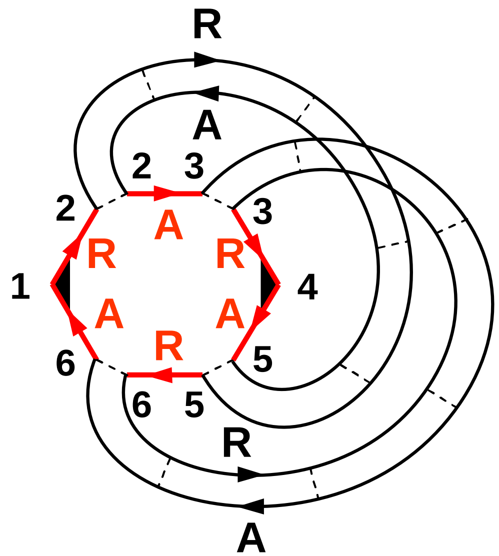


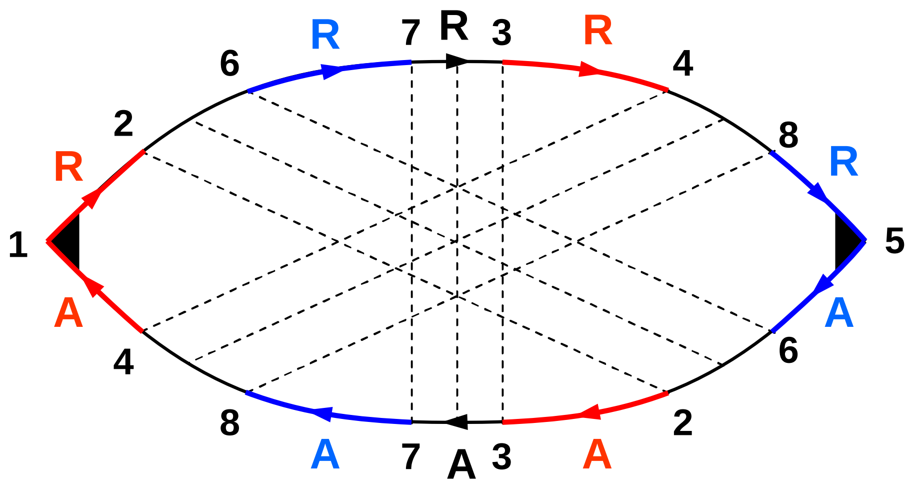


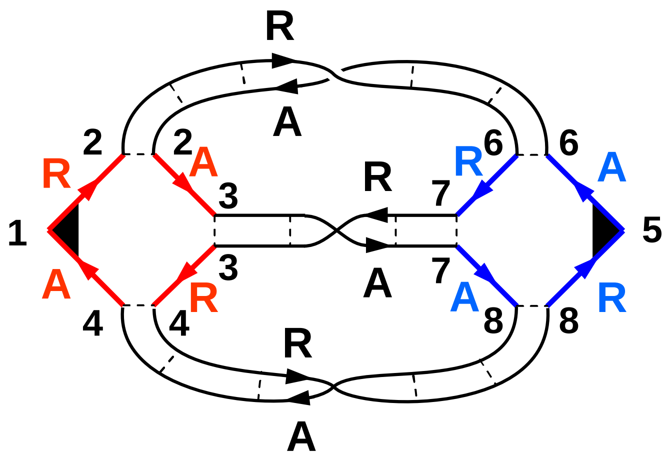


Weak localization (in magnetic field)



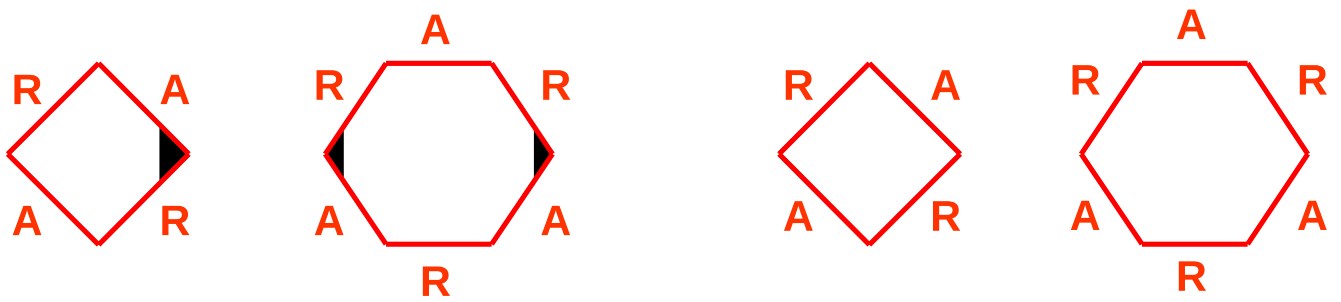






No closed loops!

