Low-dimensional disordered electronic systems

(theoretical aspects)

Part II

Alexander D. Mirlin

Research Center Karslruhe & University Karlsruhe & PNPI St. Petersburg

http://www-tkm.physik.uni-karlsruhe.de/∼mirlin/

Evers, ADM "Anderson transitions", arXiv:0707.4378

Electron-electron interaction effects: renormalization and dephasing

- **Renormalization**

Virtual processes, energy transfer $\gtrsim T$

become stronger when T is lowered; in low d singular in the limit $T \to 0$

Altshuler, Aronov '79 (review: '85) ;

Zala, Narozhny, Aleiner, Phys. Rev. B 64, 214204 (2001) ;
see also Gornyi, ADM, Phys. Rev. B 69, 045313 (2004)

Cf.: Fermi-liquid renormalizations, Kondo, Luttinger-liquid power laws, Fermi-edge singularity ...

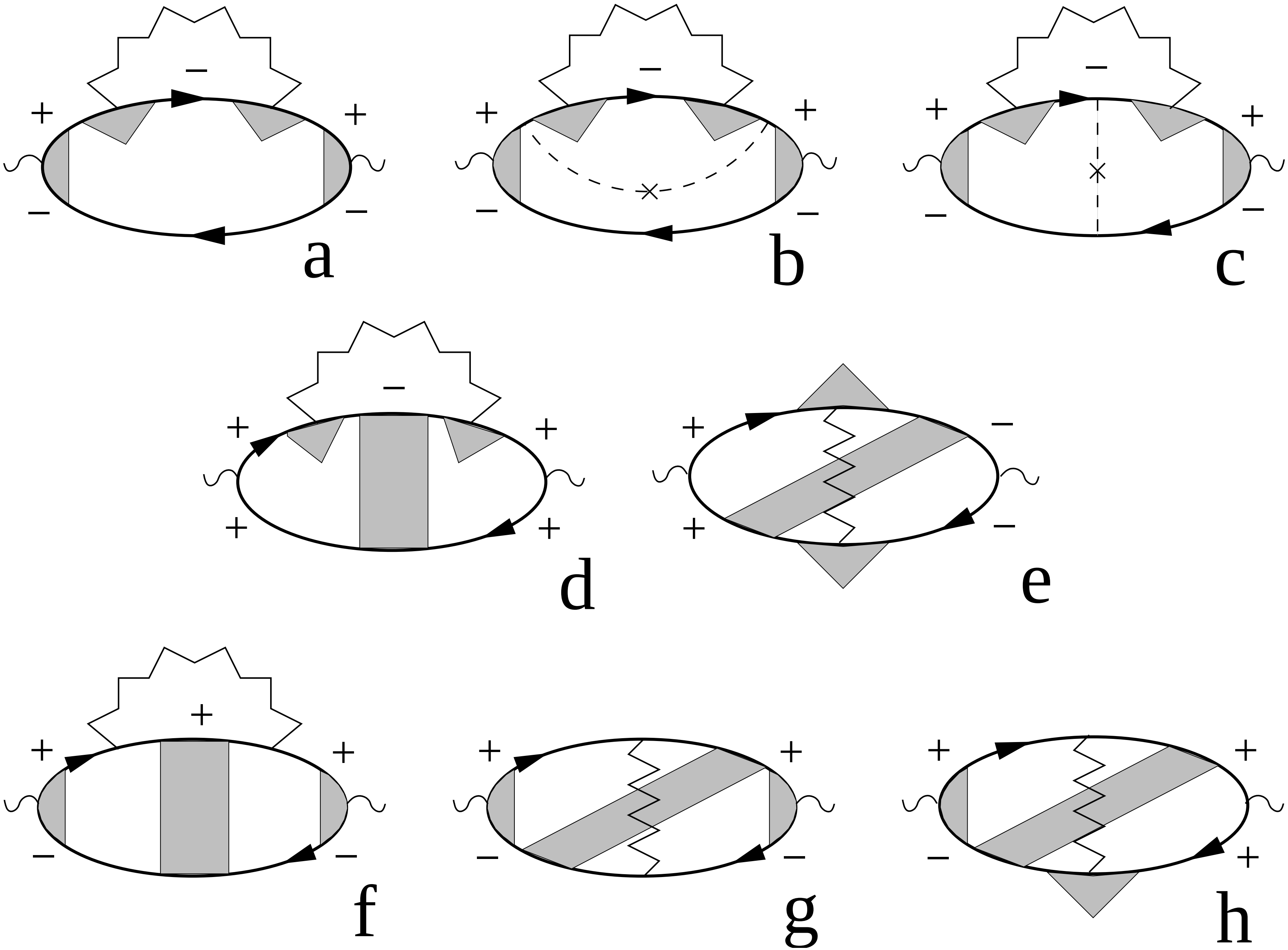
- **Dephasing**

Real inelastic scattering processes, energy transfer $\lesssim T$

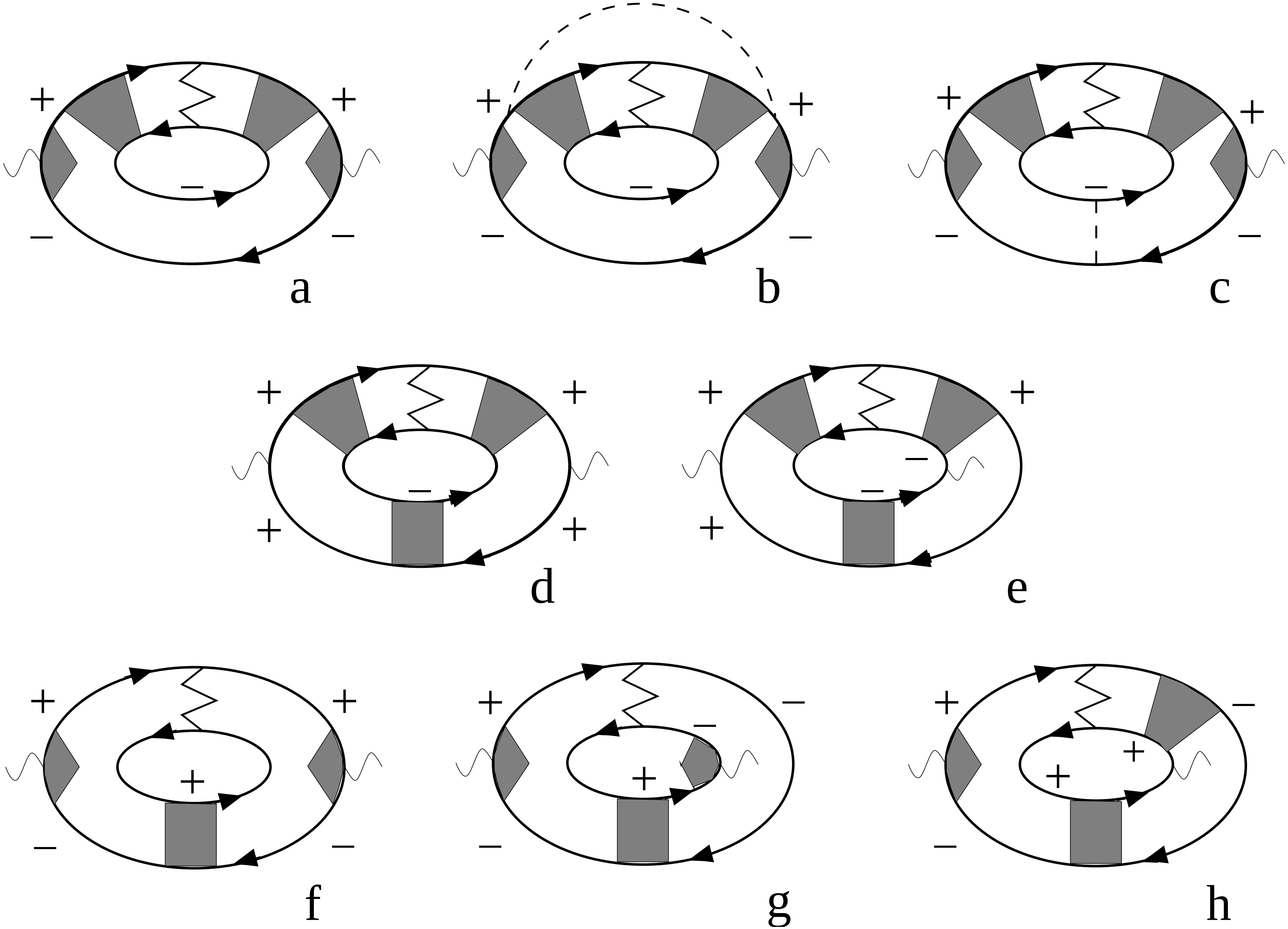
become weaker when T is lowered, vanishing as $T \to 0$

Altshuler, Aronov, Khmelnitskii '82 ;
see also Aleiner, Altshuler, Gershenson, Waves Random Media 9, 201 (1999)

Interaction correction to conductivity: Exchange



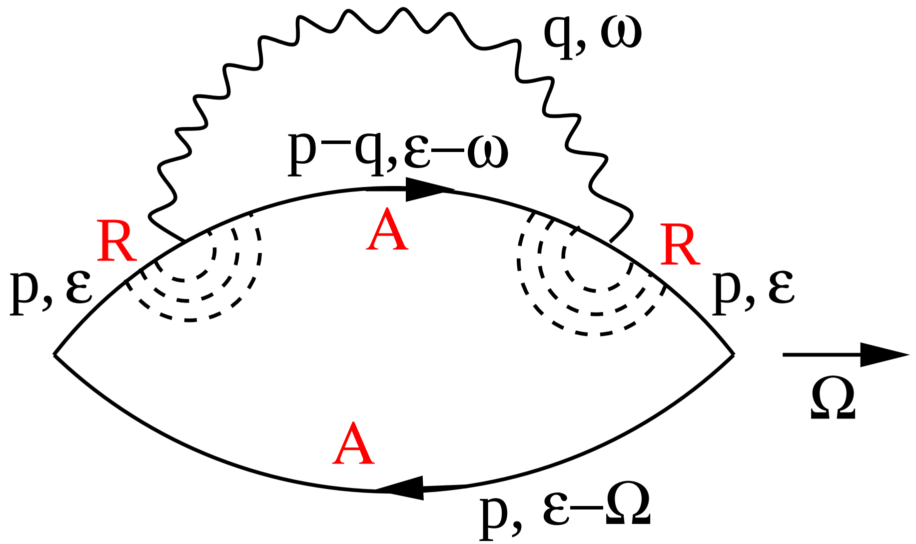
Interaction correction to conductivity: Hartree



Interaction correction to conductivity: Diffusive regime

diffusive regime: $T\tau \ll 1$

typical diagram (exchange):



$$\Delta\sigma(\Omega) \simeq \frac{2}{\Omega}\frac{e^2}{2\pi}\int (dp) v_x^2 [G_\epsilon^R(p)]^2 [G_\epsilon^A(p)]^2 \int (dq) \int_0^\Omega \frac{d\epsilon}{2\pi}\int_\epsilon^\infty \frac{d\omega}{2\pi}\frac{iU(q,\omega)}{(Dq^2 - i\omega)^2\tau^2}$$

$$\int (dp) v_x^2 [G^R G^A]^2 = 4\pi\tau^2\nu D \longrightarrow \Delta\sigma(\Omega) = \frac{\sigma_0}{\pi^2\Omega}\int_0^\Omega d\epsilon \int (dq) \int_\epsilon^\infty d\omega \frac{iU(q,\omega)}{(Dq^2 - i\omega)^2}$$

short-range interaction $U(q,\omega) \rightarrow U(0)$

$$\longrightarrow \Delta\sigma(\Omega) \sim \sigma_0 U(0) \int_{l_\Omega^{-1}}^{l^{-1}} \frac{(dq)}{Dq^2} \sim e^2\nu U(0) \times \begin{cases} \ln \Omega\tau\ , & d=2 \\ \left(\frac{\Omega}{D}\right)^{d/2-1}, & d\neq 2 \end{cases}$$

Interaction correction to conductivity (cont'd)

For $T \gg \Omega$: $\Omega \longrightarrow T$

Coulomb interaction, all exchange diagrams:

$$\Delta\sigma(\Omega) = e^2 \times \begin{cases} \dfrac{1}{2\pi^2} \ln T\tau \ , & d = 2 \\ \sim \left(\dfrac{T}{D}\right)^{d/2-1}, & d \neq 2 \end{cases}$$

Exchange + Hartree: overall factor $1 + 3\left[1 - \dfrac{\ln(1+F_0^\sigma)}{F_0^\sigma}\right]$

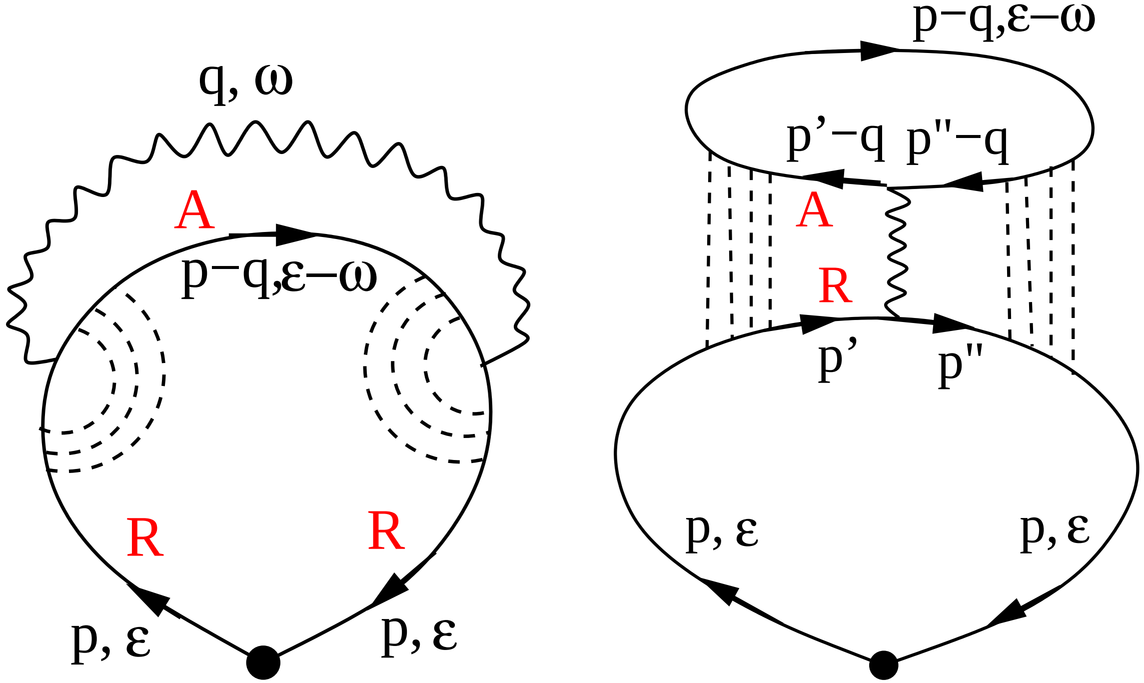
Ballistic regime $T\tau \gg 1$,   2D : $\ln T\tau \longrightarrow (E_F - T)\tau$

In the ballistic regime, prefactor of the Hartree term depends not only on F_0^σ, but also on all F_m^ρ, F_m^σ with $m \neq 0$ and on disorder range

Interaction correction to tunneling DOS

Tunneling DOS:

$$\nu(\epsilon) = -\frac{1}{\pi}\mathrm{Im}\, G^R(r,r;\epsilon)$$
$$= -\frac{1}{\pi}\mathrm{Im}\int (dp) G^R(p;\epsilon)$$



$$\frac{\Delta\nu(\epsilon)}{\nu_0} = -\frac{1}{\pi}\mathrm{Im}\int (dq)\int_\epsilon^\infty d\omega \left[U(q,\omega) - 2\overline{U(p'-p'',0)}\right]\frac{1}{(Dq^2 - i\omega)^2}$$

For short-range interaction

$$\frac{\Delta\nu(\epsilon)}{\nu_0} = -\frac{1}{\pi}\left[U(q=0) - 2\overline{U(p'-p'')}\right]\int (dq)\frac{Dq^2}{(Dq^2)^2+\epsilon^2} \sim \frac{\Delta\sigma(\Omega)|_{\Omega\sim\epsilon}}{\sigma_0}$$

However, for Coulomb interaction the TDOS correction is parametrically larger!

