Charges and Spins in Quantum Dots

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Chernogolovka 2007

Outline

- Confined (0D) Fermi liquid: Electron-electron interaction and ground state properties of a quantum dot
- Confined (0D) Fermi liquid: Transport across a quantum dot
- Kondo effect in quantum dots

Bulk Fermi Liquids

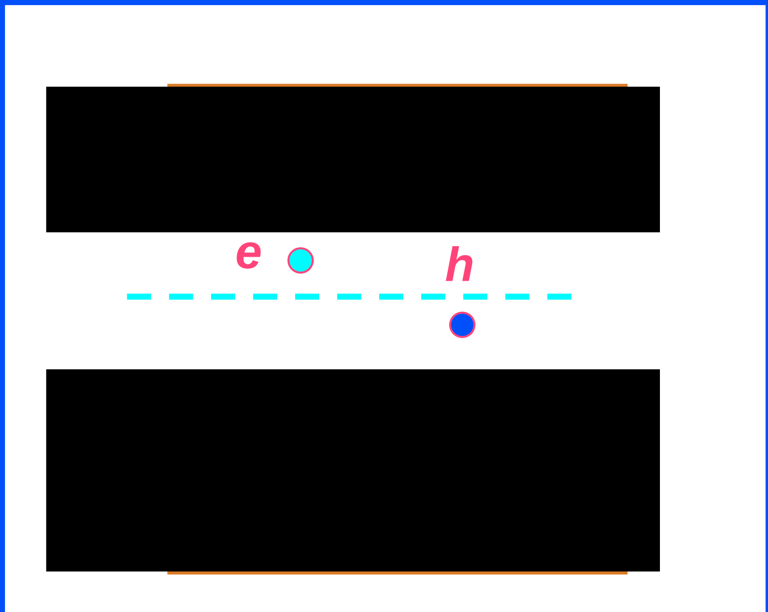
$$H = \sum_k \xi_k \psi^\dagger_{k\sigma} \psi_{k\sigma}$$

$$\xi_k = \frac{k^2}{2m} - \mu$$

$$H_{\text{int}} = \sum_q V(q) : \hat{n}_q \hat{n}_{-q} :$$

$$\hat{n}_q = \sum_\sigma \psi^\dagger_{p+q,\sigma} \psi_{p,\sigma}$$

$$r_s = e^2/\hbar v_F$$



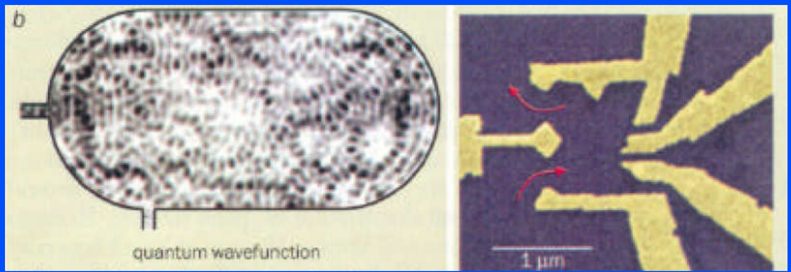
$$\xi_k \to \tilde{\xi}_k \qquad V(q) \to \tilde{V}(q)$$

interaction remains weak

(Landau 1956)

Droplets of a Fermi Liquid

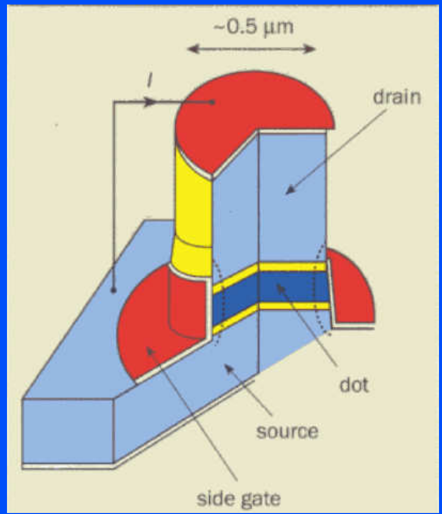
small, symmetric

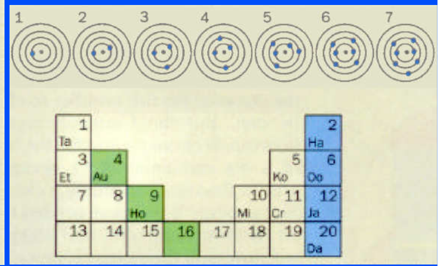


descendants of “dirty” bulk Fermi liquid

$L, l_{tr} \gg \lambda_F$

allow for statistical description



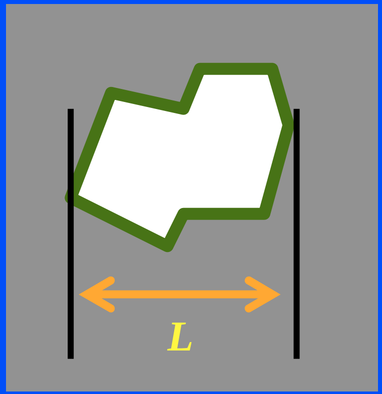


physics of “artificial atoms”

Marcus; Chang ~1996

Kouwenhoven+Tarucha ~1996

Quantum Chaos and Interactions in Quantum Dots: Energy Scales



Single-particle level spacing: $$\delta E = \frac{1}{\nu L^d}$$

Thouless energy: $$E_T = \frac{\hbar v_F}{L}$$ or $$E_T = \frac{\hbar D}{L^2}$$

Charging energy: $$E_C = \frac{e^2}{2C} \sim \frac{e^2}{L}$$

$$g \equiv E_T/\delta E = (k_F L)^{d-1}$$

Good conductor: $g \gg 1$

(Thouless1972)

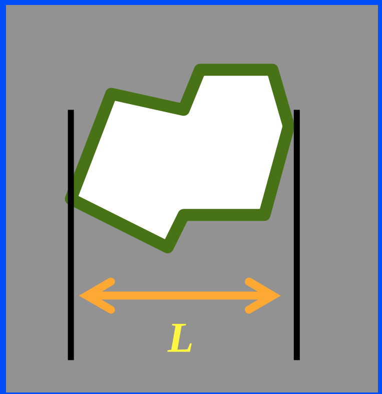
2D: $$g = k_F L \sim \sqrt{N}$$

(ratios for the ballistic case)

$$E_C/E_T \sim r_s \leq 1$$

Quantum Chaos and Interactions in Quantum Dots:
Universal Hamiltonian – Single-Particle Part

$|E| \leq E_T, \ g \gg 1 \ :$ Random Matrix Theory (RMT) limit



$$H = \sum_k \xi_k \psi^\dagger_{k\sigma} \psi_{k\sigma} \quad \rightarrow \quad H_0 = \sum_i \xi_i \psi^\dagger_{i\sigma} \psi_{i\sigma}$$

ξ_i are random, $\xi = |\xi_{i+1} - \xi_i| \sim \delta E$ repel each other

at $\xi \ll \delta E \ :$ $P(\xi) \propto \xi^\beta; \quad \beta = 1(\mathrm{GOE}) \text{ or } 2(\mathrm{GUE})$

Random eigenfunctions $\varphi_i(r) \ :$ $\langle \varphi_i^*(r_1) \varphi_j(r_2) \rangle \propto \delta_{ij}$ GUE, GOE

(Porter-Thomas statistics)

+

$\langle \varphi_i(r_1) \varphi_j(r_2) \rangle \propto \delta_{ij}$ GOE

Quantum Chaos and Interactions in Quantum Dots: Universal Hamiltonian of Interactions

$$H_{\text{int}} = \sum_q V(q) : \hat{n}_q \hat{n}_{-q} :$$

$$\hat{n}_q = \sum_\sigma \psi^\dagger_{p+q,\sigma} \psi_{p,\sigma}$$

→

$$H_{\text{int}} = \frac{1}{2} \sum_{ijkl} h_{ijkl} : \hat{n}_{il} \hat{n}_{jk} :$$

$$\hat{n}_{il} = \sum_\sigma \psi^\dagger_{i,\sigma} \psi_{l,\sigma}$$

$$h_{ijkl} = \int dr dr' \varphi_i^*(r) \varphi_l(r) V(r - r') \varphi_j^*(r') \varphi_k(r')$$

Random eigenfunctions $\varphi_i(r)$: $\langle \varphi_m^*(r_1) \varphi_n(r_2) \rangle \propto \delta_{mn}$ GUE, GOE