Hikami boxes



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Anderson localization in a nonlinear-σ-model representation

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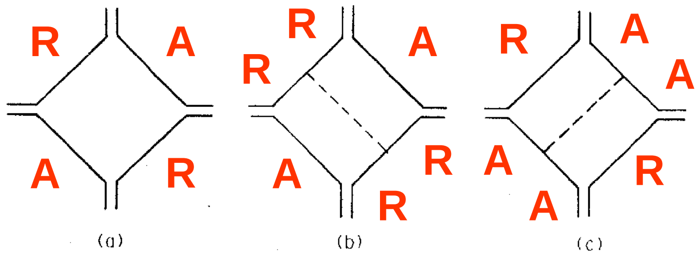
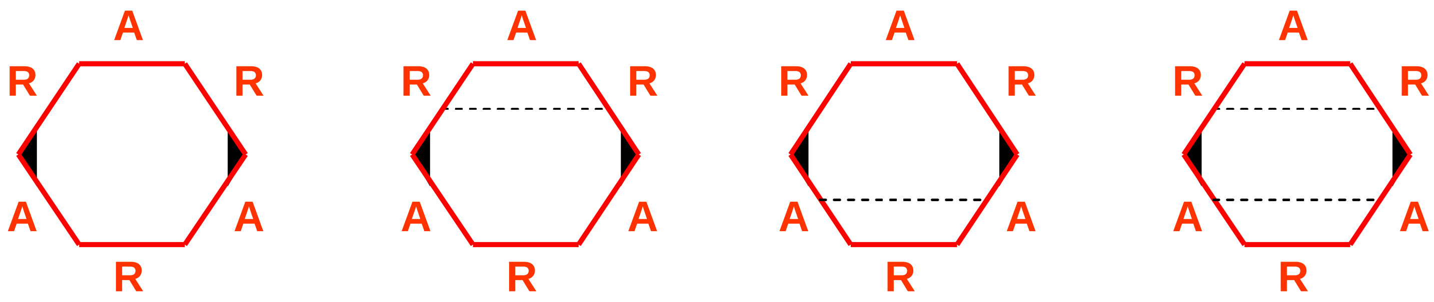
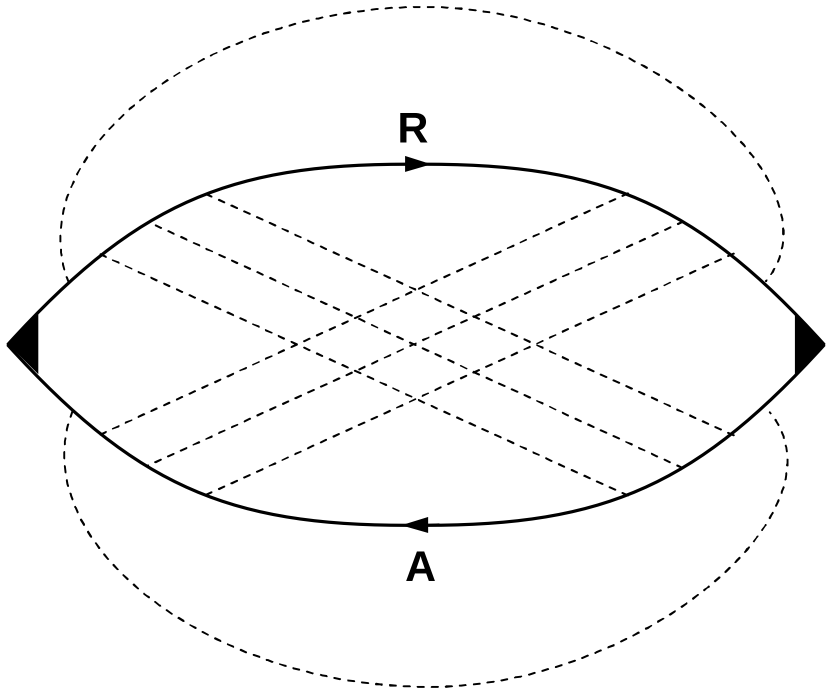


$$= \pi\nu\tau^4 D[(q_1+q_2)^2+(q_2+q_3)^2+(q_3+q_4)^2+(q_4+q_1)^2]$$

FIG. 6. Four-point vertex.