Interaction correction to TDOS: Coulomb interaction

Dynamically screened Coulomb interaction, 2D :

$$U(q,\omega) = \frac{2\pi e^2}{q + \kappa\frac{Dq^2}{Dq^2 - i\omega}}\,, \qquad \kappa = 4\pi e^2\nu \qquad \longrightarrow \text{ poor screening for } Dq^2 \ll \omega$$

Exchange correction to TDOS:

$$\frac{\Delta\nu(\epsilon)}{\nu_0} = -\frac{1}{2\pi^2}\mathrm{Im}\int q\,dq\int_\epsilon^\infty d\omega\frac{2\pi e^2}{q(Dq\kappa - i\omega)(Dq^2 - i\omega)}$$

In the range $Dq^2 \ll \omega \ll Dq\kappa$ the integral is of $\log^2$ type: $\displaystyle\int\frac{dq\,d\omega}{D\kappa q\omega}$

$$\frac{\Delta\nu(\epsilon)}{\nu_0} = -\frac{1}{8\pi^2\nu D}\left[\ln^2\frac{D\kappa^2}{\epsilon} - \ln^2 D\kappa^2\tau\right]$$

This does not happen for $\Delta\sigma$! Reason: gauge invariance

There is a range of parameters where $\Delta\sigma$ is still small, whereas $\Delta\nu$ is not!

$$\frac{\nu(\epsilon)}{\nu_0} = \exp\left\{-\frac{1}{8\pi^2\nu D}\left[\ln^2\frac{D\kappa^2}{\epsilon} - \ln^2 D\kappa^2\tau\right]\right\}$$

Finkelstein '83

Anderson transitions: Renormalization by e-e interaction

Renormalization Group: Finkelstein'83 σ-model with interaction

Anderson transitions: depending on the symmetry class and the character (short- vs. long-range) of the interaction, different situations:

- interaction RG-irrelevant; fixed points and critical indices unchanged

  Example: broken spin-rotation invariance + short-range interaction

- interaction RG-relevant; new fixed point and critical indices; phase diagram qualitatively unchanged

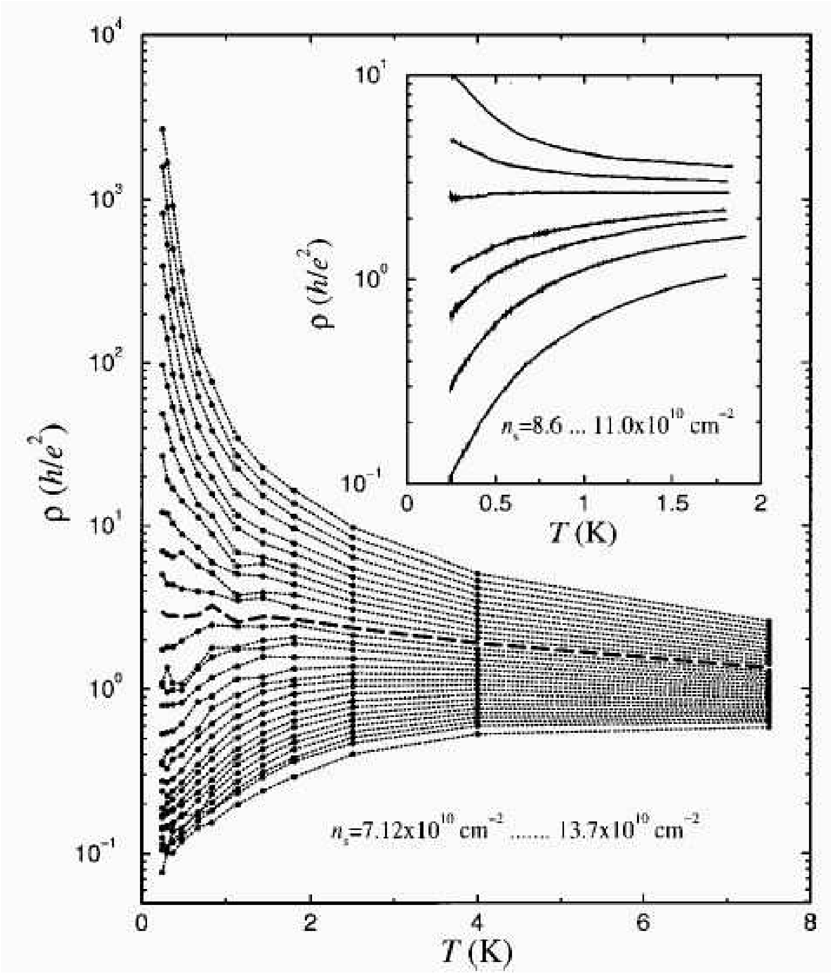
  Example: broken spin-rotation invariance + $1/r$ Coulomb interaction

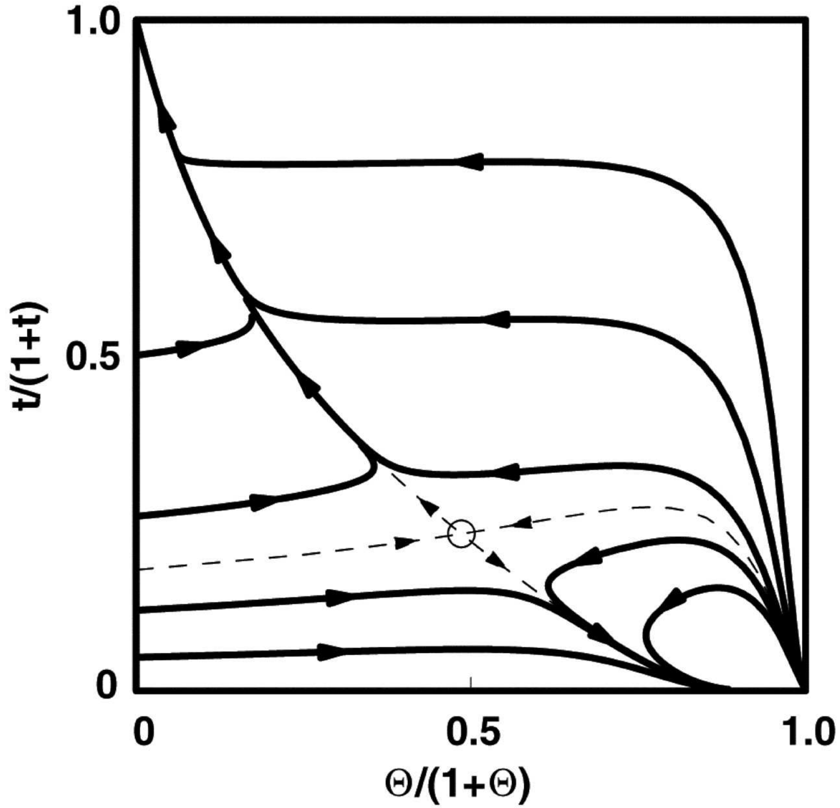
- interaction not only RG-relevant but also leads to some instabilities and new phases

  Example: preserved spin-rotation invariance + short/long-range interaction

Reviews: Finkelstein '90 ; Belitz, Kirkpatrick '94

MIT in a 2D gas with strong interaction





Kravchenko et al '94, . . .

Punnoose, Finkelstein, Science '05

(number of valleys $N \gg 1$;
in practice, $N = 2$ sufficient)

Dephasing by e-e interaction

inelastic e-e scattering processes $\longrightarrow$ electron decays, $\tau_\phi^{-1} \propto T^p$

interacting electrons $\longrightarrow$ Nyquist noise

FDT: $$\langle \varphi\varphi \rangle_{q,\omega} = -\mathrm{Im} U(q,\omega) \coth \frac{\omega}{2T}$$

Noise can be considered as classical only for $\omega \ll T$

$$U(q,\omega) = \frac{1}{U_0^{-1}(q) + \Pi(q,\omega)} \simeq \Pi^{-1}(q,\omega) = \frac{Dq^2 - i\omega}{\nu D q^2} \longrightarrow -\mathrm{Im} U \simeq \frac{\omega}{\nu D q^2}$$

$$\tau_\phi^{-1} \simeq \int_{?}^{l_T^{-1}} (dq) \frac{T}{\nu D q^2}$$

Equivalently: $$\tau_\phi^{-1} \sim \frac{1}{\nu D^{d/2}} T \int_{?}^{T} d\omega\, \omega^{d/2-2}$$

2D, quasi-1D : IR (low-q) divergence!

weak localization: self-consistent cutoff: $q_{\min} = l_\phi^{-1}$, $\omega_{\min} = \tau_\phi^{-1}$

$$\tau_\phi^{-1} \sim \left(\frac{T}{\nu D^{1/2}} \right)^{2/3}, \quad \text{quasi-1D}$$

$$\tau_\phi^{-1} \sim \frac{T}{g} \ln g\ , \quad \text{2D}$$

Altshuler, Aronov, Khmelnitskii ’82

Dephasing by e-e interaction (cont'd)

- **IR divergence:** cutoff depends on geometry of the phenomenon.

Aharonov-Bohm oscillations: harmonics decay as $\exp\{-2\pi n R/l_\phi^{\mathrm{AB}}\}$

trajectories should encircle the ring! $\longrightarrow$ **cutoff:** $q_{\min} \sim R^{-1}$

$$1/\tau_\phi^{\mathrm{AB}} \sim \frac{TR}{\nu D}\,, \qquad\qquad l_\phi^{\mathrm{AB}} = \left(D\tau_\phi^{\mathrm{AB}}\right)^{1/2} \sim \frac{\nu^{1/2} D}{T^{1/2} R^{1/2}}$$

Ludwig, ADM '04

- **Near Anderson transition:** dynamical scaling

finite frequency ω $\longrightarrow$ length scale $l_\omega \propto \omega^{-1/z}$

$\longrightarrow$ smearing of the transition, width $\propto \omega^{\zeta}$ $\qquad \zeta = 1/\nu z$

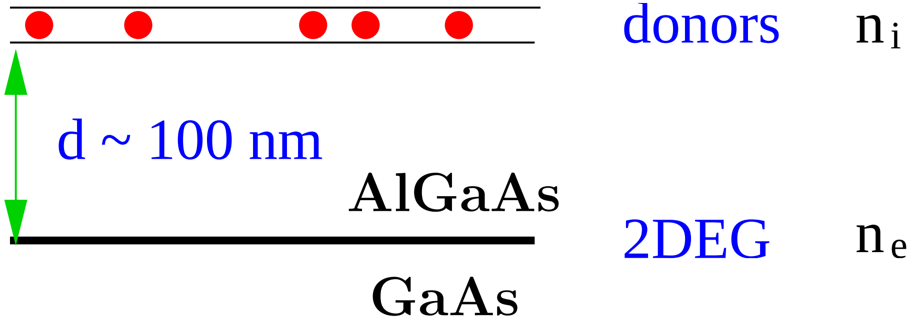
e-e interaction $\longrightarrow$ dephasing $\longrightarrow$ length scale $l_\phi \propto T^{-1/z_T}$

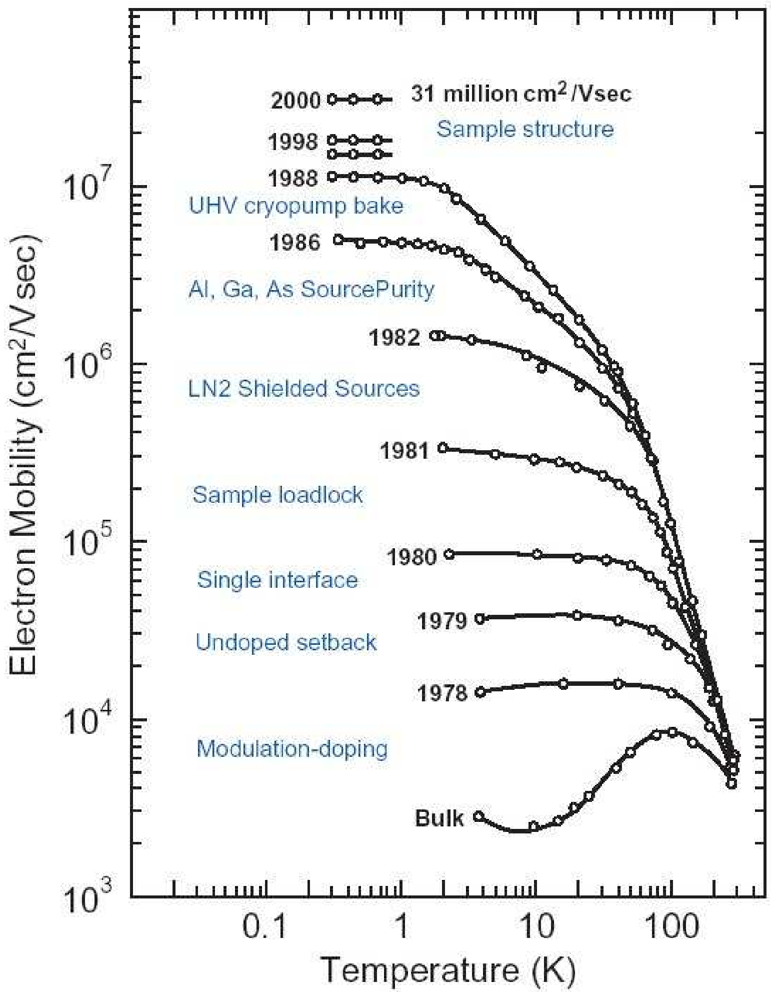
$\longrightarrow$ smearing of the transition, width $\propto T^{\kappa}$ $\qquad \kappa = 1/\nu z_T$

2D electron gas in strong magnetic fields

Standard realization: GaAs/AlGaAs heterostructures

Disorder: charged donors





Typical experimental parameters:

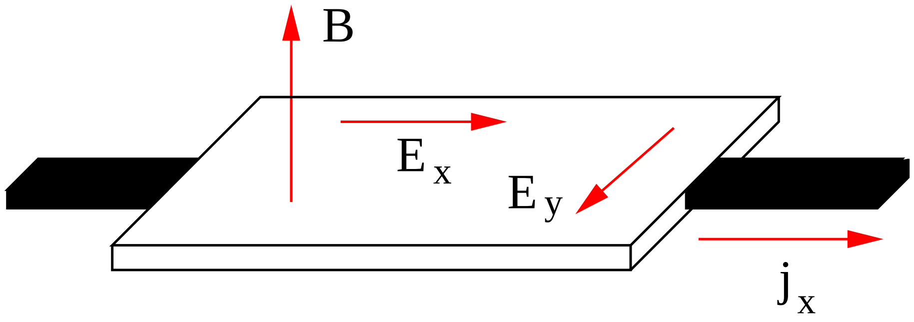
$$n_e,\ n_i \sim (1 \div 3) \cdot 10^{11}\ \mathrm{cm}^{-2}, \qquad d \sim 100\ \mathrm{nm}$$