$\langle \varphi_i(r_1) \varphi_j(r_2) \rangle \propto \delta_{ij}$ GOE

$$\langle h_{ijkl} \rangle = A \delta_{il} \delta_{jk} + B \delta_{ik} \delta_{jl} + C \delta_{ij} \delta_{kl}$$

- A: $E_C \sim \frac{e^2}{L}$ — charging
- B: $E_S \sim r_s \delta E$ — exchange
- C: $|\Lambda| \ll \delta E$ — Cooper

Important only in supercond. dots, $\Lambda < 0$

Quantum Chaos and Interactions in Quantum Dots:
Universal Hamiltonian of Interactions

$$H_{\rm int} = \frac{1}{2}\sum_{ijkl} h_{ijkl} :\hat{n}_{il}\hat{n}_{jk}:$$

$$\hat{n}_{il} = \sum_{\sigma} \psi^{\dagger}_{i,\sigma}\psi_{l,\sigma}$$

$$h_{ijkl} = \int dr dr' \varphi_i^*(r)\varphi_l(r)V(r-r')\varphi_j^*(r')\varphi_k(r')$$

$$\langle h_{ijkl}\rangle = A\delta_{il}\delta_{jk} + B\delta_{ik}\delta_{jl} + C\delta_{ij}\delta_{kl}$$

No other matrix elements with non-zero averages in the limit $g \equiv E_T/\delta E \gg 1$

$$h_{ijkl} = \langle h_{ijkl}\rangle + \delta h_{ijkl}$$

$\langle h_{ijkl}\rangle$: finite part, does not fluctuate

δh_{ijkl}: fluctuates, average is zero, variance is $\propto 1/g$

Full Form of the Universal Hamiltonian

$$H = H_0 + H_{\text{int}}$$

$$H_0 = \sum_i \xi_i \psi^\dagger_{i\sigma}\psi_{i\sigma}$$

$$\xi = |\xi_{i+1} - \xi_i| \sim \delta E$$

random

not random

$$H_{\text{int}} = E_C(\hat{N} - N_0)^2 - E_S\hat{\mathbf{S}}^2 + \Lambda \hat{T}^\dagger \hat{T}$$

$$\hat{T} = \sum_i \psi^\dagger_{i,\uparrow}\psi^\dagger_{i,\downarrow}$$

Cooper pairs operator

$$\Lambda = \Lambda(E_T)$$

$$\hat{N} = \sum_{i,\sigma} \psi^\dagger_{i,\sigma}\psi_{i,\sigma}$$

number of electrons

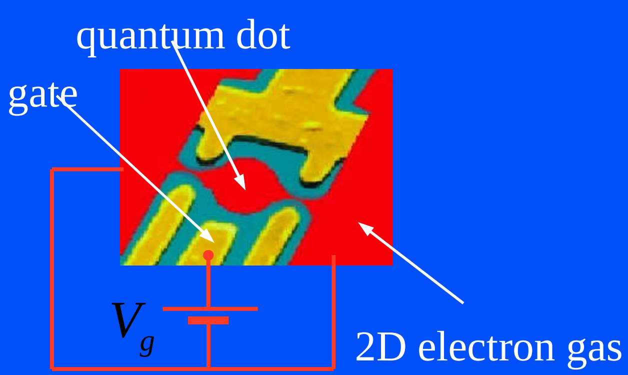
$$E_C = e^2/2C$$

$$\hat{\mathbf{S}} = \sum_{\sigma\sigma'} \mathbf{s}_{\sigma\sigma'} \sum_i \psi^\dagger_{i,\sigma}\psi_{i,\sigma'}$$

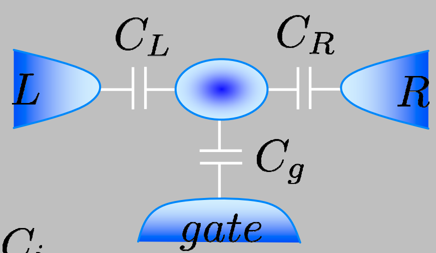
spin of the dot

$$E_S \sim r_s \delta E$$

Kurlyand, Aleiner, Altshuler,`00;
Aleiner, Brouwer, LG (review) 02



$$N_0 = C_g V_g / e$$



$$C = \sum C_i$$

Corrections to the Universal Hamiltonian

Leading correction (only planar dots):

$$H_1^{1/g} \propto \frac{\delta E}{\sqrt{g}} \qquad (g \gg 1)$$

does not limit quasiparticle lifetime

Blanter, Mirlin, Muzykantskii, 1997

Smaller correction (planar dots, 3D grains):

$$H_2^{1/g} \propto \frac{\delta E}{g}$$

causes transitions between the levels

$$\boxed{\frac{1}{\tau_\varepsilon} \sim \delta E \cdot \left(\frac{\varepsilon}{E_T}\right)^2} \qquad (E_T \gg \varepsilon \gg \delta E)$$

Sivan, Imry, Aronov, 1994; Blanter 1996

Equilibrium Properties of a Quantum Dot

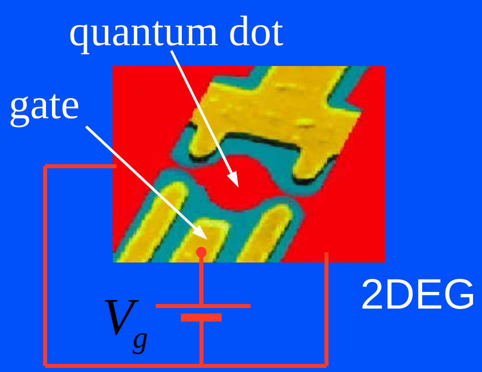
$$H_0 = \sum_i \xi_i \psi^\dagger_{i\sigma} \psi_{i\sigma}$$

$$H_{\text{int}} = E_C(\hat{N} - N_0)^2 - E_S \hat{\mathbf{S}}^2 + \Lambda \hat{T}^\dagger \hat{T}$$