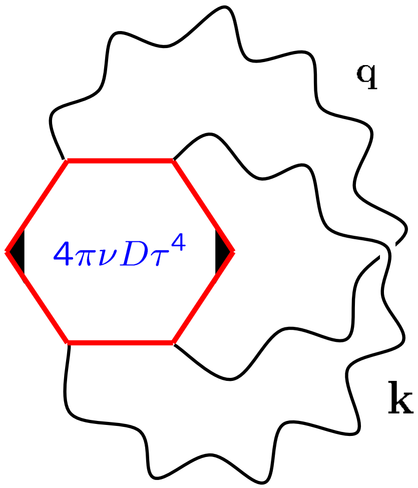
Hikami boxes

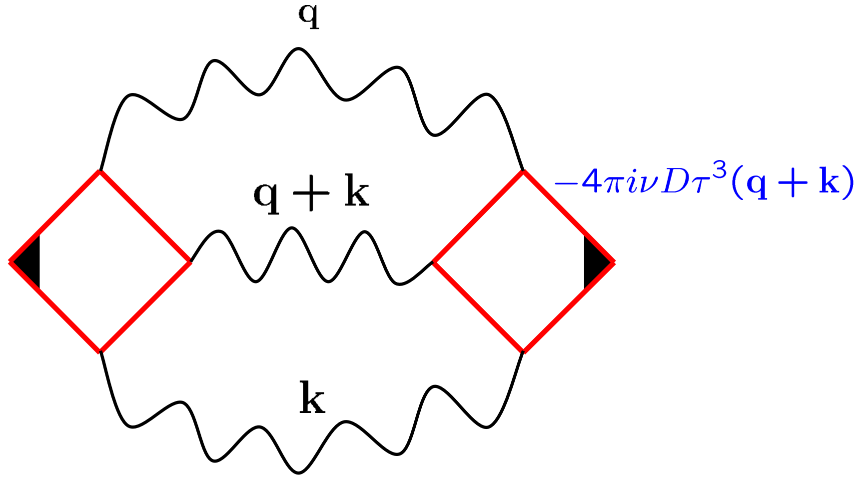




Interacting diffusive modes

Gor'kov, Larkin, Khmelnitskii (1979)





$$\frac{\delta\sigma_6}{\sigma_0}=\frac{1}{2(\pi\nu)^2}\int\frac{(d\mathbf{q})(d\mathbf{k})}{(Dq^2-i\omega)(Dk^2-i\omega)}$$

$$\frac{\delta\sigma_{44}}{\sigma_0}=-\frac{1}{(\pi\nu)^2 d}\int\frac{D(\mathbf{q}+\mathbf{k})^2\,(d\mathbf{q})(d\mathbf{k})}{(Dq^2-i\omega)(Dk^2-i\omega)(D(\mathbf{q}+\mathbf{k})^2-i\omega)}$$

2D geometry: $$\frac{\delta\sigma}{\sigma_0}=-\frac{\ln(1/\omega\tau)}{(4\pi^2\nu D)^2}=-\frac{\ln(L/l)}{8(\pi^2\nu D)^2}$$

$$g=\frac{\sigma_0}{e^2/h}=4\pi\nu D$$

Unitary localization length: $L_{\mathrm{loc}}^{\mathrm{unit}}\sim l\,e^{(\pi g/2)^2}$

Nonlinear σ-models

Zoology of σ-models

Low-energy effective theory formulated in terms of the matrix Q-field ($Q^2 = 1$)

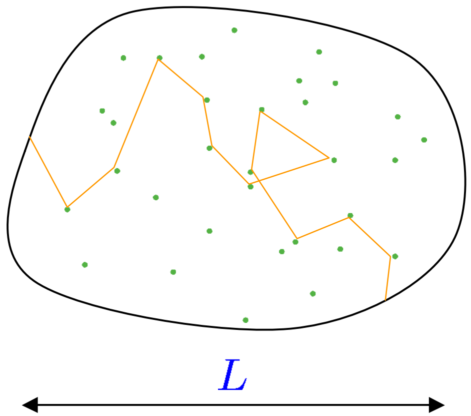
Typical form: $$S[Q] = \frac{\pi\nu}{8} \,\mathrm{str} \int d\mathbf{r} \left[D(\nabla Q)^2 + 2i\omega\Lambda Q \right] \qquad Q = U^{-1}\Lambda U$$

	spaces	Λ	intract.	non-eq.	exact sol.
SUSY	FB, RA	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{RA}$	–	–	0D, Q1D
replica	*replica, im. time*	sign ε	+	–	–
Keldysh	*time*, Keldysh	$\begin{pmatrix} 1 & 2F(E) \\ 0 & -1 \end{pmatrix}_{K}$	+	+	–

+ PH to handle cooperons

SUSY σ-model in 0D: reduction to RMT

Efetov (1982)



$$S[Q] = \frac{\pi\nu}{8}\,\mathrm{str}\int d\mathbf{r}\left[D(\nabla Q)^2 + 2i\omega\Lambda Q\right]$$