$\longrightarrow$ $k_F d \sim 10 \gg 1$

$\longrightarrow$ weak smooth disorder

$\longrightarrow$ high mobility

Magnetotransport



resistivity tensor:

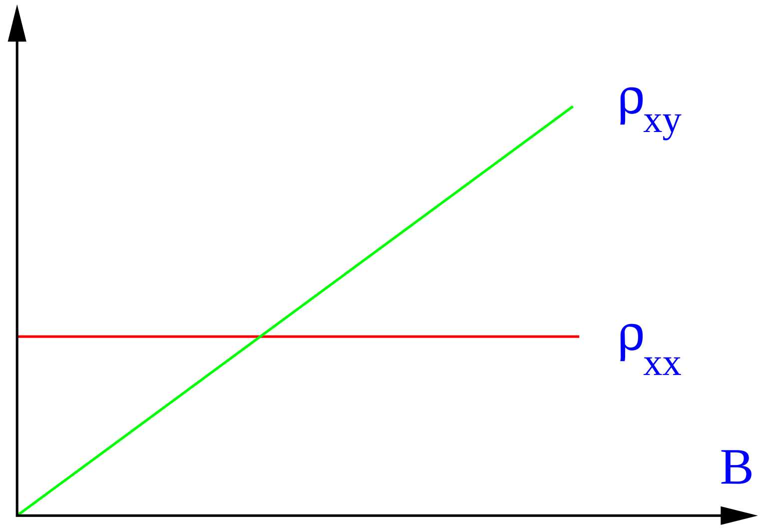
$$\rho_{xx} = E_x / j_x$$

$$\rho_{yx} = E_y / j_x$$ **Hall resistivity**

classically (Drude–Boltzmann theory):

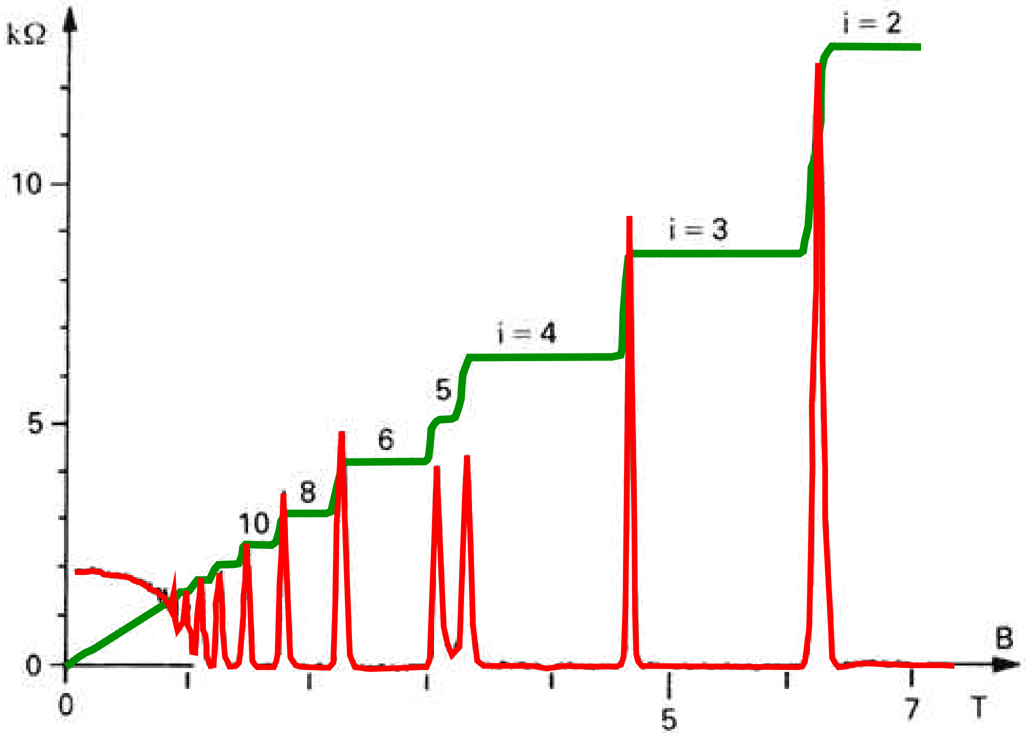
$$\rho_{xx} = \frac{m}{e^2 n_e \tau}$$ **independent of** B

$$\rho_{yx} = -\frac{B}{n_e e c}$$

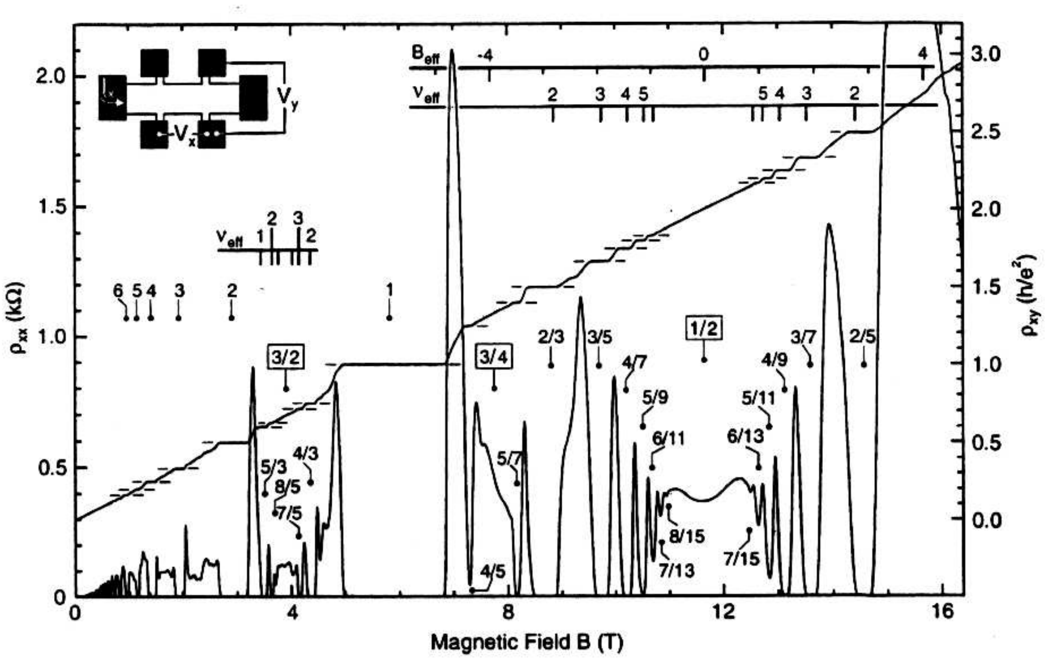


Quantum transport in strong magnetic fields

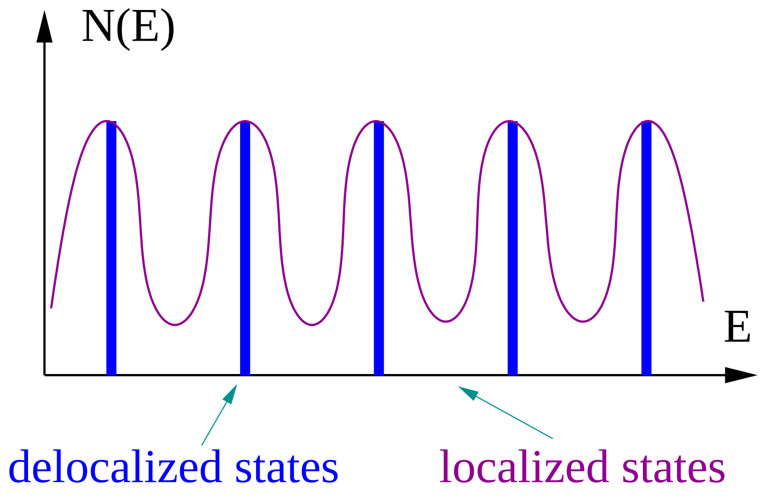
Integer Quantum Hall Effect (IQHE)



Fractional Quantum Hall Effect (FQHE)



Basics of IQHE



2D Electron in transverse magnetic field

$\longrightarrow$ Landau levels $E_n = \hbar\omega_c(n + 1/2)$

ω_c - cyclotron frequency

$\nu = \Phi_0 \frac{n}{B} = \frac{N_e}{N_\Phi}$ - filling factor

$\Phi_0 = \frac{hc}{e}$ - flux quantum

disorder $\longrightarrow$ Landau levels broadened

Anderson localization $\longrightarrow$ only states in the band center delocalized

E_F in the range of localized states $\longrightarrow$ $\begin{cases} \text{plateau in } \sigma_{xy} \\ \sigma_{xx} = 0 \end{cases}$

Quantization of σ_{xy} – ?

Hall conductivity quantization

Laughlin'81, Halperin'82

$$E_H = \frac{1}{2\pi r}\frac{1}{c}\frac{d\Phi}{dt}$$

$$\sigma_{xy} = \frac{j}{E_H} = \frac{\Delta Q}{\Phi_0/c}$$

ΔQ - charge transported as $\Phi \longrightarrow \Phi + \Phi_0$

Gauge invariance $\longrightarrow$ states are not changed

Localized states are not influenced

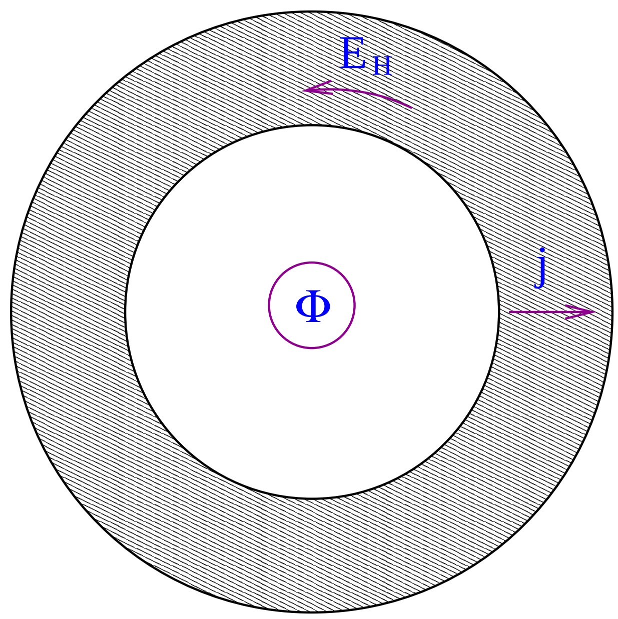
Delocalized state may transform
into another delocalized state

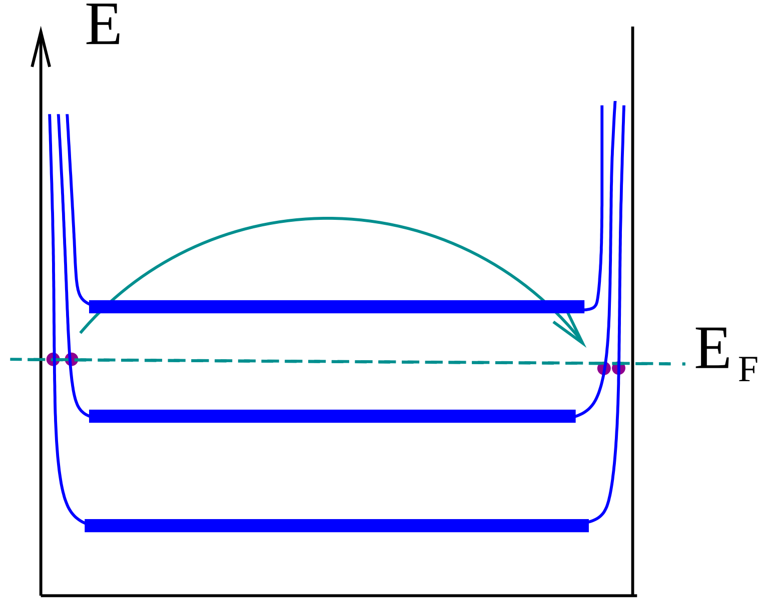
$\Longrightarrow$ $k = 0, 1, 2, \ldots$ electrons are transported

k - number of filled delocalized bands

$\longrightarrow$ $\Delta Q = ke$

$$\Longrightarrow \quad \sigma_{xy} = k\frac{e^2}{h}$$

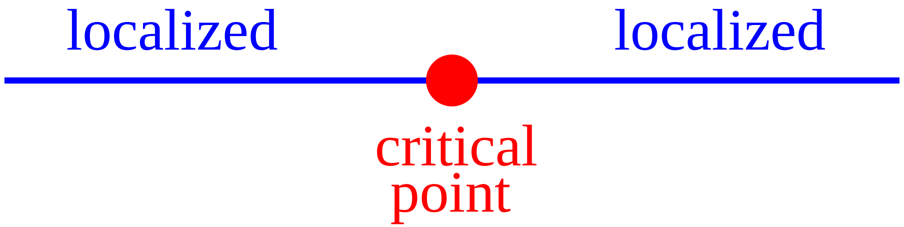


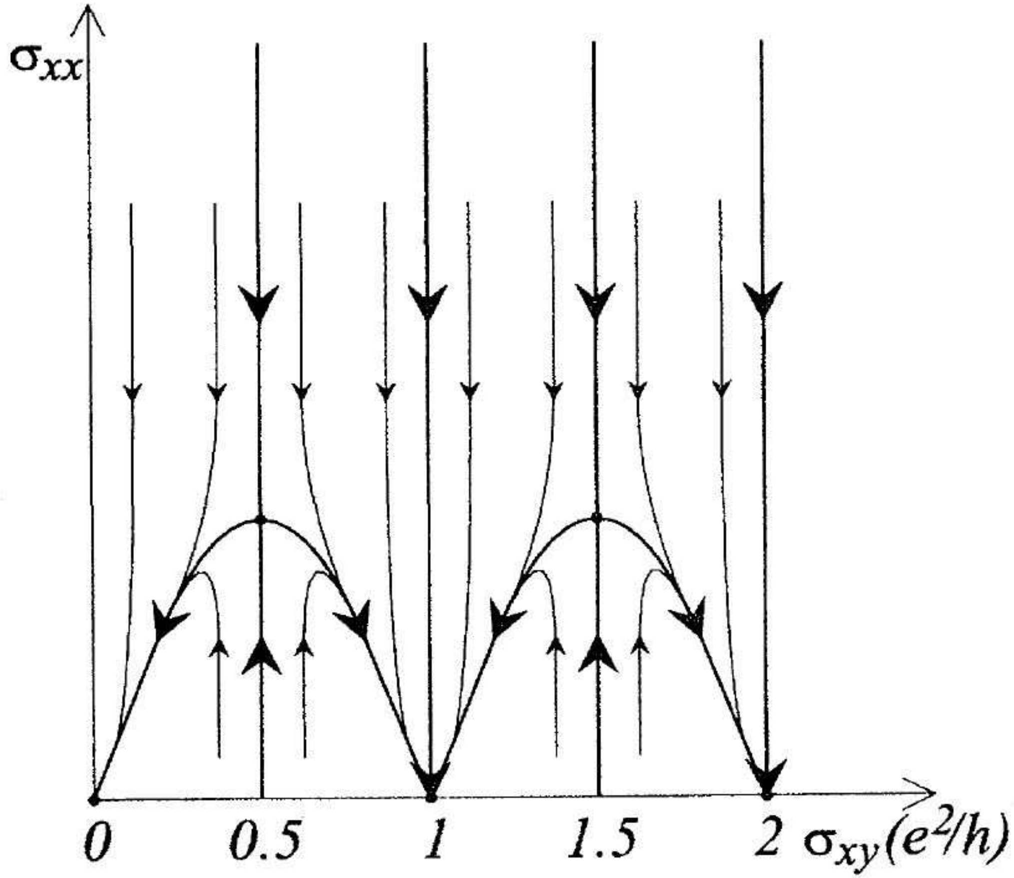


IQH transition

IQHE flow diagram

Khmelnitskii' 83, Pruisken' 84





Field theory (Pruisken):