$E_S = 0$

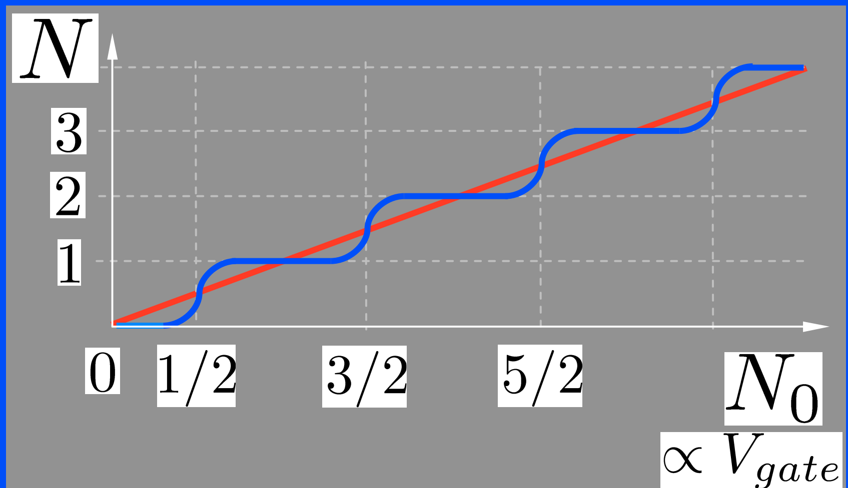
Charge *vs*. gate voltage

$\delta E \ll T$ → Electrostatics



$\Lambda = 0$

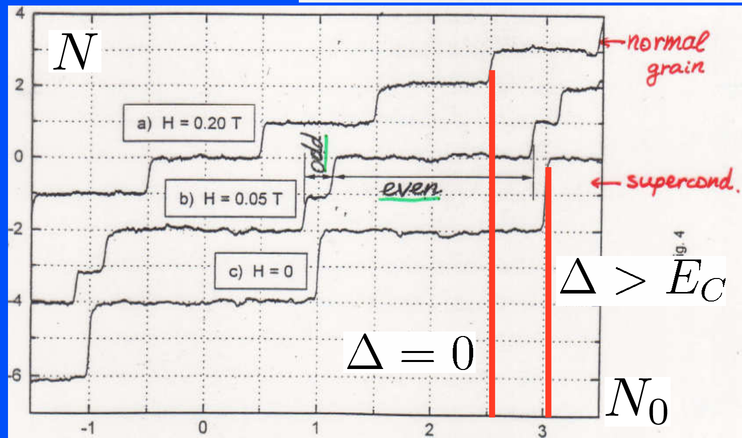
$\Lambda \neq 0$



$E_C \sim \Delta \gg \delta E$

$\frac{\Delta}{e} \simeq 180\,\mu V$ $T = 28\,mK$

$\frac{1}{e}(\Delta - E_c) \simeq 10\,\mu V$



P. Lafarge et al (SACLAY) 1991-1993

Matveev, LG, Shekhter (theo review) 1994

$E_C \gg \Delta \sim \delta E;\ T = 0$

Matveev, Larkin, 1997

Delft, Ralph – Phys Rep (review) 2001

Spin States of a Quantum Dot

$$H_0 = \sum_i \xi_i \psi^\dagger_{i\sigma}\psi_{i\sigma}$$

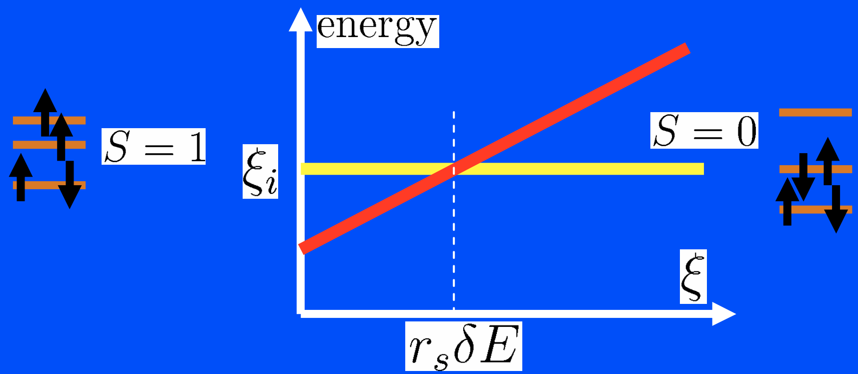
$$H_{\text{int}} = E_C(\hat{N} - N_0)^2 - E_S\hat{\mathbf{S}}^2$$

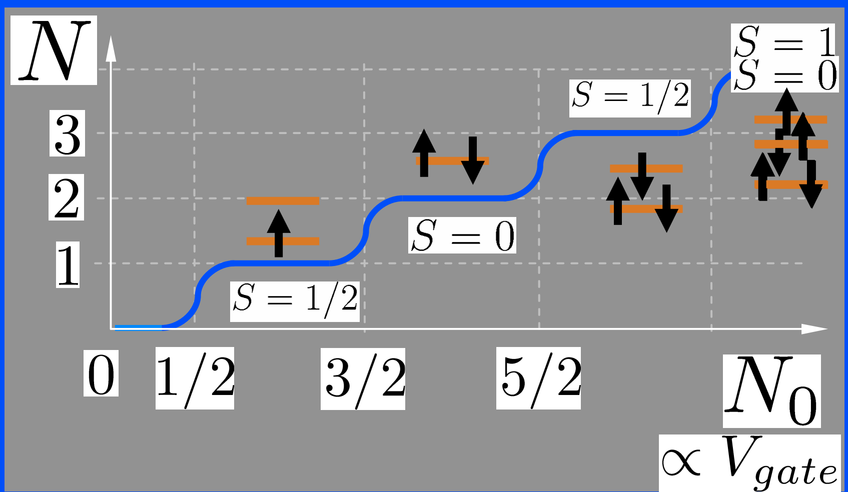
$$\delta E \gg T$$

$E_S = 0$

$E_S \sim r_s \delta E > 0$

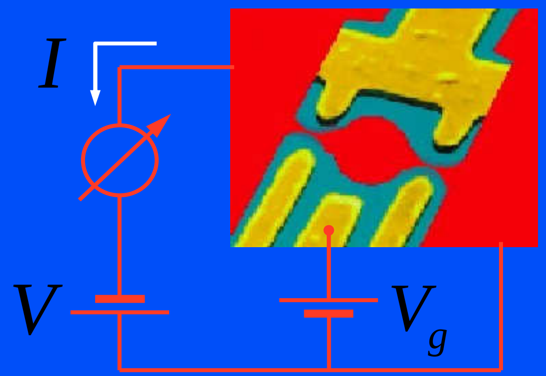
$\xi \equiv \xi_{i+1} - \xi_i \sim \delta E,$
random (large dots)
or controllable (small dots)



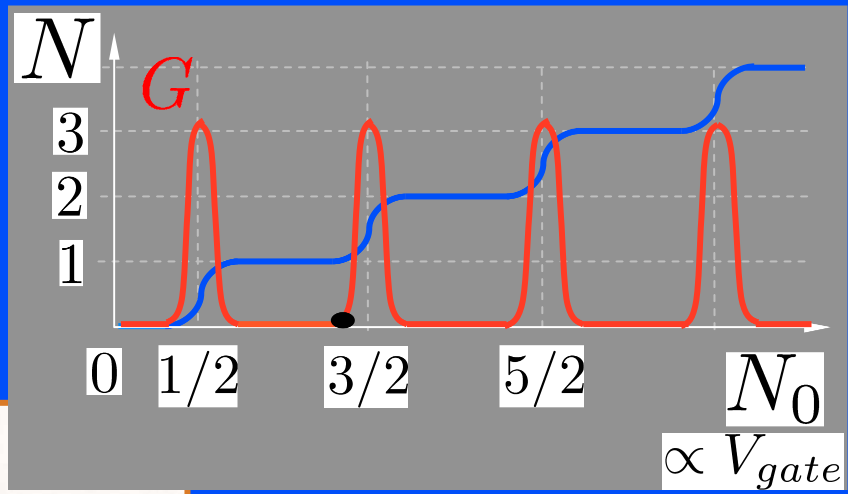


theory:
Oreg, Brouwer, Halperin`99;
Ullmo, Baranger, LG 2000;
Usaj, Barabger, 2002
experiments:
Tarucha group (small dots) 2001
Ensslin group (larger dots) 2001
Marcus group (large dots) 2001

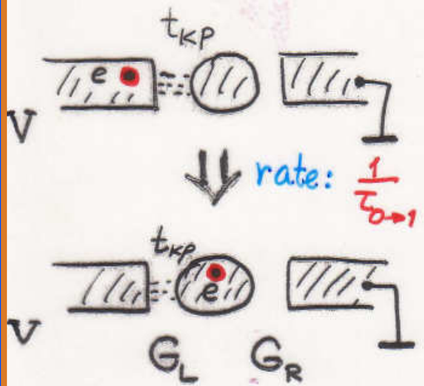
Transport through a Quantum Dot: "Classical" Coulomb Blockade



$\delta E \ll T$



Tunneling rate



Probability of no extra electron on the dot: W_0

$W_0 + W_1 = 1$

$$f(x) = \frac{x}{e^{x/T} - 1}$$

$$\frac{1}{\tau_{0\to 1}} = W_0 \frac{2\pi}{\hbar} \sum_{k,p} |t_{kp}|^2 n_k (1 - n_p)\, \delta(\varepsilon_k + eV + E_0 - \varepsilon_p - E_1)$$

$$= W_0 \cdot \frac{1}{e^2} \cdot G_L\, f(E_1 - E_0 - eV)$$

$x = E_1 - E_0 - eV$

(E_1, E_0:) electrostatic energies

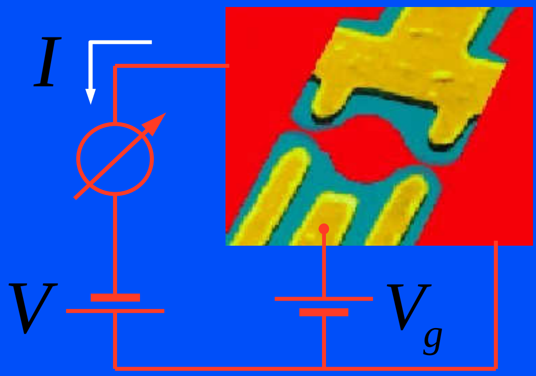
"Classical" Coulomb Blockade

$$f(x) = \frac{x}{e^{x/T} - 1}$$

In the equilibrium (Gibbs):

$$W_0 = \frac{f(E_0 - E_1)}{f(E_0 - E_1) + f(E_1 - E_0)}$$

$$W_1 = \frac{f(E_1 - E_0)}{f(E_0 - E_1) + f(E_1 - E_0)}$$



Balance Equations (currents through L and R junctions equal each other):

$$G_L\left[W_0 f(E_1 - E_0 - eV) - W_1 f(E_0 - E_1 + eV)\right] =$$

$$= G_R\left[W_1 f(E_0 - E_1) - W_0 f(E_1 - E_0)\right]$$

current through the right junction

Linear conductance ($eV \to 0$):