Diffusive modes: $-D\nabla^2\phi_n(\mathbf{r}) = \gamma_n\phi_n(\mathbf{r}), \quad \mathbf{n}\nabla\phi_n\big|_\Gamma = 0$

eigenvalues: $\gamma_0 = 0$ — zero mode

$\gamma_1 \sim D/L^2 = E_{\mathrm{Th}}$

At $\omega \ll E_{\mathrm{Th}}$, contribution of nonzero modes is suppressed, and $Q(\mathbf{r}) \to Q$

$$S[Q] = \frac{i\pi\omega\nu}{4}\int d\mathbf{r}\,\mathrm{str}\,\Lambda Q = \frac{i\pi\omega\nu V}{4}\,\mathrm{str}\,\Lambda Q = \frac{i\pi\omega}{4\Delta}\,\mathrm{str}\,\Lambda Q$$

The same action can be derived for RMT

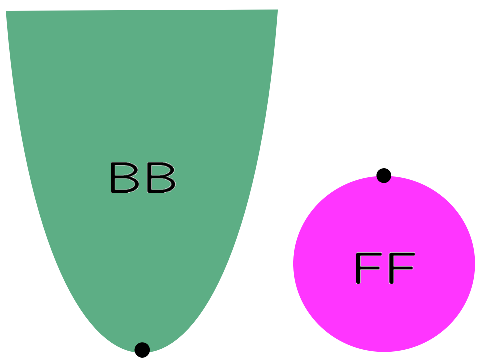
SUSY σ-model for RMT

Efetov (1982)

GUE: $$R(\omega) = \frac{1}{2} - \frac{1}{8} \int \mathrm{str}(kQ_{RR})\, \mathrm{str}(kQ_{AA}) \exp\left(-\frac{i\pi\omega}{2\Delta} \mathrm{str}\, \Lambda Q \right) DQ$$

$$Q = U_\eta^{-1} \left(\begin{array}{cc|cc} \cos\theta_F & 0 & ie^{i\varphi_F} \sin\theta_F & 0 \\ 0 & \cosh\theta_B & 0 & e^{i\varphi_B} \sinh\theta_B \\ \hline -ie^{-i\varphi_F} \sin\theta_F & 0 & -\cos\theta_F & 0 \\ 0 & -e^{-i\varphi_B} \sinh\theta_B & 0 & -\cosh\theta_B \end{array} \right)_{RA} U_\eta$$

$$U_\eta = \begin{pmatrix} u^{-1} & 0 \\ 0 & v^{-1} \end{pmatrix}_{RA}, \quad u = \exp \begin{pmatrix} 0 & \eta \\ \eta^* & 0 \end{pmatrix}_{FB}, \quad v = \exp \begin{pmatrix} 0 & \sigma \\ \sigma^* & 0 \end{pmatrix}_{FB}$$



$$R(\omega) = 1 - \frac{\sin^2(\pi\omega/\Delta)}{(\pi\omega/\Delta)^2}$$

SUSY σ-model: Crossover between ensembles

From GOE to GUE: $H = H_s + i\alpha H_a$ Mehta, Pandey (1983)

$H_s^T = H_s$ $H_a^T = -H_a$

$$S[Q] = \frac{i\pi\omega}{4\Delta}\,\underset{\text{RA}}{\underset{\uparrow}{\mathrm{str}\,\Lambda Q}} - \frac{N\alpha^2}{8}\,\underset{\text{PH}}{\underset{\uparrow}{\mathrm{str}[Q,\tau_3]^2}}$$

Q is an 8×8 supermatrix acting in FB $\otimes$ RA $\otimes$ PH

$$Q = \begin{array}{ccc} & \text{D} & \text{C} \\ & \searrow & \swarrow \\ & \begin{pmatrix} Q_{pp} & Q_{ph} \\ Q_{hp} & Q_{hh} \end{pmatrix} & \\ & \nearrow & \nwarrow \\ & \text{C} & \text{D} \end{array}$$

Large symmetry breaking

Cooperon modes become massive

$$S[Q] = \frac{i\pi\omega}{2\Delta}\,\mathrm{str}\,\Lambda Q_{pp}$$

$$R(x) = 1 - \frac{\sin^2 x}{x^2} + \int_1^\infty d\lambda \frac{\sin \lambda x}{\lambda} e^{-s\lambda^2} \int_0^1 d\mu\, \mu \sin(\mu x) e^{s\mu^2}$$

$x = \omega/\pi\Delta$, $s = 2N\alpha^2$ Altland, Iida, Efetov (1993)