σ-model with topological term

$$S = \int d^2r \left\{ -\frac{\sigma_{xx}}{8} \mathrm{Tr}(\partial_\mu Q)^2 + \frac{\sigma_{xy}}{8} \mathrm{Tr}\epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q \right\}$$

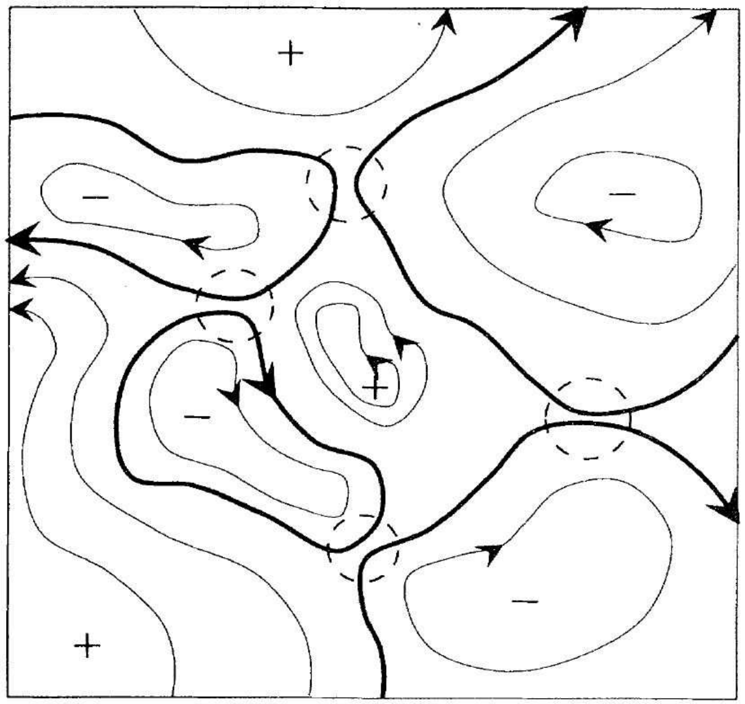
Critical behavior: Modeling and numerics

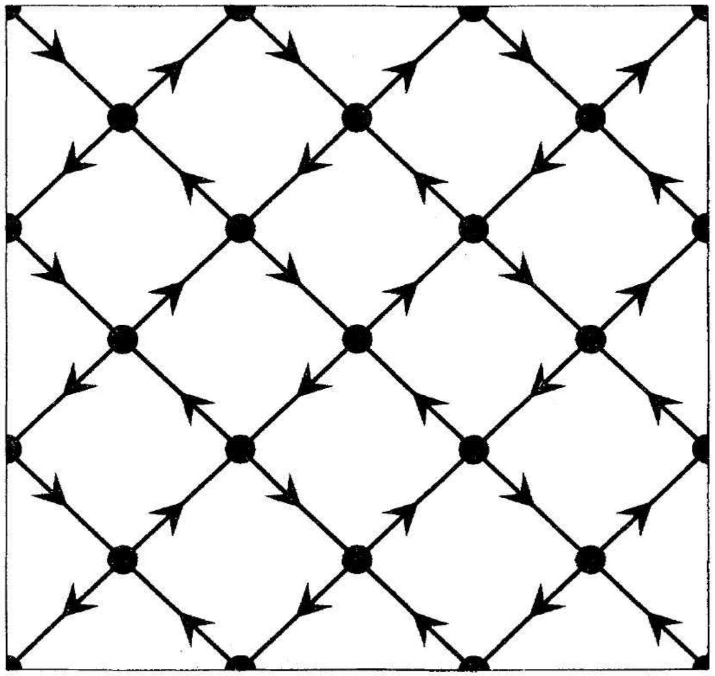
Models:

a) white-noise disorder projected to the lowest Landau level

b) Chalker-Coddington network (models a long-range disorder)

Chalker, Coddington '88

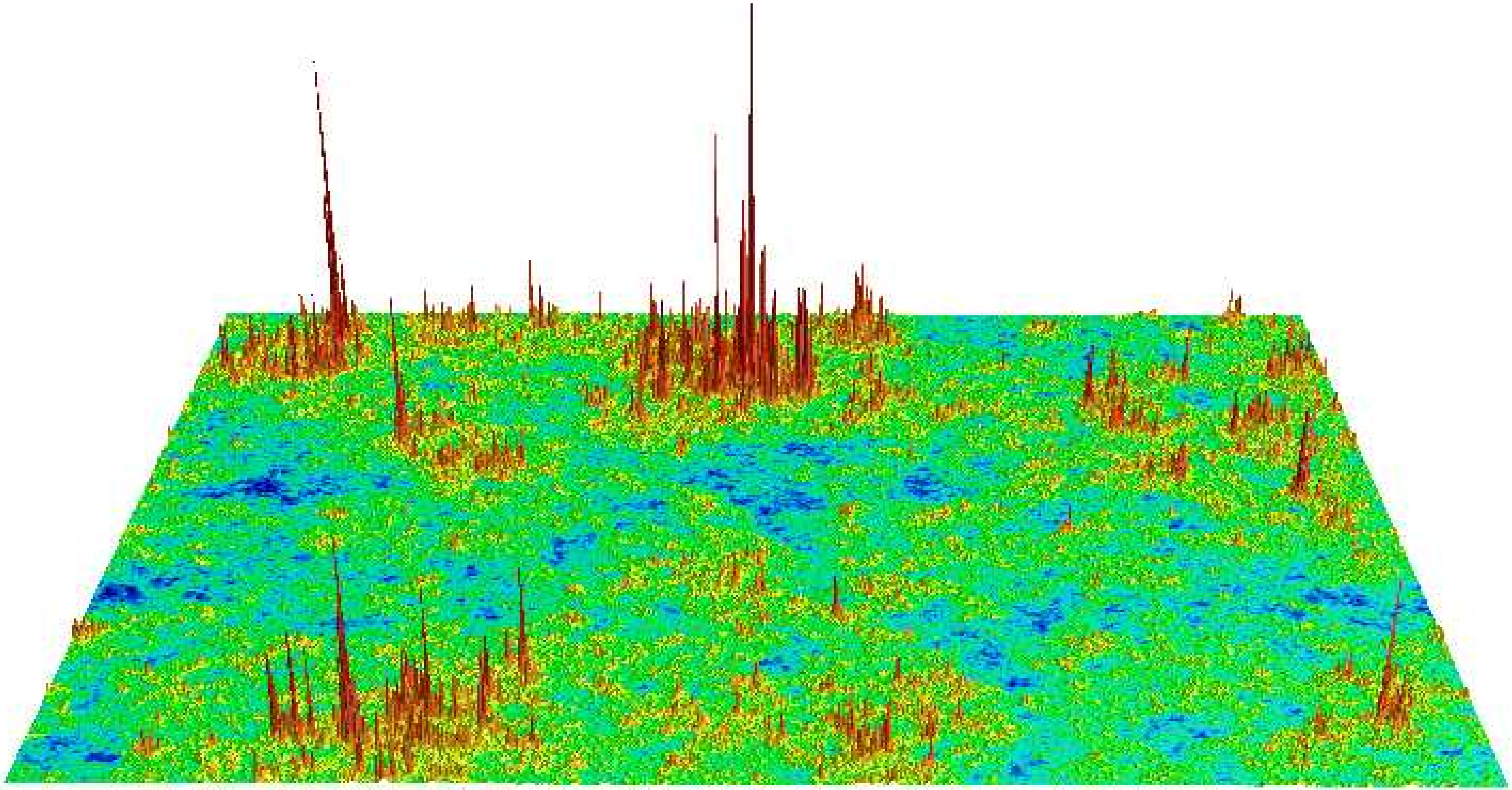




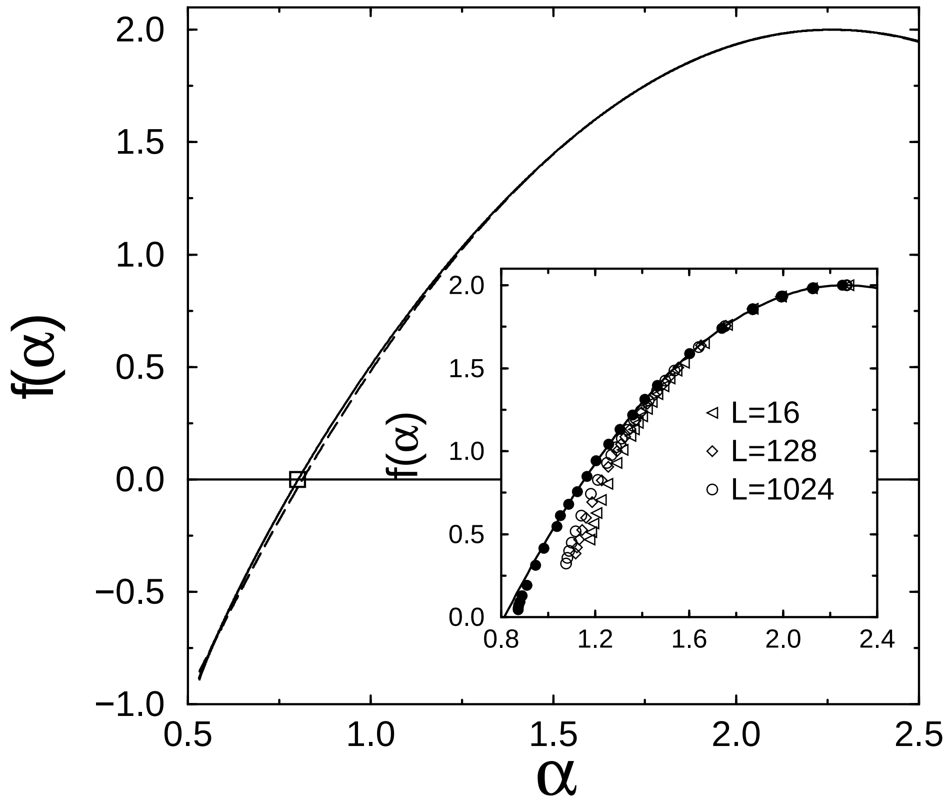
Localization length exponent $\xi \propto |E - E_c|^{-\nu}$ $\nu = 2.35 \pm 0.03$

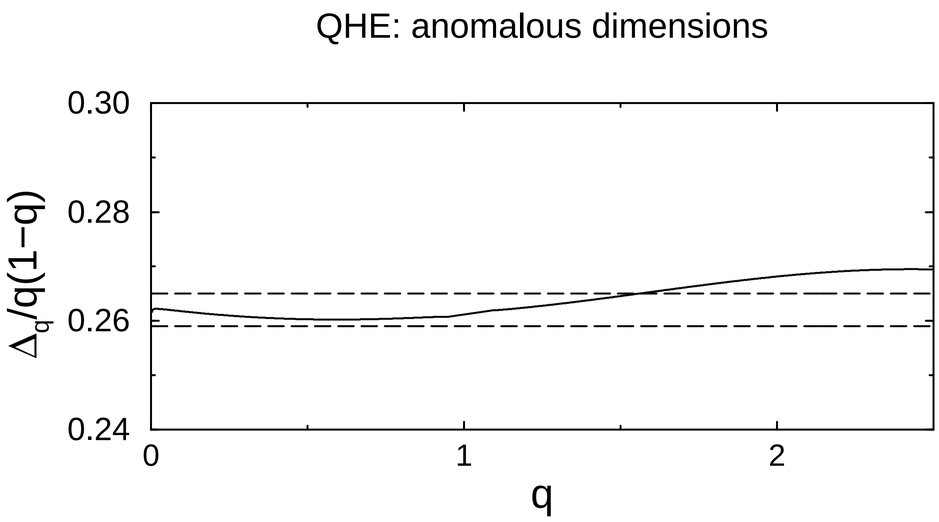
Huckestein, Kramer '90, ...

Multifractal wave functions at the Quantum Hall transition



Multifractality at the Quantum Hall critical point





⟶ spectrum is parabolic with a very high (1%) accuracy:

$$f(\alpha) = 2 - \frac{(\alpha-\alpha_0)^2}{4(\alpha_0-2)}, \qquad \Delta_q = (\alpha_0-2)q(1-q) \quad \text{with} \quad \alpha_0 - 2 = 0.262 \pm 0.003$$

Evers, Mildenberger, ADM '01

important for identification of the CFT of the Quantum Hall critical point

Critical exponents: analytical approaches

Numerics summary:

$\nu = 2.35 \pm 0.03$ $\quad$ $(= 7/3\ ?)$ $\quad$ **localization length**

$\Delta_q = (\alpha_0 - 2)q(1-q)$ $\quad$ $\alpha_0 - 2 = 0.262 \pm 0.003$ $\quad$ **multifractality**

Analytics - ?

- **attempts to identify the Conformal Field Theory**

  Zirnbauer '99 ; Bhaseen, Caux, Kogan, Tsvelik '00 ; Tsvelik '07

  models of the Wess-Zumino-Novikov-Witten type term

  lines of critical points – good to accomodate a "strange" value of $\alpha_0 - 2$

  $\nu = ?$ $\quad$ Universality - ?