$\varepsilon = E_1 - E_0$ depends on V_g

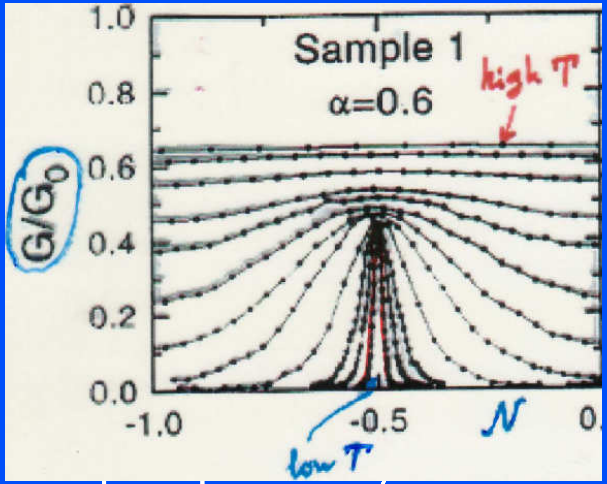
$$G = \frac{G_L G_R}{G_L + G_R} \cdot \frac{f(\varepsilon) f'(-\varepsilon) + f'(\varepsilon) f(-\varepsilon)}{f(\varepsilon) + f(-\varepsilon)}$$

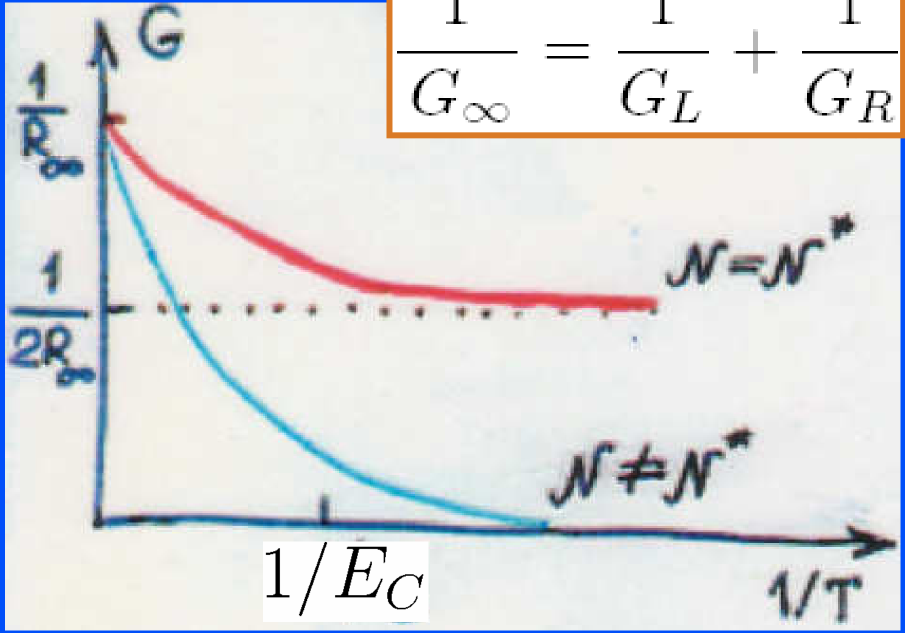
“Classical” Coulomb Blockade: Linear Conductance

$$G = \lim_{V \to 0} dI/dV = G_\infty \frac{E_C(N_0 - N_0^*)/T}{\sinh[2E_C(N_0 - N_0^*)/T]}$$

$N_0 = N_0^*$ corresponds to the Coulomb blockade peak

$\delta E \ll T \ll E_C$ Peak width: $|N_0 - N_0^*| \lesssim T/E_C$

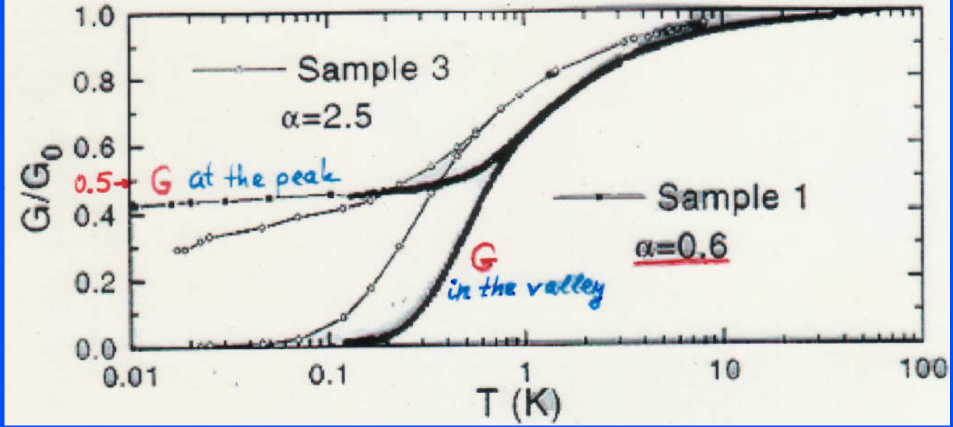




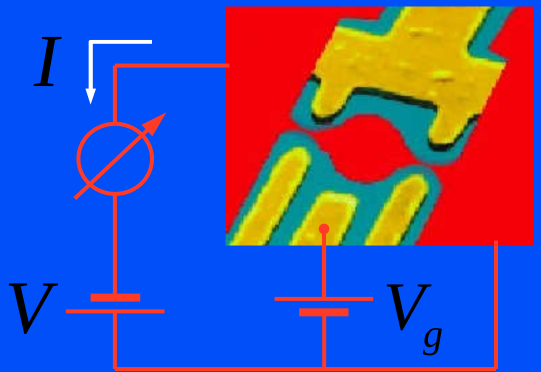
$$\frac{1}{G_\infty} = \frac{1}{G_L} + \frac{1}{G_R}$$

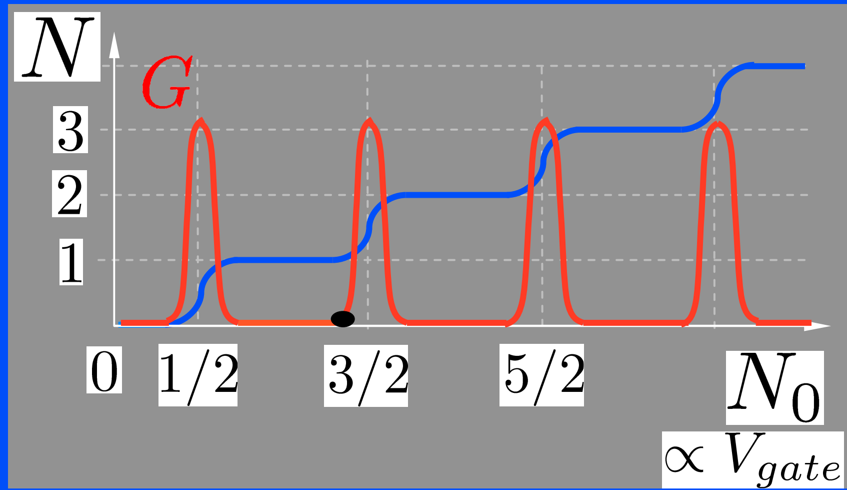
$1/E_C$

P. Joyez et al. (SACLAY) PRL 1997



Transport through a Quantum Dot: Resonances



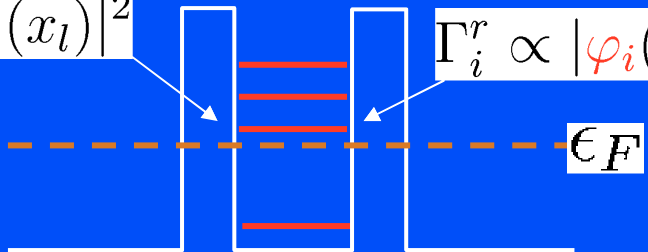


$\delta E \gg T$ separate-level resonances

$$G_i^{max} \sim \frac{e^2}{\pi\hbar T}\frac{\Gamma_i^l\Gamma_i^r}{\Gamma_i^l+\Gamma_i^r}$$