SUSY σ-model in Q1D: exact solution

Efetov, Larkin (1983)

- σ-**model**

$$S[Q(x)] = \mathrm{str}\int \left[D \left(\frac{\partial Q}{\partial x} \right)^2 + 2i\omega \Lambda Q \right] dx$$

Hamiltonian acting on $\Psi(Q)$:

$$H = -\hat{\Delta}_Q + i\omega\, \mathrm{str}\, \Lambda Q$$

- **Feynman path integral**

$$S[r(t)] = \int \left[\frac{m}{2} \left(\frac{\partial r}{\partial t} \right)^2 + U(r) \right] dt$$

Hamiltonian acting on $\Psi(r)$:

$$H = -\frac{1}{2m}\hat{\Delta}_r + U(r)$$

Laplace-Beltrami operator
on the curved supermanifold Q

$$\hat{\Delta}_Q = -\frac{(\lambda_1 - \lambda)^2}{2} \left[\frac{\partial}{\partial \lambda} \frac{1-\lambda^2}{(\lambda_1 - \lambda)^2} \frac{\partial}{\partial \lambda} + \frac{\partial}{\partial \lambda_1} \frac{\lambda_1^2 - 1}{(\lambda_1 - \lambda)^2} \frac{\partial}{\partial \lambda_1} \right] + \text{"angular terms"}$$

Relevant: finite-size $Q(x)$

λ, λ_1 — eigenvalues of the Q-matrix

Parameterizations

Q is subject to a nonlinear constraint:

$$Q^2 = 1, \qquad Q = U^{-1}\Lambda U \qquad\qquad \Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Parametrization of the curved manifold

$$Q = \Lambda f(W) = \Lambda[1 + W + W^2/2 + \ldots]$$

$$\{W, \Lambda\} = 0 \quad \longrightarrow \quad W = \begin{pmatrix} 0 & W_{12} \\ W_{21} & 0 \end{pmatrix}$$

$$f(W)f(-W) = 1$$

parametrizations

rational	$f(W) = \dfrac{1 + W/2}{1 - W/2}$	$J = 1$
sqrt	$f(W) = \sqrt{1 + W^2} + W$	cross diagram technique
exp	$f(W) = e^W$	$\rightarrow$ global parametrization

Bare diffusons & cooperons

$$S[Q] = \frac{\pi\nu}{8} \,\mathrm{str} \int d\mathbf{r} \left[D(\nabla Q)^2 + 2i\omega\Lambda Q\right] \qquad Q = \Lambda[1 + W + W^2/2 + \ldots]$$

$$S^{(2)}[W] = \frac{\pi\nu}{8} \,\mathrm{str} \int d\mathbf{r} \left[D(\Lambda\nabla W)^2 + i\omega\Lambda\Lambda W^2\right]$$

$$S^{(2)}[W] = \frac{\pi\nu}{8} \,\mathrm{str} \int d\mathbf{r} \left[-D(\nabla W)^2 + i\omega W^2\right] \qquad \langle W_{\mathbf{q}} W_{-\mathbf{q}} \rangle \stackrel{?}{=} \frac{\#}{Dq^2 - i\omega}$$

Contraction rules

$$\langle \mathrm{str}\, PW(\mathbf{r}) RW(\mathbf{r}') \rangle = \Pi(\mathbf{r},\mathbf{r}')(\mathrm{str}\, P\Lambda\, \mathrm{str}\, R\Lambda - \mathrm{str}\, P\, \mathrm{str}\, R) + (2-\beta)\Pi(\mathbf{r},\mathbf{r}')\, \mathrm{str}(P\Lambda\overline{R}\Lambda - P\overline{R})$$

$$\langle \mathrm{str}[PW(\mathbf{r})]\, \mathrm{str}[RW(\mathbf{r}')] \rangle = \Pi(\mathbf{r},\mathbf{r}')\, \mathrm{str}(P\Lambda R\Lambda - PR)$$

Diffusion kernel: $\Pi(\mathbf{r},\mathbf{r}') = \frac{1}{\pi\nu} \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')}}{Dq^2 - i\omega}$

Weak localization with σ-model

Keldysh σ-model

Derivation of the Keldysh σ-model (0/6)

$$H(t) = H_1 \cos k\varphi(t) - H_2 \sin k\varphi(t)$$

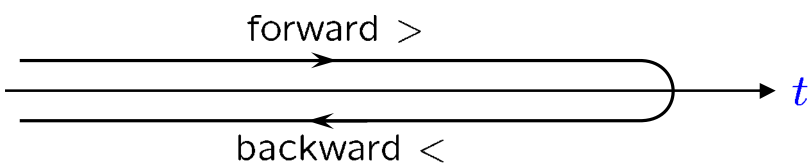
related to level sensitivity: $Nk^2 = \dfrac{\pi^2}{2\Delta^2}\left\langle \left(\dfrac{\partial E_i}{\partial \varphi}\right)^2 \right\rangle$