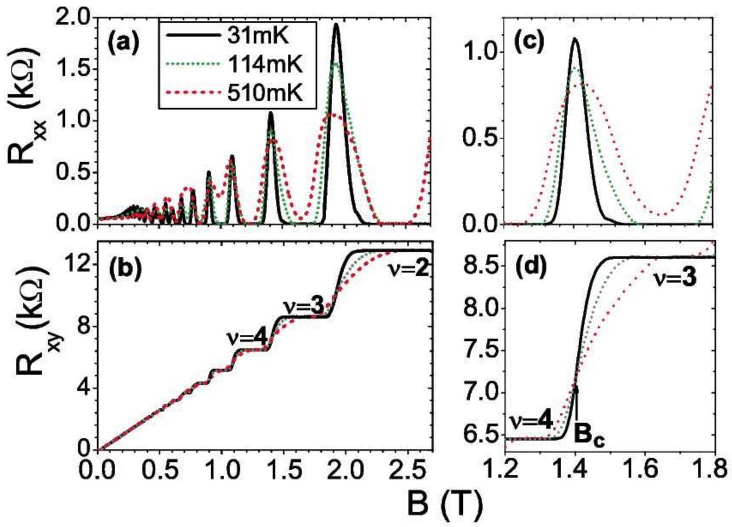
- **σ-model with topological term** $\quad$ Pruisken et al. 1984 - present

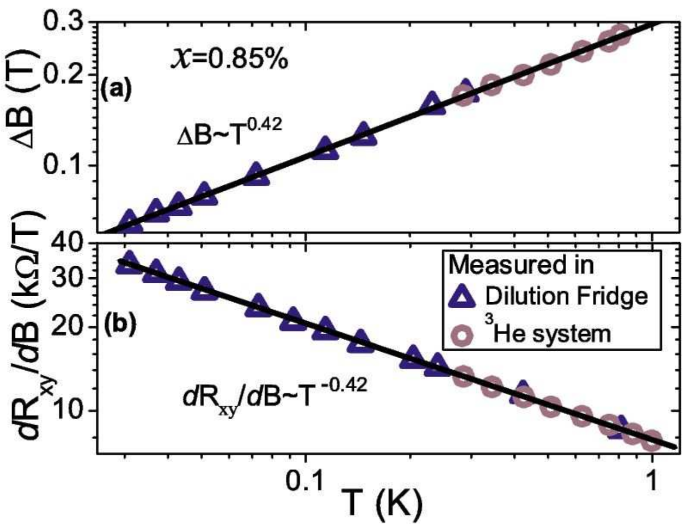
  extension of a weak-coupling RG calculation (perturbation theory + instantons) beyond the region of its validity (to $\sigma_{xx} \sim 1$)

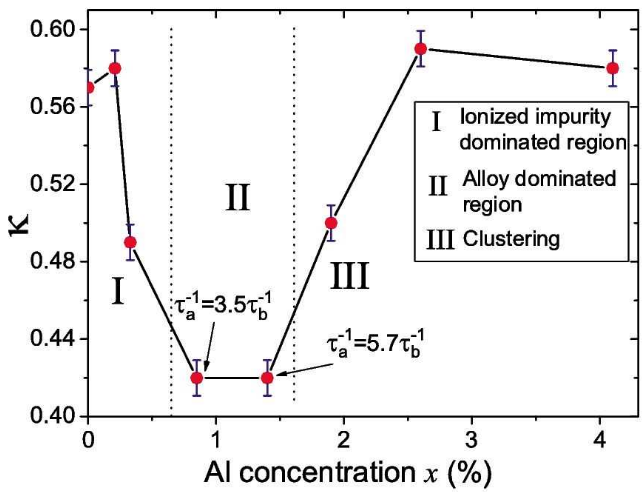
  $\longrightarrow$ $\nu \simeq 2.8,$ $\quad$ $\alpha_0 - 2 \simeq 0.14$ $\quad$ accuracy uncontrollable

Dynamical scaling: Transition width

Transition width $\quad \delta \sim T^{\kappa}\ , \quad \omega^{\zeta} \qquad \kappa,\ \zeta - ?$







Experiment: Wei et al PRL'88 ... Li et al PRL'05

measured κ depends on the nature of disorder

For short-range disorder (presumably) $\quad \kappa \simeq 0.42\ , \quad \nu \simeq 2.3$

Frequency scaling: $\zeta \simeq 0.4 \div 0.5$ $\quad$ Engel et al '93 ; Hohls et al '02

What about theory?

Dynamical scaling: Theory vs experiment

Transition width $\delta(T) \sim T^{\kappa}$ is found from $L_\phi(T) \sim \xi(\delta)$

Dephasing length $L_\phi \sim T^{-1/z_T}$ $\longrightarrow$ $\kappa = 1/z_T\nu$

analogously, for frequency scaling: $\delta(\omega) \sim \omega^{\zeta}$, $L_\omega \sim \omega^{-1/z}$, $\zeta = 1/z\nu$

- short-range interaction: interaction irrelevant in the RG sense,

  fixed point unchanged $\longrightarrow$ $\nu \simeq 2.35$, $z = 2$ unchanged $\longrightarrow \zeta \simeq 0.21$

  $z_T \simeq 1.2$ (numerics) $\longrightarrow$ $\kappa \simeq 0.36$

- Coulomb interaction: interaction relevant, fixed point changed,

  indices not known

Experiment $\longrightarrow$ $z,\ z_T \simeq 1$ ($\longrightarrow$ Coulomb interaction relevant)

but $\nu = 2.3$ (as for non-interacting system)

$\longrightarrow$ persistent puzzle

More about IQHE and its relatives

- **Dirac fermions, graphene:**

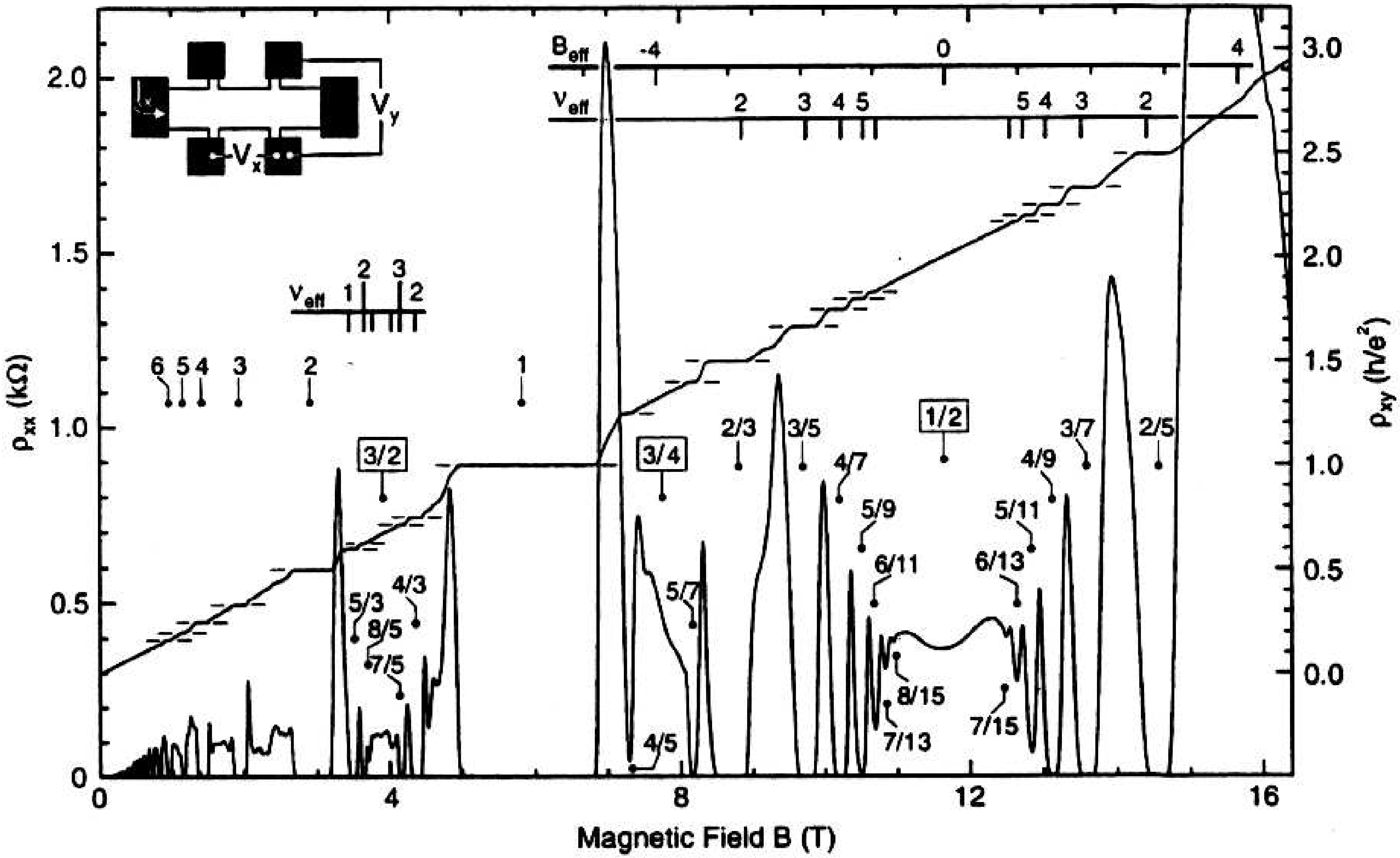
anomalous IQHE ; relation of conductivity at Dirac point to IQHE

- **unconventional (Bogoliubov-de Gennes) symmetry classes:**

spin and thermal QHE

$\longrightarrow$ **coming soon**

Fractional Quantum Hall Effect



Fractional Quantum Hall Effect. Theory

Coulomb interaction $\longrightarrow$ strongly correlated ground state:

incompressible quantum fluid

wave function for $\nu = 1/3$ $[\nu = 1/(2p+1)]$ R.B. Laughlin, 1983

$$\Psi = \prod_{j<k} (z_j - z_k)^3 \prod_j e^{-|z_j|^2/4l_B^2} \qquad z = x + iy$$

Attachment of zeroes (“vortices”) to particles

N_Φ flux quanta, N particles $N/N_\Phi = \nu$ filling factor

Lowest Landau level $\longrightarrow$ N_Φ zeroes

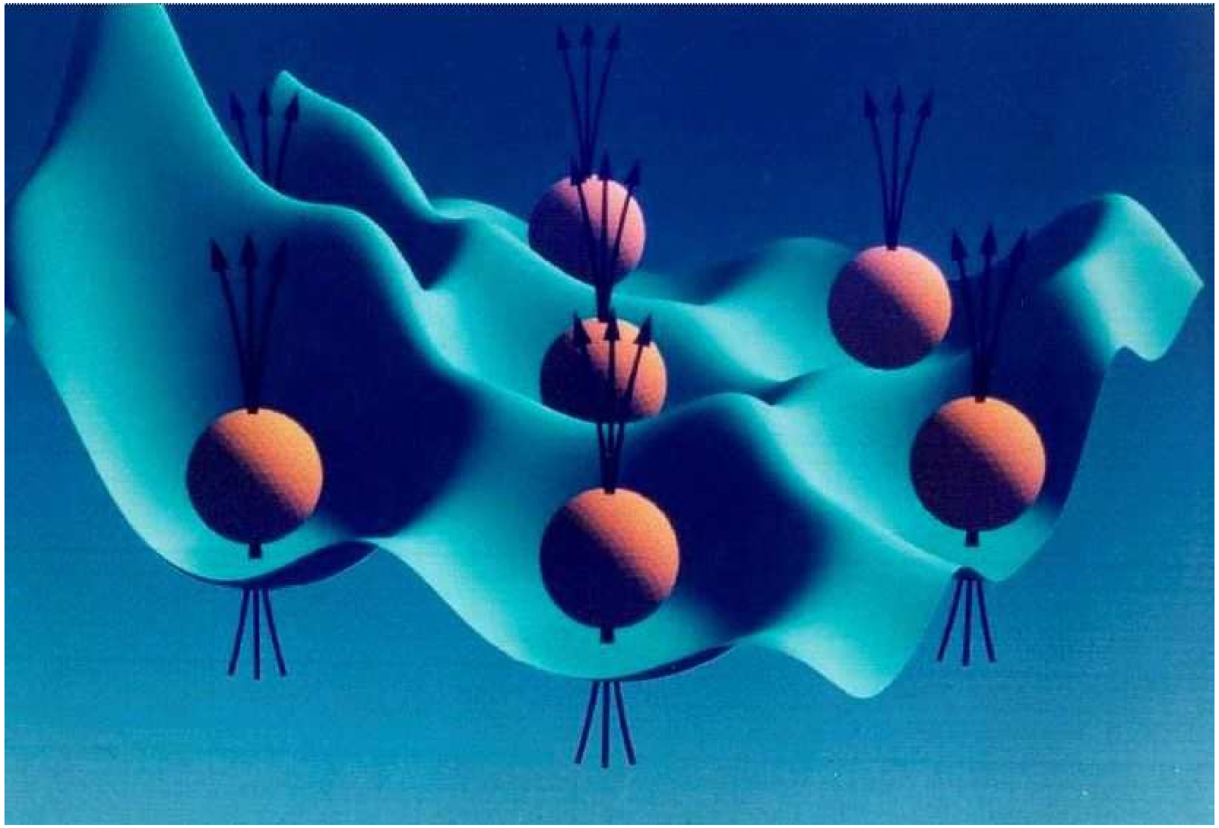
$$\longrightarrow \frac{N_\Phi}{N} = \nu^{-1} = 3$$ zeroes per particle

Fractional Quantum Hall Effect. Theory (cont'd)

Field-theoretical description: Chern-Simons theory

(Girvin, MacDonald,. . .)

electron + 3 flux quanta ⟶ "composite boson" ⟶ condensation



Excitations (quasiparticles): charge e/3

localization of quasiparticles ⟶ FQH plateaus

Hierarchy of FQHE states

Laughlin states: filling factor $\nu = 1/(2m+1)$

Experiment: FQHE also at $\nu = 2/5,\ 3/7, \ldots$

Hierarchy theory — Haldane 1983, Laughlin 1984, Halperin 1984

electrons ($+\ m$ flux quanta) $\longrightarrow$ condensation $\nu = \dfrac{1}{m}$

m – odd

$\longrightarrow$ quasiparticles $\longrightarrow$ condensation $\nu = \dfrac{1}{m \pm \frac{1}{p_1}}$

p_1 – even

$\longrightarrow$ quasiparticles $\longrightarrow$ condensation $\nu = \dfrac{1}{m \pm \frac{1}{p_1 \pm \frac{1}{p_2}}}$

p_2 – even

$\longrightarrow \ldots$

problems:

- why do $\nu = \dfrac{p}{2p \pm 1}, \quad \dfrac{p}{4p \pm 1}$ dominate in experiment?

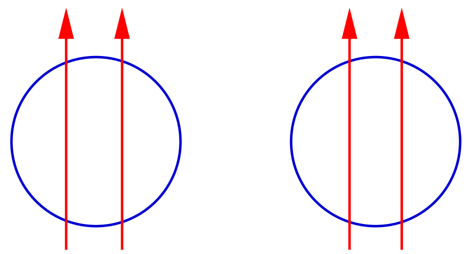
hierarchy: $1/3 \longrightarrow 2/5 \longrightarrow 3/7 \longrightarrow 4/9 \longrightarrow 5/11 \longrightarrow 6/13 \longrightarrow \ldots$

- properties of the $\nu = 1/2$ state?