$$\overline{G^{max}} > G_\infty$$

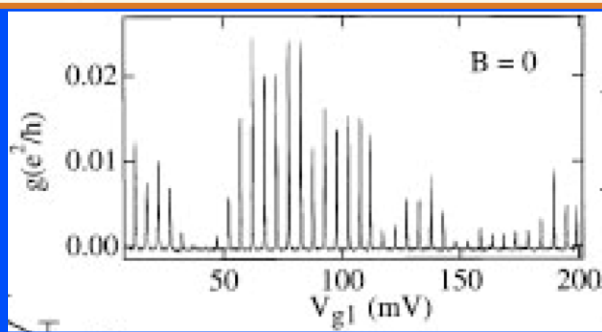
$$\Gamma_i^l \propto |\varphi_i(x_l)|^2 \qquad \Gamma_i^r \propto |\varphi_i(x_r)|^2$$



$$P(\Gamma) = \int d\Gamma_l \, d\Gamma_r \, P_{PT}(\Gamma_l) \, P_{PT}(\Gamma_r) \, \delta\left(\frac{\Gamma_l \Gamma_r}{\Gamma_l + \Gamma_r} - \Gamma\right)$$

$$T(\varepsilon) = \frac{4\Gamma_i^l\Gamma_i^r}{(\varepsilon-\varepsilon_i)^2 + (\Gamma_i^l+\Gamma_i^r)^2}$$

$$G(\varepsilon_i) = \frac{2e^2}{2\pi\hbar}\int d\varepsilon \frac{\partial f_F}{\partial \varepsilon} T(\varepsilon)$$



Folk et al 1996

Transport through a Quantum Dot: Statistics of Coulomb Blockade Peaks

Non-Gaussian Distribution of Coulomb Blockade Peak Heights in Quantum Dots

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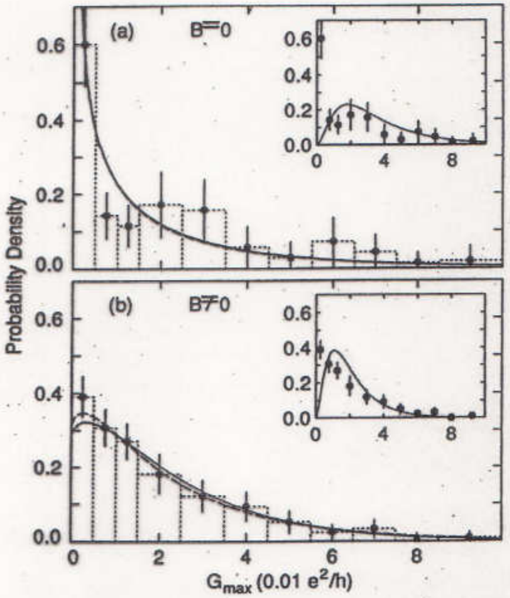
(Received 26 July 1995)

We have observed a strongly non-Gaussian distribution of Coulomb blockade conductance peak heights for tunneling through quantum dots. At zero magnetic field, a low-conductance spike dominates the distribution; the distribution at nonzero field is distinctly different and still non-Gaussian. The observed distributions are consistent with theoretical predictions based on single-level tunneling and the concept of "quantum chaos" in a closed system weakly coupled to leads.



G. 4. Histograms of conductance peak heights for (a) $B =$ and (b) $B \neq 0$. Data are scaled to unit area; there are peaks for $B = 0$ and 216 peaks for $B \neq 0$; the statistical ror bars are generated by bootstrap resampling. Note the n-Gaussian shape of both distributions and the strong spike ar zero in the $B = 0$ distribution. Fits to the data using both e fixed pincher theory (solid) and the theory averaged over ncher variation (dashed) are excellent. The insets show fits $\chi_6^2(\alpha)$—a more Gaussian distribution—averaged over the ncher variation; the fit is extremely poor.

$$G_{max} = \frac{e^2}{h}\frac{\pi\bar{\Gamma}}{2kT}\alpha$$

$$P(\alpha) = \sqrt{\frac{2}{\pi\alpha}}\, e^{-2\alpha}$$ GOE ($B=0$)

$$P(\alpha) = 4\alpha\left[K_0(2\alpha) + K_1(2\alpha)\right]e^{-2\alpha}$$ GUE ($B>B_c$)

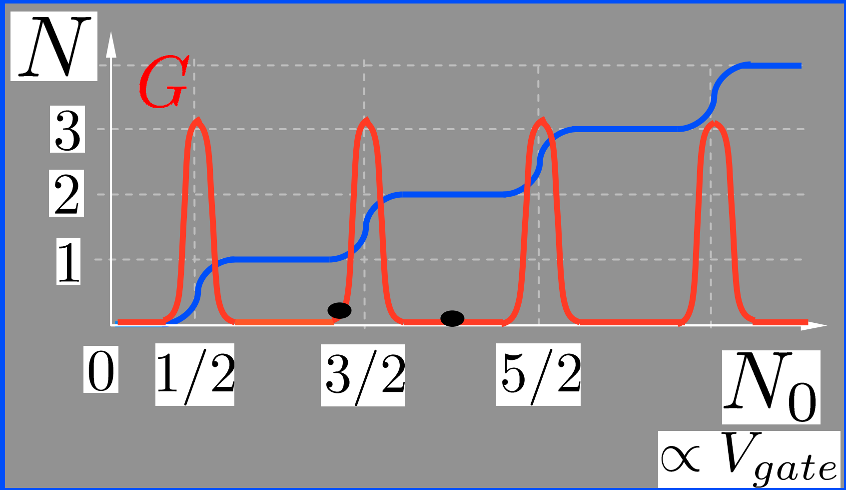
(Jalabert, Stone, Alhassid, 1992)

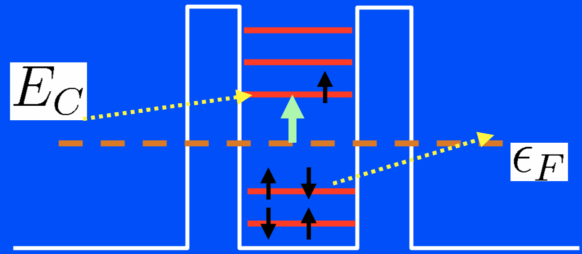
$$\Phi_c^{exp} > \Phi_0/\sqrt{E_T/\delta E}$$

Crossover GOE → GUE:

Falko, Efetov, 1996

Coulomb Blockade Valleys: Inelastic Transport





electron-hole excitation is left behind

$$A_{\rm in} \propto \frac{t_L t_R}{E_C}$$

Initial electron energy: $\delta E \ll \varepsilon \ll E_C$

Available phase space: $\propto \varepsilon^2$

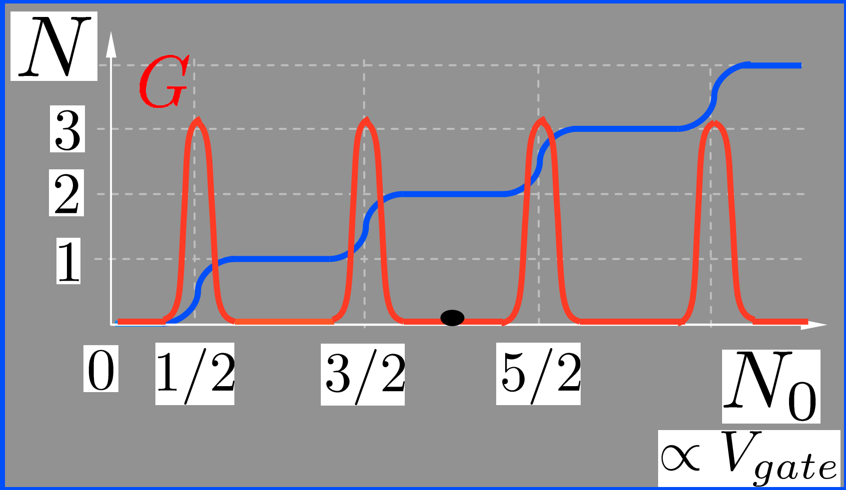
Linear conductance $\varepsilon \sim T$

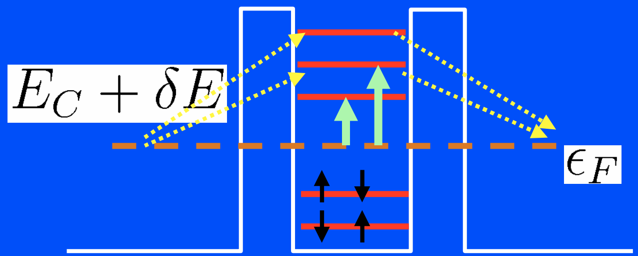
$$G_{\rm in} \sim |A_{\rm in}|^2 T^2 \sim \frac{G_L G_R}{e^2/\hbar}\left(\frac{T}{E_C}\right)^2$$