Schrödinger equation:

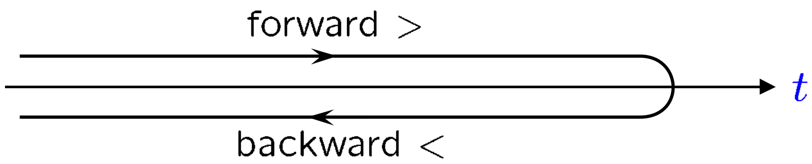
$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H(t)|\psi(t)\rangle \qquad \Longrightarrow \qquad |\psi(t)\rangle = T \exp\left(-i\int_0^t H(t')dt'\right)|0\rangle$$

Physical observable:

$$\langle\psi(t)|A(t)|\psi(t)\rangle = \langle 0|\, \underbrace{\hat{T}^{-1}\exp\left(i\int_0^t H(t')dt'\right)}_{\text{backward evolution}}\, A(t)\, \underbrace{\hat{T}\exp\left(-i\int_0^t H(t')dt'\right)}_{\text{forward evolution}}\, |0\rangle$$



Derivation of the Keldysh σ-model (1/6)



- Partition function via the functional integral over Grassmannian fields $\psi(t)$:

$$Z = \int D\psi D\psi^* \exp\left\{ i \int_{\rightleftarrows} dt\ \psi^\dagger(t) \left[i\frac{\partial}{\partial t} - H(t) \right] \psi(t) \right\}$$

- Introduction of the Keldysh 2×2 space:

$$\Psi(t) = \begin{pmatrix} \psi_>(t) \\ \psi_<(t) \end{pmatrix}$$

$$Z = \int D\Psi D\Psi^* \exp\left\{ i \int_{-\infty}^{\infty} dt\ \Psi^\dagger(t) \left[i\frac{\partial}{\partial t} - H(t) \right] \sigma_3 \Psi(t) \right\}$$

- Introduction of the 2×2 Particle-Hole space to treat GOE symmetry:

$$\hat{\Psi}(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} \Psi(t) \\ \Psi^*(t) \end{pmatrix}$$

$$Z = \int D\Psi D\Psi^* \exp\left\{ i \int_{-\infty}^{\infty} dt\ \hat{\Psi}^\dagger(t) \left[i\tau_3\frac{\partial}{\partial t} - H(t) \right] \sigma_3 \hat{\Psi}(t) \right\}$$

Derivation of the Keldysh σ-model (2/6)

$$Z = \int D\Psi D\Psi^* \exp\left\{ i \int_{-\infty}^{\infty} dt\ \hat{\Psi}^\dagger(t) \left[i\tau_3 \frac{\partial}{\partial t} - H_1 \cos k\varphi(t) - H_2 \sin k\varphi(t) \right] \sigma_3 \hat{\Psi}(t) \right\}$$

- Averaging over H_1 and H_2 with the Gaussian weight

$$\mathcal{P}(H) \propto \exp\left(-\frac{\pi^2}{4N\Delta^2} \operatorname{tr} H^2 \right) \qquad \langle H_{ij} H_{kl} \rangle = \frac{N\Delta^2}{\pi^2} (\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl})$$

$$\left\langle \exp\left\{ -i \sum_{ij} \int dt\ \hat{\Psi}_i^\dagger(t) H_{1ij} \cos k\varphi(t) \sigma_3 \hat{\Psi}_j(t) \right\} \right\rangle = \exp\left\{ i S_{\text{dis}}^{(1)} \right\}$$

$$i S_{\text{dis}}^{(1)} = -\frac{1}{2} \left\langle \left(\sum_{ij} \int dt\ \hat{\Psi}_i^\dagger(t) H_{1ij} \cos k\varphi(t) \sigma_3 \hat{\Psi}_j(t) \right)^2 \right\rangle$$

$$i S_{\text{dis}}^{(1)} = -\frac{N\Delta^2}{2\pi^2} \sum_{ij} \int dt\ dt' \left[\left(\hat{\Psi}_i^\dagger \cos k\varphi \sigma_3 \hat{\Psi}_j \right)_t \left(\hat{\Psi}_j^\dagger \cos k\varphi \sigma_3 \hat{\Psi}_i \right)_{t'} + \left(\hat{\Psi}_i^\dagger \cos k\varphi \sigma_3 \hat{\Psi}_j \right)_t \left(\hat{\Psi}_i^\dagger \cos k\varphi \sigma_3 \hat{\Psi}_j \right)_{t'} \right]$$