Composite fermions

Jain 1989:

Electron + 2 flux quanta $\longrightarrow$ composite fermion (CF)



CF's are subjected to the effective magnetic field

$$B_{eff} = B - B_{1/2} \qquad\qquad B_{1/2} = 2\Phi_0 n$$

- **half filling:** $\nu = 1/2 \longrightarrow B = B_{1/2} \longrightarrow B_{eff} = 0$
- **near $\nu = 1/2$:** $\nu_{eff} \equiv \dfrac{\Phi_0 n}{|B_{eff}|} \longrightarrow \nu = \dfrac{\nu_{eff}}{2\nu_{eff} \pm 1}$

$$\nu_{eff} = p \qquad \longleftrightarrow \qquad \nu = \frac{p}{2p \pm 1}$$

IQHE of CF's — **dominant FQHE-series**

Field-theoretical formulation

Chern-Simons (CS) gauge theory

Lopez, Fradkin 1991; Halperin, Lee, Read 1993

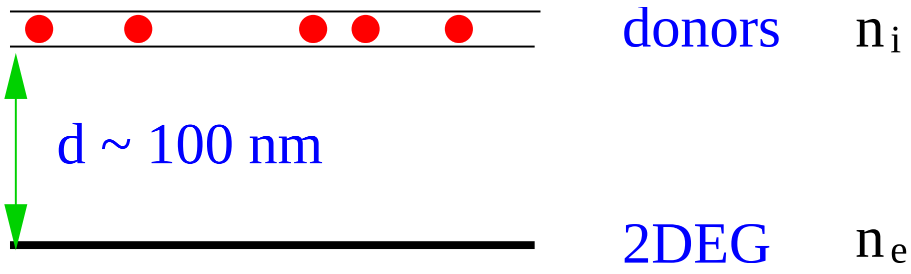
$$
\begin{aligned}
L &= L_0 && \leftarrow \text{free particle in } B_{\text{eff}} \\
&+ \int d^2r \left[a_0 \psi^* \psi + \frac{-i}{2m} (\psi^* \partial_i \psi - \partial_i \psi^* \cdot \psi - i a_i \psi^* \psi) a_i \right] \\
&- \frac{1}{4\pi} \int d^2r\, a_0 \epsilon^{ij} \partial_i a_j && \leftarrow \text{Chern} - \text{Simons term} \\
&- \frac{1}{2} \int d^2r\, d^2r' \psi^*(\mathrm{r}) \psi(\mathrm{r}) V_{\text{Coul}}(\mathrm{r} - \mathrm{r}') \psi^*(\mathrm{r}') \psi(\mathrm{r}')
\end{aligned}
$$

CF's – quasiparticles describing lower-energy physics of a strongly correlated electron system near $\nu = 1/2$

differ from electrons in many respects:

- effective mass – set by e-e interaction, $m_* \sim 10 \div 15\, m_e$ (GaAs)
- effective magnetic field B_{eff}
- effective disorder: random magnetic field
- gauge-field interaction

Effect of disorder



Typical experimental parameters:

$n_e \sim \; n_i \sim (1 \div 2) \cdot 10^{11} \, \mathrm{cm}^{-2}$

$d \sim 100$ nm $\quad \longrightarrow \quad k_F d \sim 15 \gg 1 \quad$ weak smooth disorder

Bare potential of impurity $\quad V(q) = \frac{2\pi e^2}{\varepsilon q} e^{-qd}$

$\longrightarrow$ screening by CF's interacting via Coulomb and Chern-Simons $\longrightarrow$

- screened potential $\quad V(q) = \frac{2\pi}{m^*} e^{-qd}$
- effective magnetic field $\quad b(q) = 4\pi e^{-qd}$

$k_F d \gg 1 \quad \longrightarrow \quad$ random magnetic field constitutes the dominant mechanism of scattering

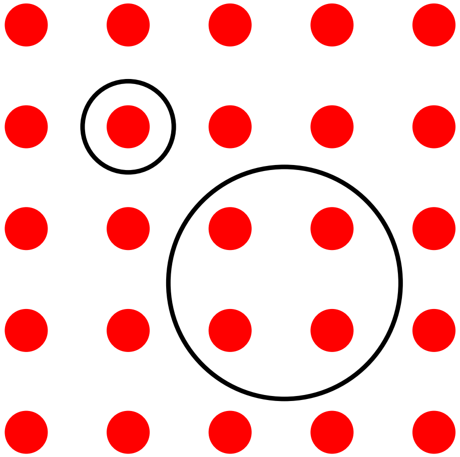
$$\langle h(q) h(-q) \rangle = (4\pi)^2 n_i e^{-2qd}$$

How to "observe" composite fermions?

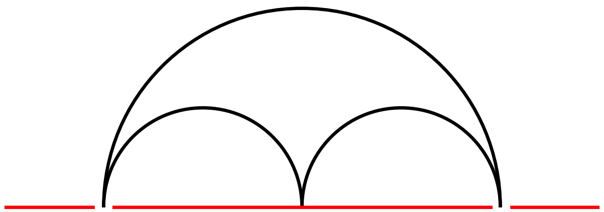
$B_{\text{eff}} \;\rightarrow\;$ effective cyclotron radius $R_c^{\text{eff}} = \dfrac{\hbar c\sqrt{4\pi n_e}}{eB_{\text{eff}}}$

$B_{\text{eff}} \ll B \quad \longrightarrow \quad R_c^{\text{eff}} \gg R_c$

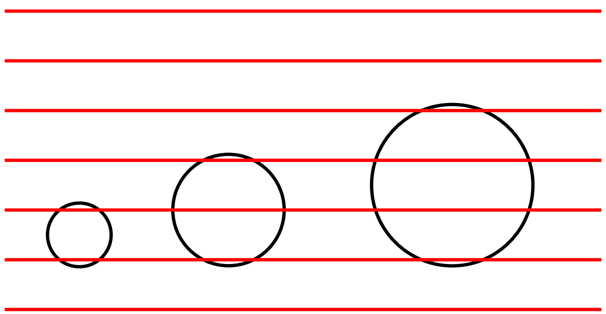
Geometric resonances:



antidot arrays



magnetic focussing

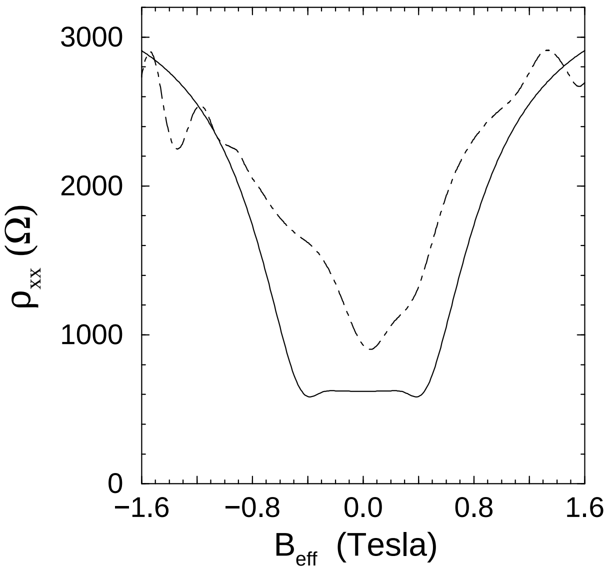


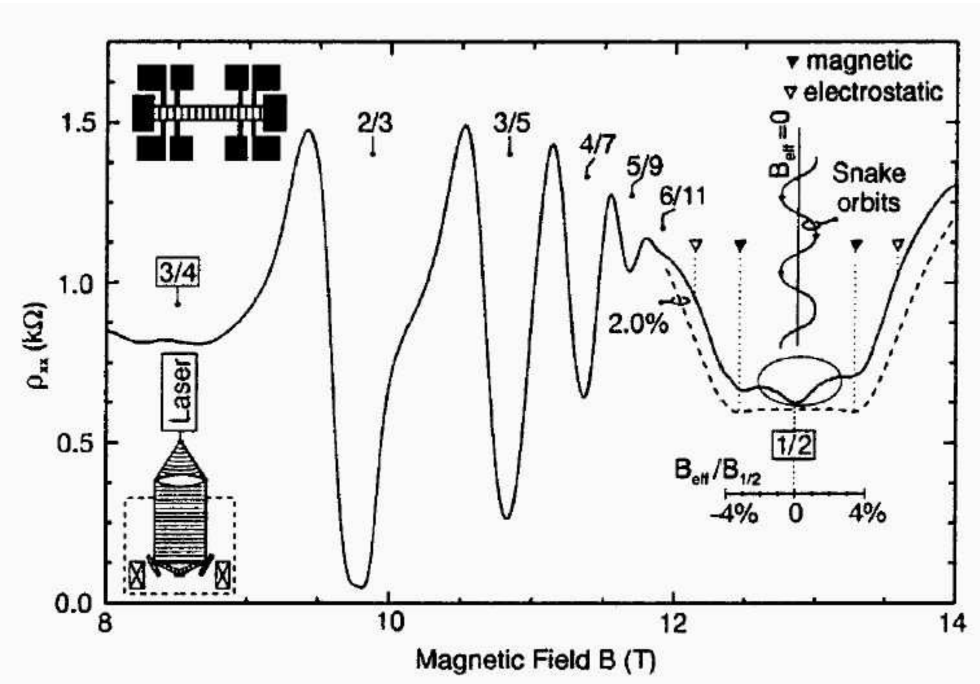
lateral superlattice or
surface acoustic wave

Experiments: Willett, Störmer, Smet, Goldman

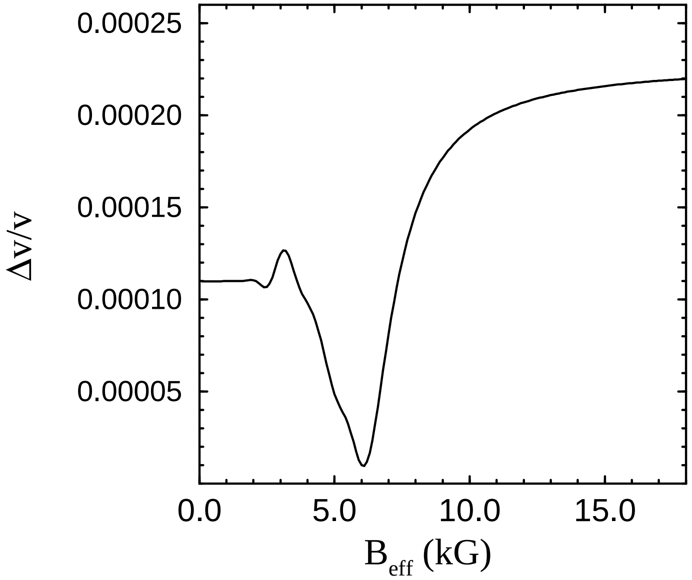
Composite fermions: Experimental confirmations

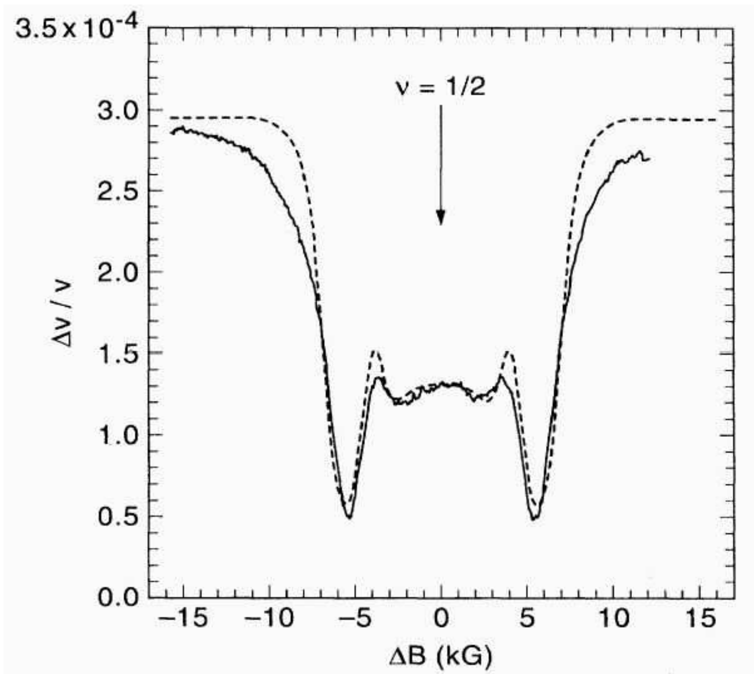
modulated structure





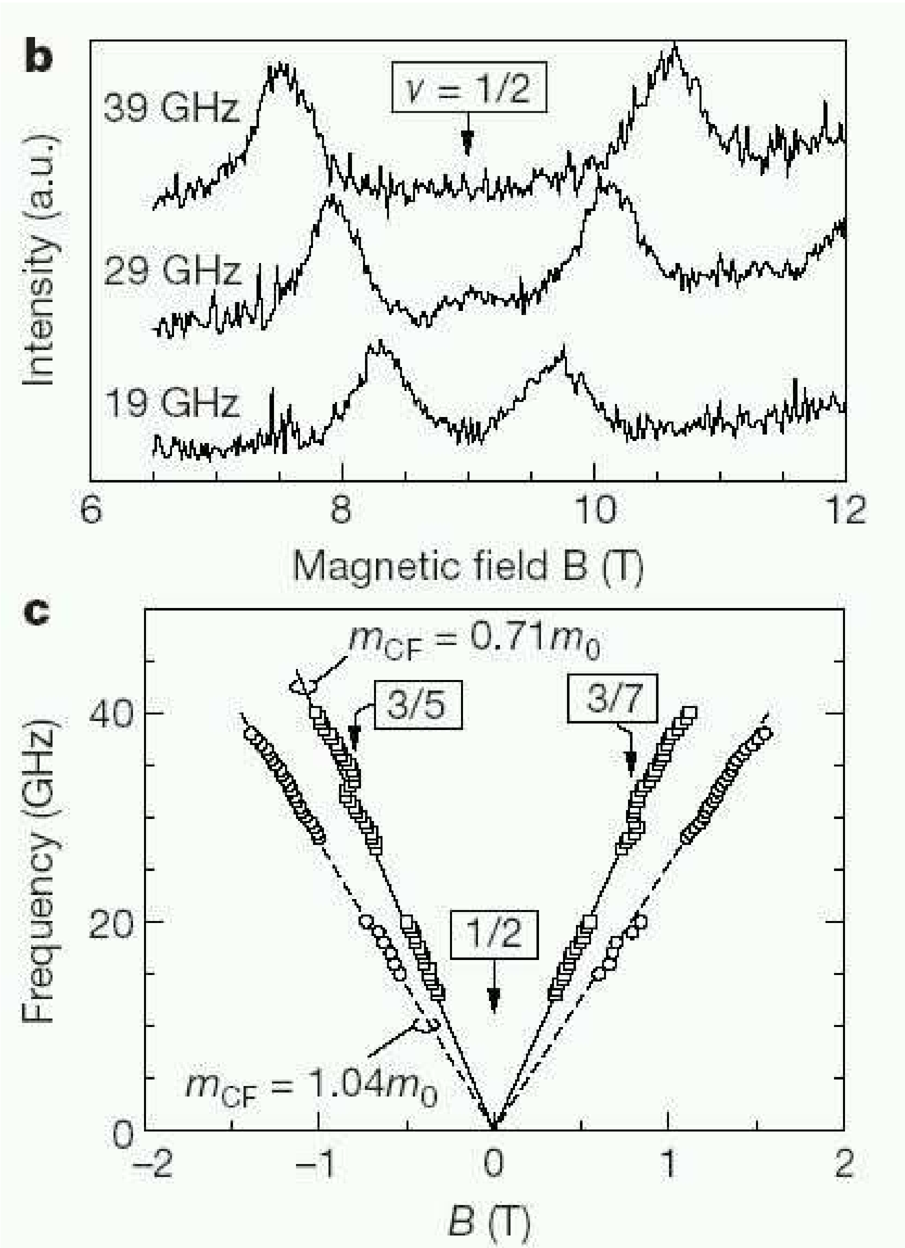
surface acoustic wave

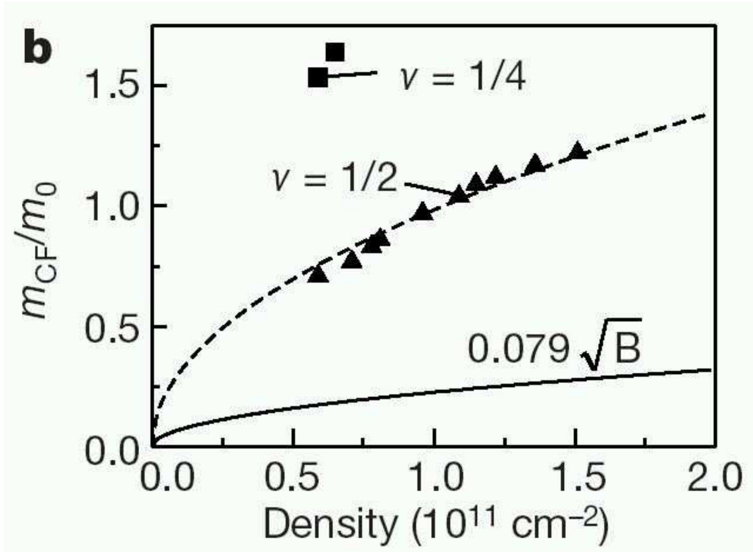




Composite fermions: Cyclotron resonance

Kukushkin, Smet, von Klitzing, Wegscheider, Nature ’02





CF effective mass

set by Coulomb interaction:

$$k_F^2/m_* \sim e^2/r \quad \longrightarrow \quad m_* \propto n^{1/2}$$

Composite fermions: Gauge field interaction

Chern-Simons interaction + RPA

$\longrightarrow$ transverse gauge-field fluctuations singular at low q, ω

$$D_\perp(\omega, q) = \frac{1}{\chi q^2 - i(k_F/2\pi)\,\omega/q} \qquad \chi = 1/8\pi m^*$$