Averin, Odintsov 1988

$$G_{\rm act} \sim G_\infty \exp(-E_C/T) \lesssim G_{\rm in} \longrightarrow T \lesssim \frac{E_C}{\ln[G_q/(G_L+G_R)]}$$

Coulomb Blockade Valleys: Elastic Transport





$$A_j \propto \frac{\varphi_j(x_l)\varphi_j^*(x_r)}{E_C + |\delta\xi_j|}$$

partial amplitudes are random

$$A = \sum_j A_j$$

$$1 \leq j \lesssim E_C/\delta E$$

$$T(\epsilon_F) = |A|^2 = \left|\sum_j A_j\right|^2 = \sum_j A_j A_j^* + \sum_{i \neq j} A_i A_j^*$$

survives averaging

$$\langle |A_j|^2 \rangle \propto \frac{\Gamma_L \Gamma_R}{E_C^2}$$

$$G_{el} = \frac{e^2}{\pi\hbar} T(\epsilon_F)$$

(Landauer formula)

$$G_{el} \sim \frac{\hbar}{e^2} G_l G_r \frac{\delta E}{E_C}$$ small, fluctuates

Averin, Nazarov 1990; Aleiner, LG 1996; Cronnenwett et al (exp) 1997

Mesoscopic Fluctuations of Elastic Transport

$$T(\epsilon_F) = |A|^2 = \left|\sum_j A_j\right|^2 = \sum_j A_j A_j^* + \sum_{i \neq j} A_i A_j^*$$

$N \sim E_C/\delta E$ terms

$N^2 - N$ terms

$$\mathrm{var}\, G_{\mathrm{el}} \sim \sqrt{N^2 - N}\langle |A_j|^2\rangle$$

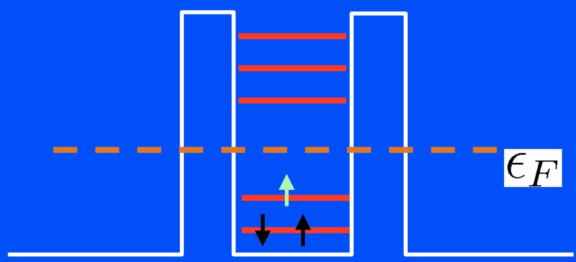
$$\langle G_{\mathrm{el}}\rangle \sim N\langle |A_j|^2\rangle$$

$$\mathrm{var}\, G_{\mathrm{el}} \sim \langle G_{\mathrm{el}}\rangle \sim \frac{\hbar}{e^2} G_L G_R \frac{\delta E}{E_C}$$

$$G_{\mathrm{act}} \sim G_\infty \exp(-E_C/T) \lesssim G_{\mathrm{in}} \longrightarrow T \lesssim \frac{E_C}{\ln[G_q/(G_L + G_R)]}$$

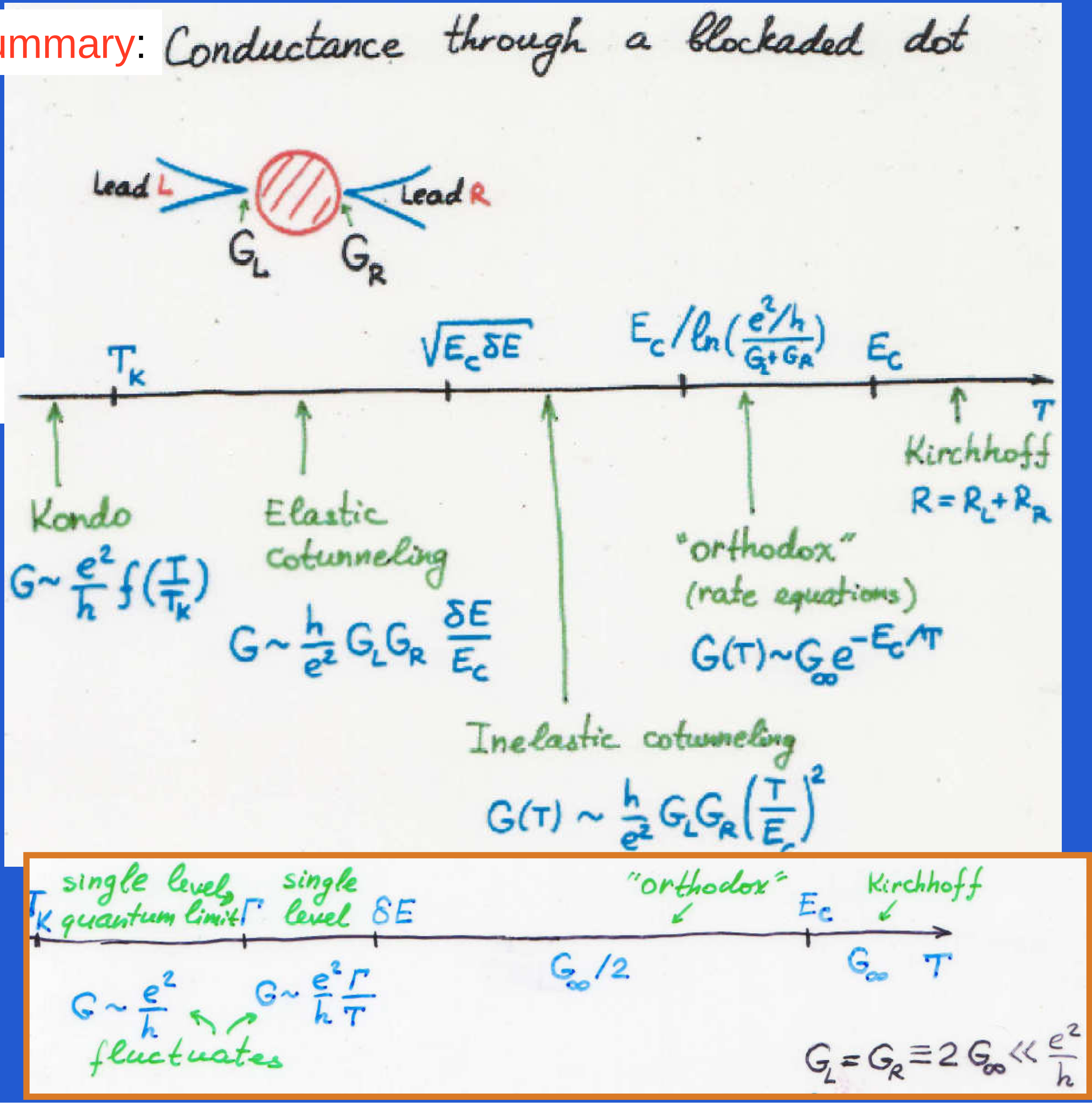
$$G_{\mathrm{in}} \sim G_{\mathrm{el}} \text{ at } T \sim \sqrt{E_C \delta E}$$

Odd valley:



looks like Anderson Impurity Model → **Kondo effect**

Summary:

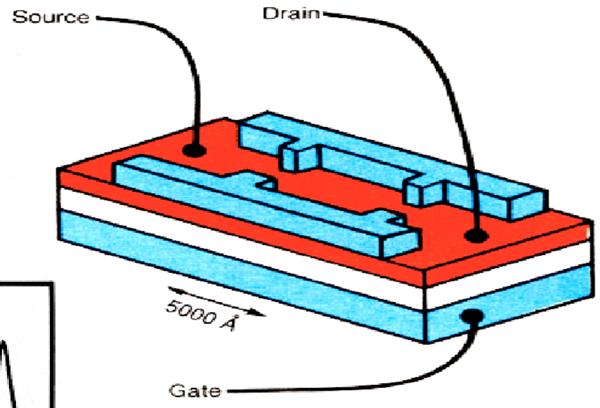


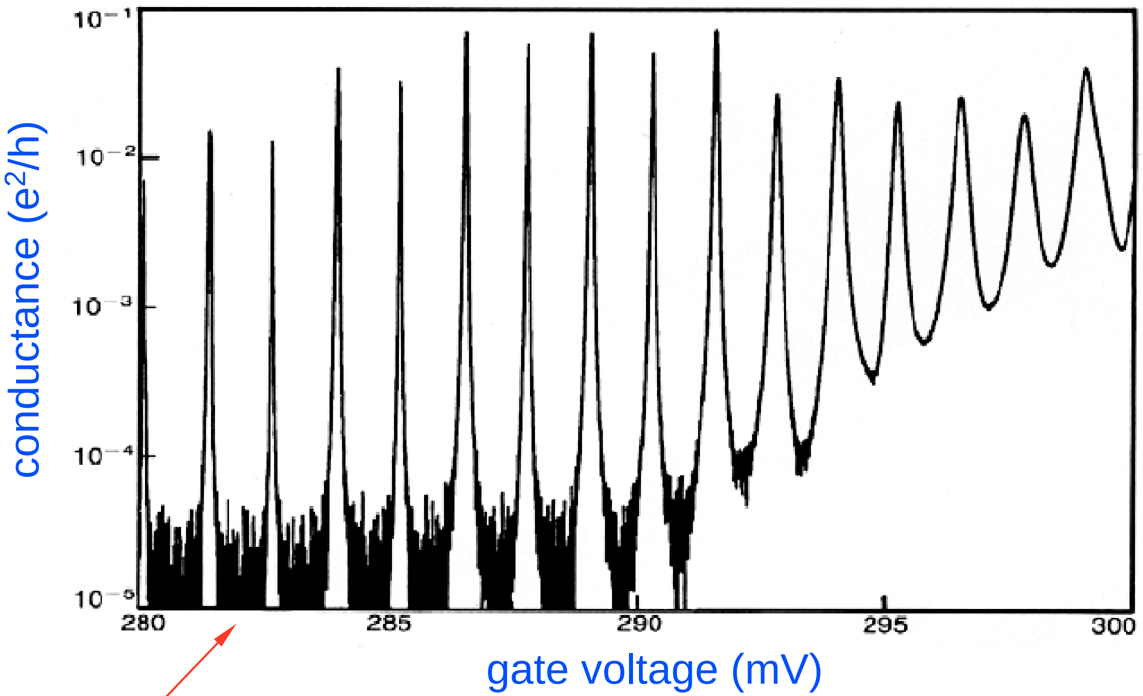
Valleys:

Peaks:

Small quantum dots (~ 500 nm)

M. Kastner, Physics Today (1993)
E.B. Foxman et al., PRB (1993)

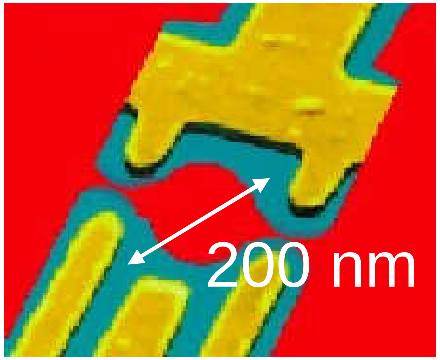




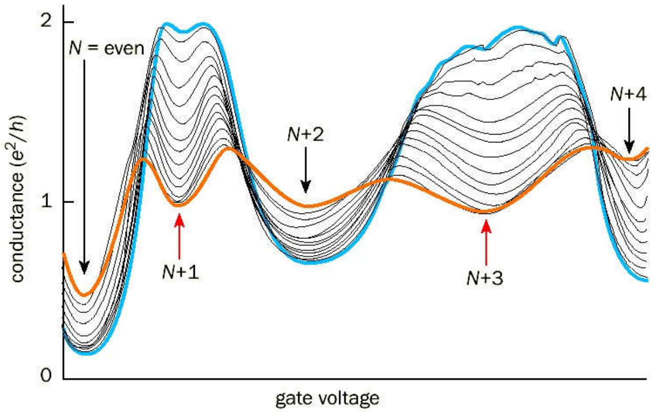
$$G \propto e^{-E_C/T}$$

Even smaller quantum dots (~ 200 nm)

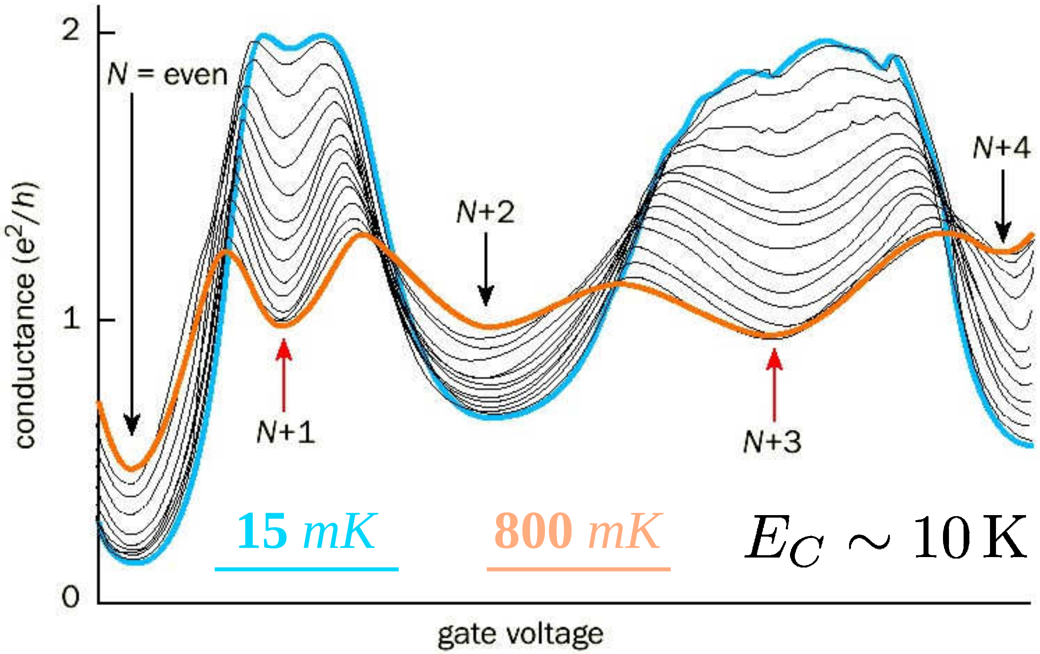
1998
- D. Goldhaber-Gordon et al. (MIT-Weizmann)
- S.M. Cronenwett et al. (TU Delft)
- J. Schmid et al. (MPI @ Stuttgart)



van der Wiel et al. (2000)