Derivation of the Keldysh σ-model (3/6)

After averaging over H_1:

$$iS_{\text{dis}}^{(1)} = -\frac{N\Delta^2}{\pi^2}\sum_{ij}\int dt\, dt' \left(\hat{\Psi}_i^\dagger \sigma_3 \hat{\Psi}_j\right)_t \left(\hat{\Psi}_j^\dagger \sigma_3 \hat{\Psi}_i\right)_{t'} \cos k\varphi(t) \cos k\varphi(t')$$

After averaging over H_2:

$$iS_{\text{dis}}^{(2)} = -\frac{N\Delta^2}{\pi^2}\sum_{ij}\int dt\, dt' \left(\hat{\Psi}_i^\dagger \sigma_3 \hat{\Psi}_j\right)_t \left(\hat{\Psi}_j^\dagger \sigma_3 \hat{\Psi}_i\right)_{t'} \sin k\varphi(t) \sin k\varphi(t')$$

Total:

$$iS_{\text{dis}} = -\frac{N\Delta^2}{\pi^2}\sum_{ij}\int dt\, dt' \overbrace{\left(\hat{\Psi}_i^\dagger \sigma_3 \hat{\Psi}_j\right)_t}^{\text{scalar}} \left(\hat{\Psi}_j^\dagger \sigma_3 \hat{\Psi}_i\right)_{t'} \cos k[\varphi(t) - \varphi(t')]$$

$$iS_{\text{dis}} = +\frac{N\Delta^2}{\pi^2}\sum_{ij}\int dt\, dt' \,\text{tr}\overbrace{\left(\sigma_3 \hat{\Psi}_j(t) \hat{\Psi}_j^\dagger(t')\right)}^{\text{matrix}} \left(\sigma_3 \hat{\Psi}_i(t') \hat{\Psi}_i^\dagger(t)\right) \cos k[\varphi(t) - \varphi(t')]$$

Derivation of the Keldysh σ-model (4/6)

$$iS_{\text{dis}} = +\frac{N\Delta^2}{\pi^2}\sum_{ij}\int dt\, dt'\, \text{tr}\left(\sigma_3\hat{\Psi}_j(t)\hat{\Psi}_j^\dagger(t')\right)\left(\sigma_3\hat{\Psi}_i(t')\hat{\Psi}_i^\dagger(t)\right)\cos k[\varphi(t)-\varphi(t')]$$

- Hubbard-Stratonovich transformation:

$$\exp\{iS_{\text{dis}}\} = \int DQ(t,t')\exp\left\{-\frac{N}{4}\int dt\, dt'\cos k[\varphi(t)-\varphi(t')]\,\text{tr}\, Q_{tt'}Q_{t't}\right\}$$
$$\times\exp\left\{\frac{N\Delta}{\pi}\int dt\, dt'\cos k[\varphi(t)-\varphi(t')]\sum_i \text{tr}\, Q_{tt'}\sigma_3\hat{\Psi}_i(t')\hat{\Psi}_i^\dagger(t)\right\}$$

- Fermionic action becomes Gaussian:

$$iS[\hat{\Psi}] = -\int dt\sum_i\hat{\Psi}_i^\dagger(t)\tau_3\sigma_3\frac{\partial}{\partial t}\hat{\Psi}_i(t) - \frac{N\Delta}{\pi}\int dt\, dt'\cos k[\varphi(t)-\varphi(t')]\sum_i\hat{\Psi}_i^\dagger(t)Q_{tt'}\sigma_3\hat{\Psi}_i(t')$$

$$iS[\hat{\Psi}] = -\frac{N\Delta}{\pi}\int dt\, dt'\sum_i\hat{\Psi}_i^\dagger(t)G^{-1}(t,t')\sigma_3\hat{\Psi}_i(t')$$

$$G^{-1}(t,t') = \frac{\pi}{N\Delta}\tau_3\delta(t-t')\frac{\partial}{\partial t'} + \cos k[\varphi(t)-\varphi(t')]Q_{tt'}$$

Derivation of the Keldysh σ-model (5/6)

$$G^{-1}(t,t') = \frac{\pi}{N\Delta}\tau_3\delta(t-t')\frac{\partial}{\partial t'} + \cos k[\varphi(t)-\varphi(t')]Q_{tt'}$$

- Gaussian integration over fermions:

$$\int \prod_i D\hat{\Psi}_i \exp\left\{-\sum_i \hat{\Psi}_i^\dagger G^{-1}\hat{\Psi}_i\right\} = \left(\int D\hat{\Psi}_1 \exp\left\{-\hat{\Psi}_1^\dagger G^{-1}\hat{\Psi}_1\right\}\right)^N = \left(\det G^{-1}\right)^{N/2}$$