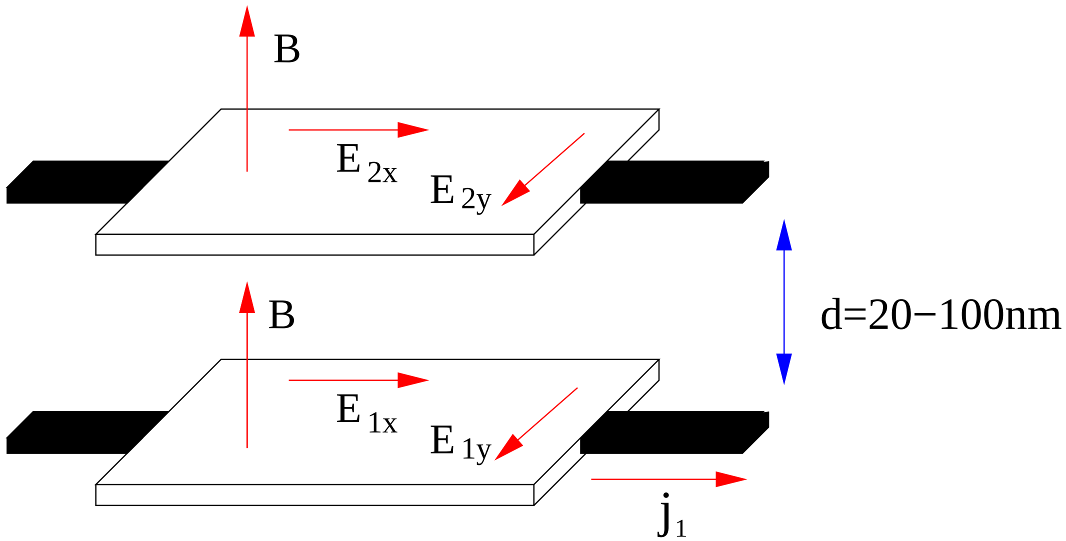
$\longrightarrow$ enhanced (compared to Fermi liquid) inelastic scattering rate

Manifestations:

- quantum correction to resistivity due to interplay of interaction and disorder
- dephasing rate
- Coulomb drag in double-layer systems

Coulomb drag

Response of the "passive" layer (2) to a current in the "active" layer (1) mediated by the Coulomb Interaction.



transresistivity (drag resistivity):

$$\rho^D_{\alpha\beta} = -E_{2\alpha}/j_{1\beta} \simeq \rho^{(1)}_{\alpha\gamma}\sigma^D_{\gamma\delta}\rho^{(2)}_{\delta\beta}$$

transconductivity:

more convenient for diagrammatics

$$\sigma^D_{\alpha\beta} = -j_{2\alpha}/E_{1\beta}$$

$$\rho^D_{xx} = \frac{\hbar^2}{e^2 n_1 n_2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{2T\sinh^2(\omega/2T)}\int\frac{d^2q}{(2\pi)^2}\, q_x^2\, |U(\omega,\mathrm{q})|^2 \mathrm{Im}\Pi_1(\omega,\mathrm{q})\mathrm{Im}\Pi_2(\omega,\mathrm{q})$$

Zheng, MacDonald '93; Jauho, Smith '93; Kamenev, Oreg '95; Flensberg *et al.* '95

$U(\omega,\mathrm{q})$ – interaction, $\Pi_i(\omega,\mathrm{q})$ – density-density response function

Composite fermions: Coulomb drag

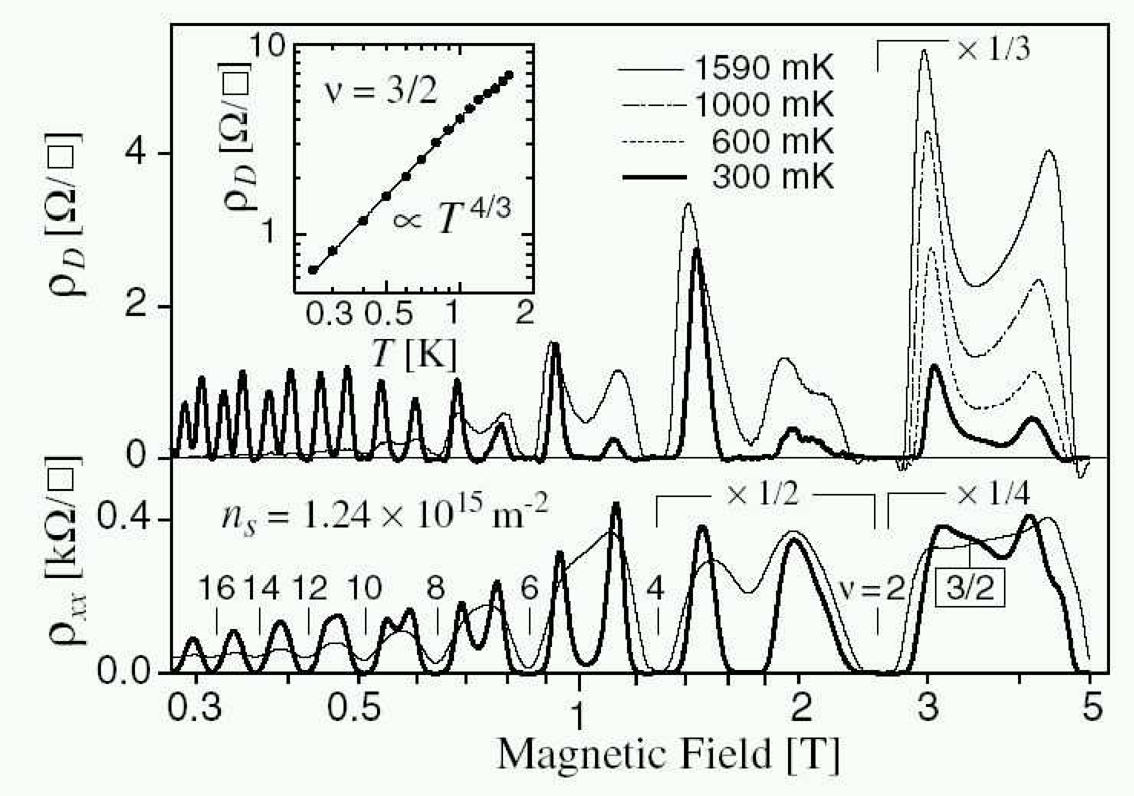
Electrons in low B (Fermi liquid): characteristic $\omega \sim T$, $q \sim d^{-1}$

$\longrightarrow$ $\rho^D \propto T^2$ Fermi-liquid inelastic scattering rate

CF's at $\nu = 1/2$: singular interaction $D_\perp(\omega, q) = \dfrac{1}{\chi q^2 - i(k_F/2\pi)\,\omega/q}$

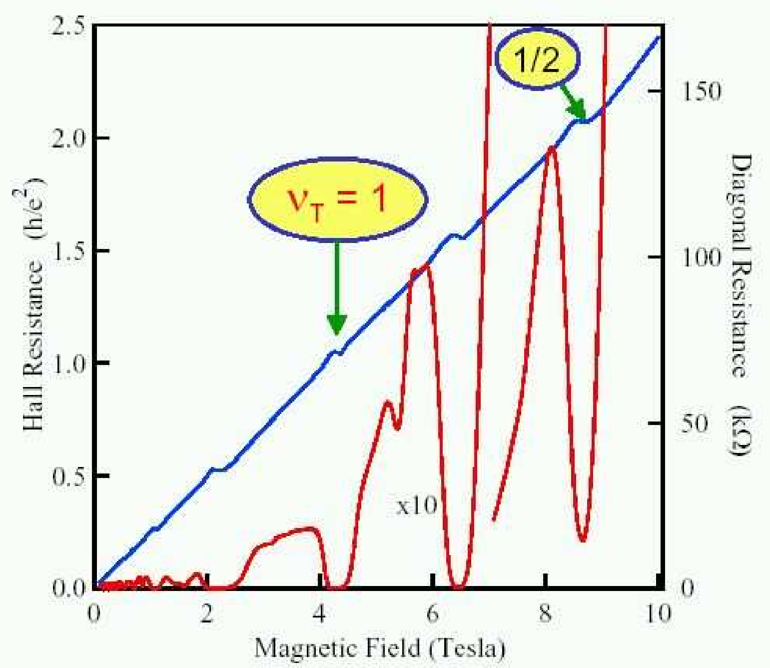
$\longrightarrow$ $q \sim T^{1/3}$ $\longrightarrow$ $\rho^D \propto T^{4/3}$

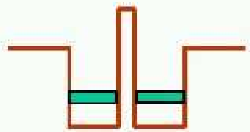
Kim, Millis '97 ; Sakhi '97 ; Ussishkin, Stern '97



Muraki et al '04

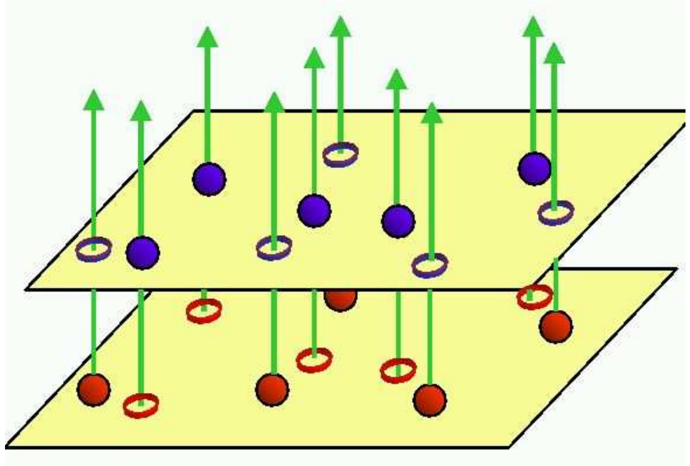
Strongly coupled bilayers





$\nu_T = 1 = \frac{1}{2} + \frac{1}{2}$

$\nu_T = \frac{1}{2} = \frac{1}{4} + \frac{1}{4}$



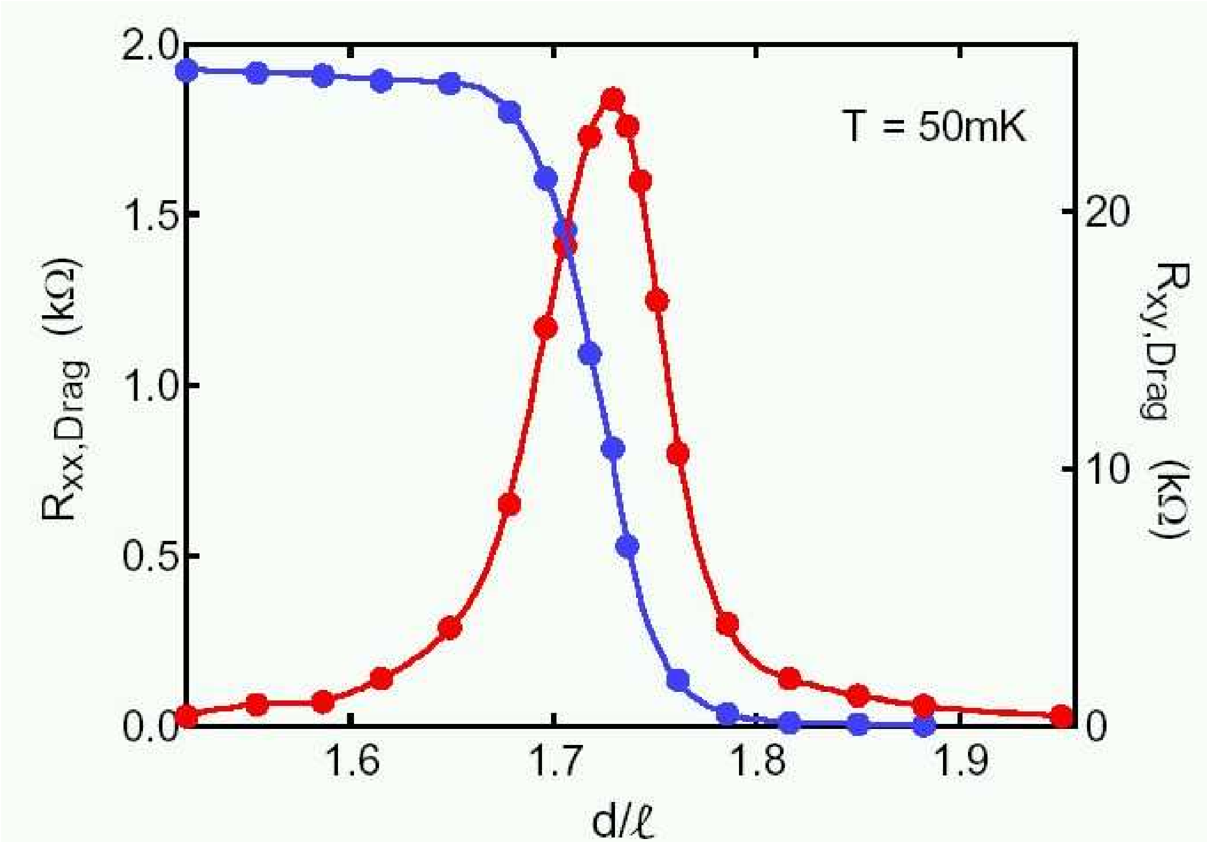
Quantized Hall state in a strongly coupled bilayer

Spontaneous interlayer phase coherence:

$$\Psi(z_1, z_2, \ldots) = \prod_{i<j}(z_i - z_j) \prod_j e^{-|z_j|^2/4l_B^2} \prod_k \frac{1}{\sqrt{2}}(|\uparrow\rangle + e^{i\phi}|\downarrow\rangle)$$

From two composite-fermion states to excitonic condensate

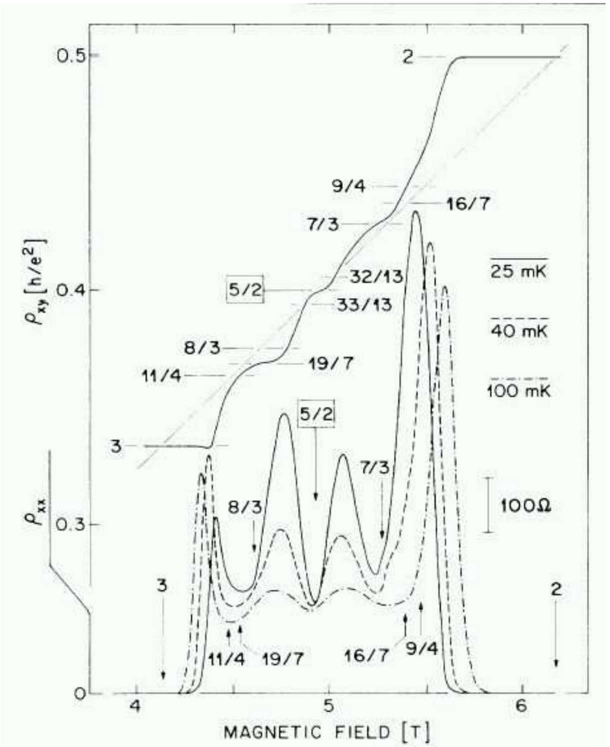
Coulomb drag – indicator of spontaneous phase coherence