- Resulting Q action:

$$iS[Q] = \frac{N}{2}\operatorname{Tr}\ln G^{-1} - \frac{N}{4}\int dt\, dt' \cos k[\varphi(t)-\varphi(t')]\operatorname{tr} Q_{tt'}Q_{t't}$$

- Saddle point: $G = Q$

  in the limit $N \to \infty$, $G^{-1} = Q$

$$\boxed{Q^2 = 1}$$

$$\int Q_{t_1 t}Q_{t t_2}dt = \delta(t_1 - t_2)\,\mathbf{1}_{4\times 4}$$

From this point, we have a σ-model

Derivation of the Keldysh σ-model (6/6)

$$G^{-1}(t,t') = \frac{\pi}{N\Delta}\tau_3\delta(t-t')\frac{\partial}{\partial t'} + \cos k[\varphi(t)-\varphi(t')]Q_{tt'}$$

$$iS[Q] = \frac{N}{2}\,\mathrm{Tr}\ln G^{-1} - \frac{N}{4}\int dt\, dt' \cos k[\varphi(t)-\varphi(t')]\,\mathrm{tr}\, Q_{tt'}Q_{t't}$$

- Action. Leading terms: $S = N \times 0$
- Action. Subleading terms

$$G^{-1}(t,t') \approx Q_{tt'} + \frac{\pi}{N\Delta}\tau_3\delta(t-t')\frac{\partial}{\partial t'} - \frac{k^2}{2}[\varphi(t)-\varphi(t')]^2 Q_{tt'}$$

$$\frac{\pi}{2\Delta}\int dt\, dt'\ \mathrm{tr}\,\underbrace{\tau_3\delta(t-t')\frac{\partial}{\partial t'}}Q_{t't} - \frac{Nk^2}{4}\int dt\, dt'\,[\varphi(t)-\varphi(t')]^2\,\mathrm{tr}\,Q_{tt'}Q_{t't}$$

$$+\frac{Nk^2}{8}\int dt\, dt'\,[\varphi(t)-\varphi(t')]^2\,\mathrm{tr}\,Q_{tt'}Q_{t't}$$

energy operator
$\hat{E} = i\partial/\partial t$

related to level sensitivity: $Nk^2 = \dfrac{\pi^2}{2\Delta^2}\left\langle\left(\dfrac{\partial E_i}{\partial\varphi}\right)^2\right\rangle$

Keldysh σ-model

M. S. (2003)

Low-energy effective theory is formulated in terms of the matrix Q-field ($Q^2 = 1$)

$Q_{tt'}^{\alpha\beta} \in$

- **time** space: continuous index t
- 2×2 **Keldysh** space (σ_i)
- 2×2 **Particle-Hole** space (τ_i)

$$S[Q] = \frac{\pi i}{2\Delta} \operatorname{Tr} \tau_3 \hat{E} Q - \frac{\pi \Gamma}{8\Delta} \operatorname{Tr}[\varphi(t), Q]^2$$

E-term

responsible for the RMT energy **level statistics** encoded in the rich structure of $Q_{EE'}$

Altland & Kamenev (2000)

kinetic term

accounts for **interlevel transitions** due to time-dependent Hamiltonian $H[\varphi(t)]$

$$\left\langle \left(\frac{\partial E_i}{\partial \varphi} \right)^2 \right\rangle = \frac{2\Gamma\Delta}{\pi}$$

Keldysh σ-model

M. S. (2003)

Low-energy effective theory is formulated in terms of the matrix Q-field ($Q^2 = 1$)

$Q_{tt'}^{\alpha\beta} \in$

- **time** space: continuous index t
- 2×2 **Keldysh** space (σ_i)
- 2×2 **Particle-Hole** space (τ_i)

$$S[Q] = \frac{\pi i}{2\Delta} \mathrm{Tr}\, \tau_3 \hat{E} Q + \frac{\pi \Gamma}{8\Delta} \int dt\, dt' \left[\varphi(t) - \varphi(t')\right]^2 \mathrm{tr}\, Q_{tt'} Q_{t't}$$

E-term

responsible for the RMT energy **level statistics** encoded in the rich structure of $Q_{EE'}$

Altland & Kamenev (2000)

kinetic term

accounts for **interlevel transitions** due to time-dependent Hamiltonian $H[\varphi(t)]$

$$\left\langle \left(\frac{\partial E_i}{\partial \varphi} \right)^2 \right\rangle = \frac{2\Gamma\Delta}{\pi}$$