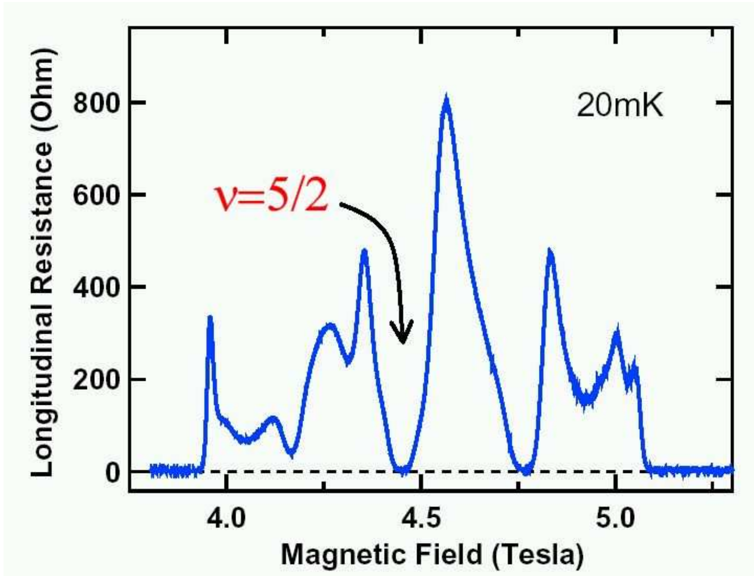


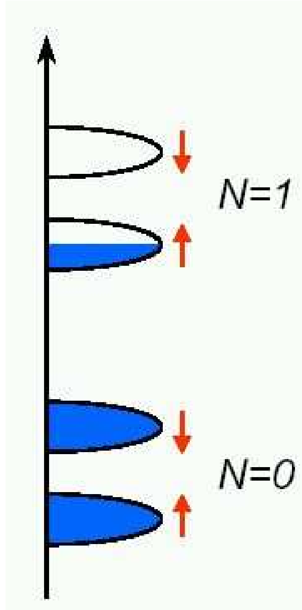
Quantum phase transition $1/2+1/2 \longrightarrow 1$ with decreasing layer separation

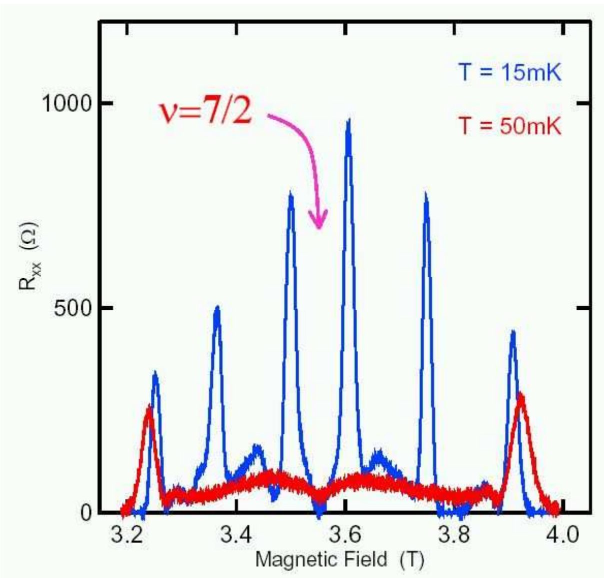
$\nu = 5/2$ & $7/2$

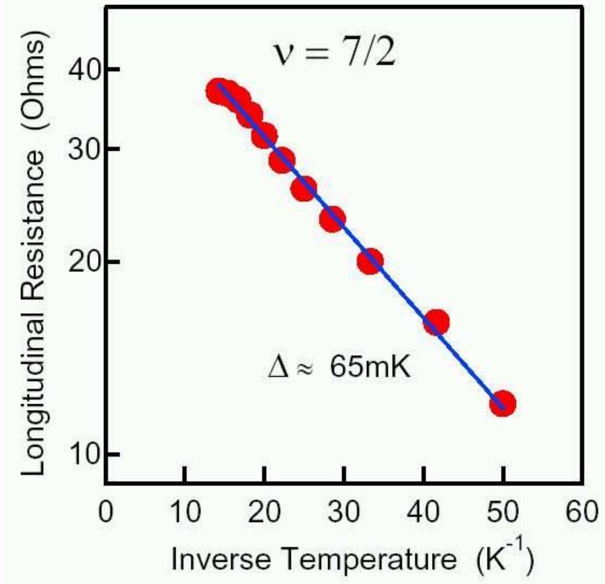
First observation: Willett et al '87





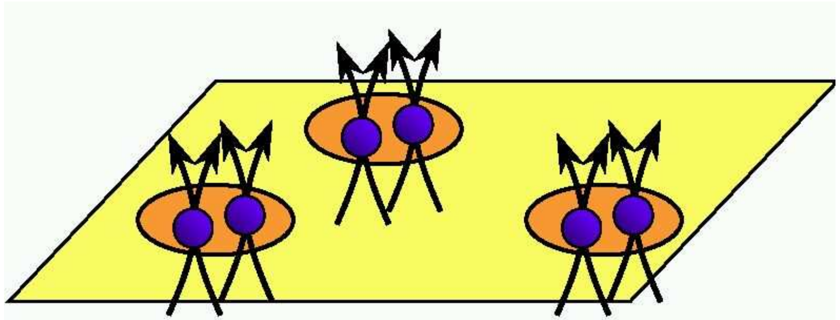






FQHE states with even denominator?!

Composite fermions: Pairing



Transverse ("magnetic") gauge-field interaction

$\longrightarrow$ attraction of CF's with opposite velocities

$\longrightarrow$ possibility of superconducting pairing!

state fully spin polarized $\longrightarrow$ p-wave pairing

Moore, Read '91 ; Read, Green '00

$$\Psi(z_1, z_2, \ldots) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^2 \prod_j e^{-|z_j|^2/4l_B^2}$$

$$\mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) = \frac{1}{2^{N/2}(N/2)!}\,\mathcal{A}\left\{\frac{1}{z_1 - z_2}\,\frac{1}{z_3 - z_4}\,\frac{1}{z_{N-1} - z_N}\right\}$$ Pfaffian

(square root of determinant) $\mathcal{A}$ – antisymmetrization

BCS-type superconducting state with pairing function $1/z$

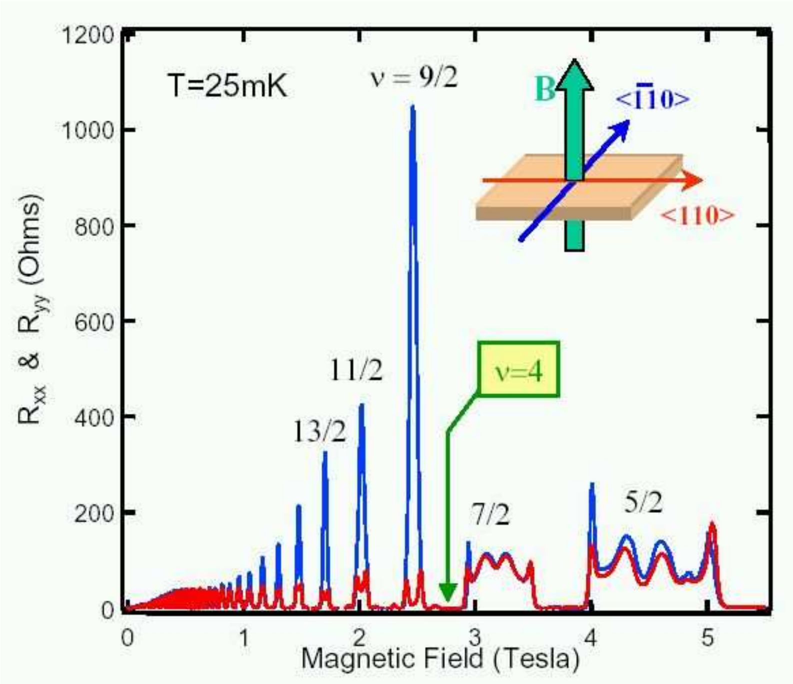
Excitations: non-abelian statistics

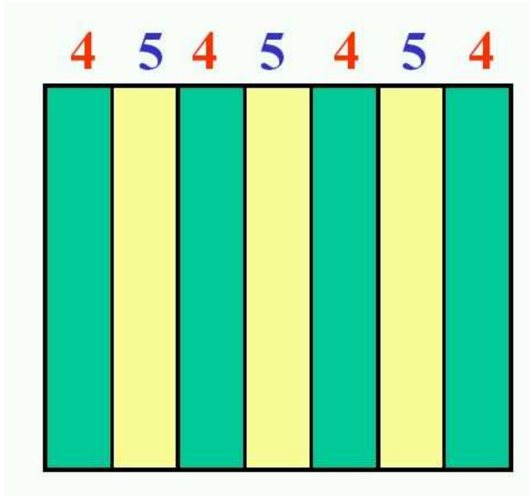
High Landau levels: Electronic liquid crystals

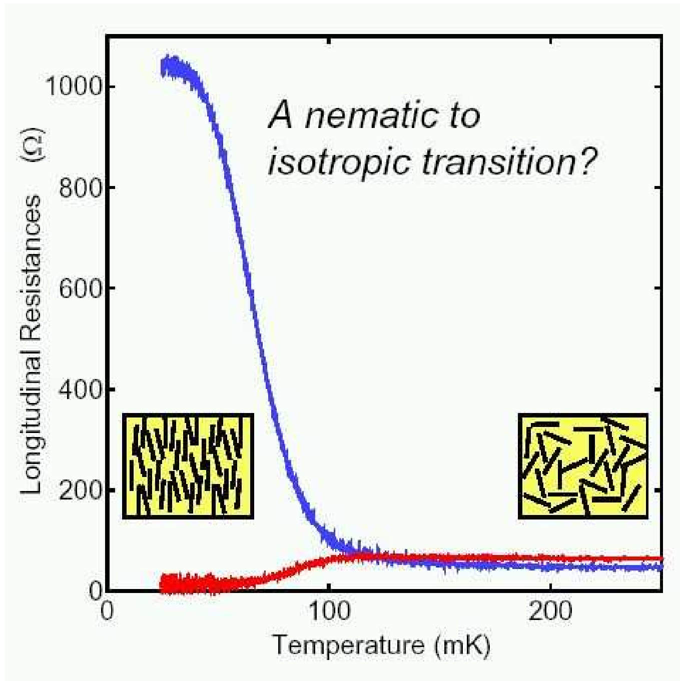
$N = 0$ ($\nu = 1/2,\ 3/2$) – composite fermions

$N = 1$ ($\nu = 5/2,\ 7/2$) – paired composite fermions

$N \geq 2$ ($\nu = 9/2, \ldots$) – ?







Strong transport anisotropy indicates formation of a striped phase

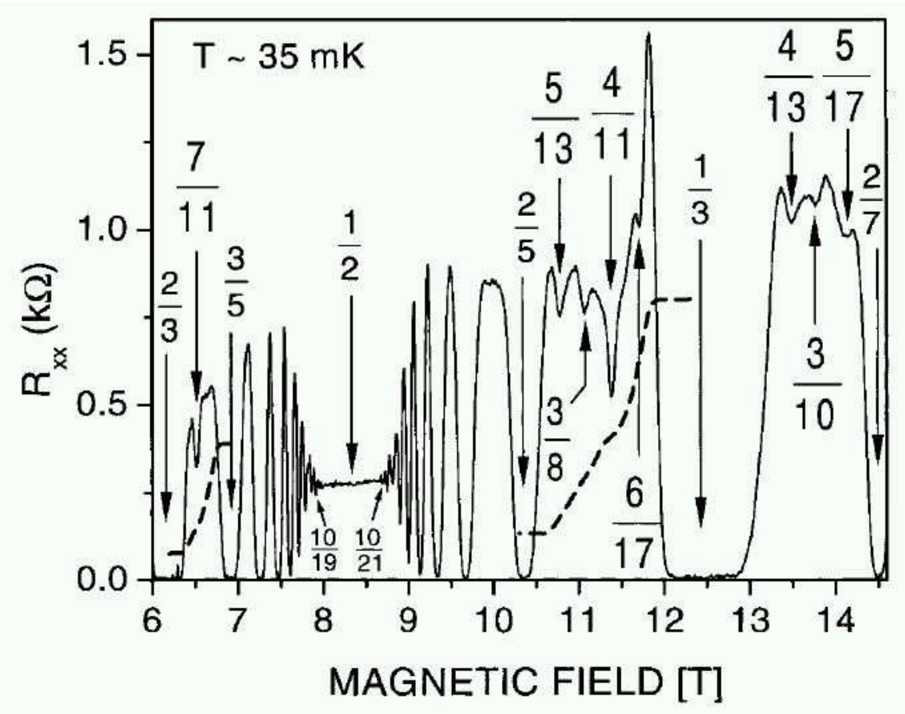
First experiment: Lilly et al (Eisenstein group, Caltech) '99

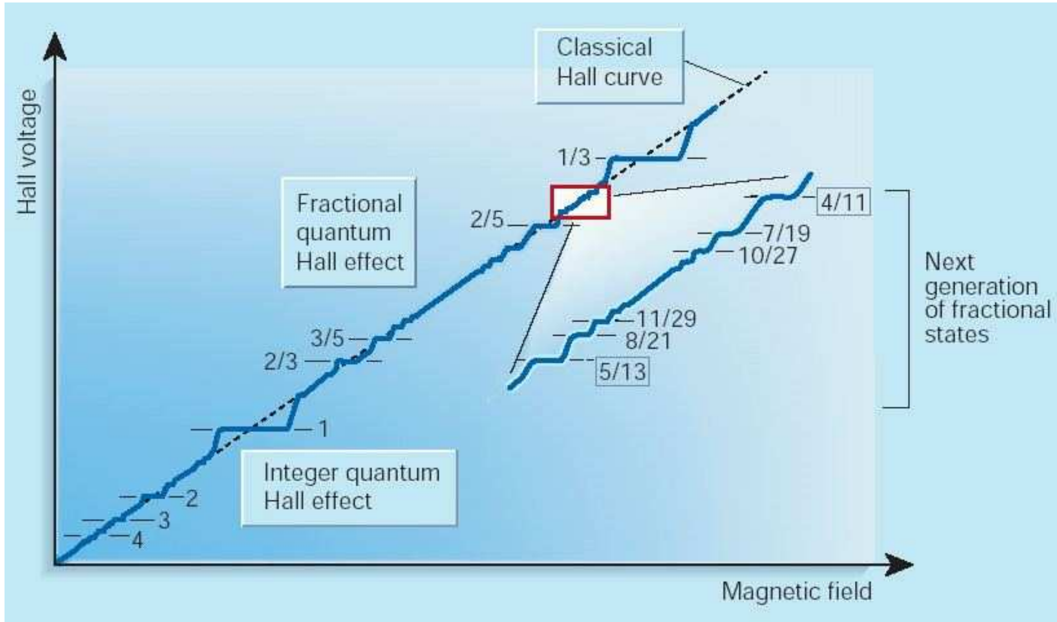
Theory: Koulakov, Fogler, Shklovskii '96 ; Chalker, Moessner '96

striped phase in high Landau levels favored by exchange interaction

Composite fermions: Second generation

Pan, Stormer, Tsui, Pfeiffer, Baldwin, West '03





$$\nu^{-1} = 2 + \nu_{\text{cf}}^{-1}$$

$\nu_{\text{cf}} = 1 + 1/2 = 3/2 \longrightarrow \nu = 3/8$ **zero effective field**

$\nu_{\text{cf}} = 1 + p/(2p \pm 1) = \longrightarrow \nu = \dfrac{3p \pm 1}{8p \pm 3}$ **IQHE of 2nd generation CF's**