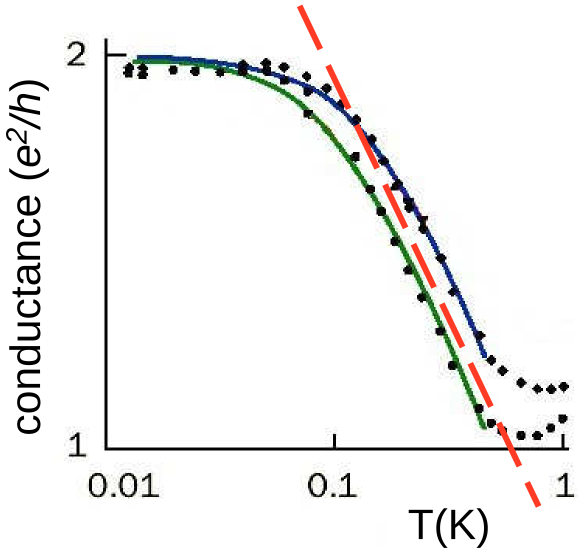
Low-*T* Conductance Anomaly



$$T \ll E_C$$

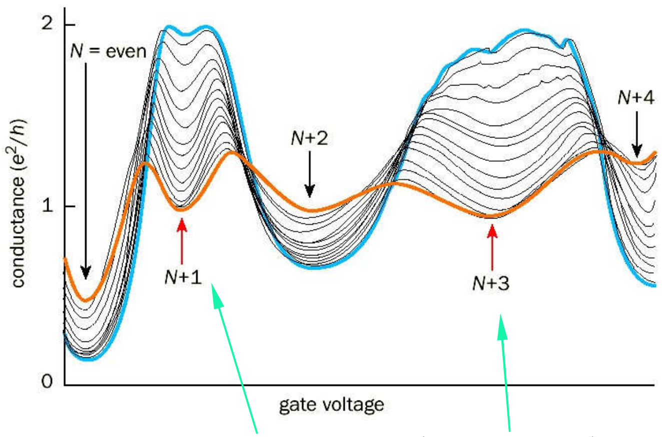
$$G \propto e^{-E_C/T}$$

$$G \propto \ln(E_C/T)$$

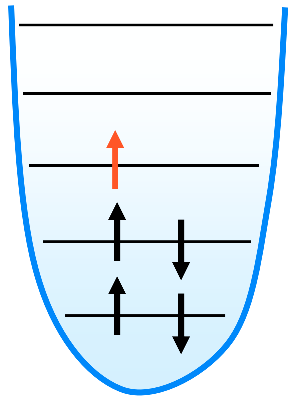


Activation conductance theory fails qualitatively in every other valley

Anomalous low-temperature behavior



$$G \propto \ln\left(E_C/T\right)$$



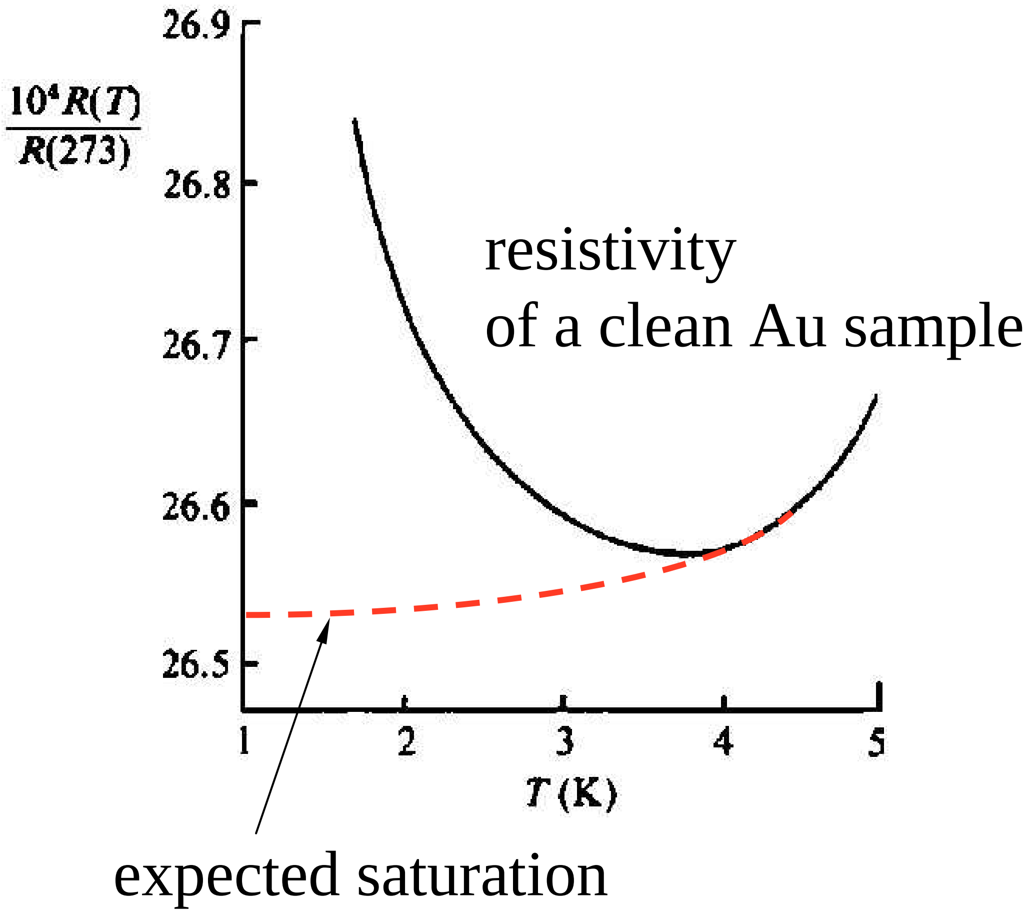
for N = odd all possible electronic configurations have $S \neq 0$

T- dependence $\begin{cases} N = \text{even: normal (decrease at } T \rightarrow 0) \\ N = \text{odd: anomalous (increase at } T \rightarrow 0) \end{cases}$

$$\left.\begin{matrix} S \neq 0 \\ G \propto \ln\left(E_C/T\right) \end{matrix}\right\} \overset{?}{=} \text{Kondo physics}$$

Anomalous behavior of metallic resistivity

de Haas et al. **(1934)**



Kondo effect

Jun Kondo (1964)

$$H_{\rm Kondo} = H_0 + J(\mathbf{s}\cdot\mathbf{S})$$

local spin density of conduction electrons → $\mathbf{s}$

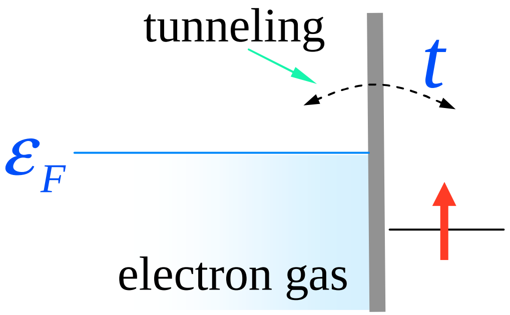
magnetic impurity → $\mathbf{S}$

correction to resistivity grows at $T \to 0$

$$\delta\rho \propto n_{\rm imp}\left[J^2 + J^3 \ln(\epsilon_F/T)\right]$$

A problem: how to deal with singularities at *T* → 0?

The Origin of Exchange Interaction

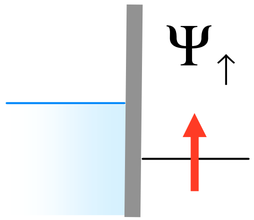


Anderson impurity model

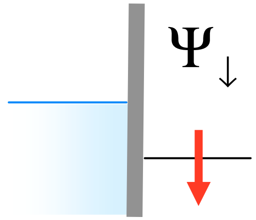
strong on-cite repulsion: $H_d = U\left(N-1\right)^2,\ N = n_\uparrow + n_\downarrow$

impurity level is **singly** occupied: $\langle N \rangle = 1$

$t = 0$: doubly degenerate ground state



$\Psi_\uparrow = |\text{electron gas}\rangle \otimes |\uparrow\rangle$



$\Psi_\downarrow = |\text{electron gas}\rangle \otimes |\downarrow\rangle$

Finite t : tunneling → exchange

example:

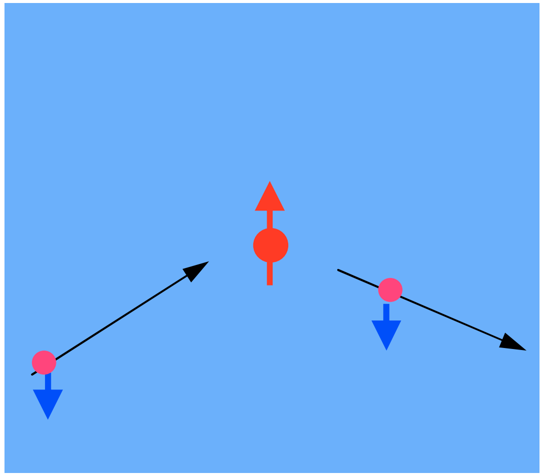
$$H_{\text{exchange}} = J\left(\mathbf{s}\cdot\mathbf{S}\right)$$

conduction electrons (→ $\mathbf{s}$)

impurity (→ $\mathbf{S}$)

$$J \propto t^2/U > 0$$

Electron Scattering in the Perturbation Theory



$$\mathcal{H} = \frac{J_0}{2} \sum_{\mathbf{p}\mathbf{p}'\sigma\sigma'} \mathbf{S} \cdot \mathbf{s}_{\sigma\sigma'} c^\dagger_{\mathbf{p},\sigma} c_{\mathbf{p}',\sigma'}$$

Scattering amplitude in the Born approximation:

$$A^{(1)}_{\mathbf{p}\sigma \to \mathbf{p}'\sigma'} \propto J_0$$

$$w^{(1)}_{\mathbf{p}\sigma \to \mathbf{p}'\sigma'} \propto J_0^2 \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})$$

Kondo (1964) correction:

$$A^{(2)}_{\mathbf{p}\sigma \to \mathbf{p}'\sigma'} \propto 2 \int_{-D}^{0} d\varepsilon'' \frac{\nu J_0^2}{\varepsilon - \varepsilon''} - \int_{0}^{D} d\varepsilon'' \frac{\nu J_0^2}{\varepsilon'' - \varepsilon}$$

$$\propto \nu J_0^2 \ln \frac{D}{|\varepsilon|}$$

$$w^{(1)+(2)}_{\mathbf{p}\sigma \to \mathbf{p}'\sigma'} \propto \left| A^{(1)}_{\mathbf{p}\sigma \to \mathbf{p}'\sigma'} + A^{(2)}_{\mathbf{p}\sigma \to \mathbf{p}'\sigma'} \right|^2 \cdot \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})$$

Electron Scattering in the Leading-Log Approx

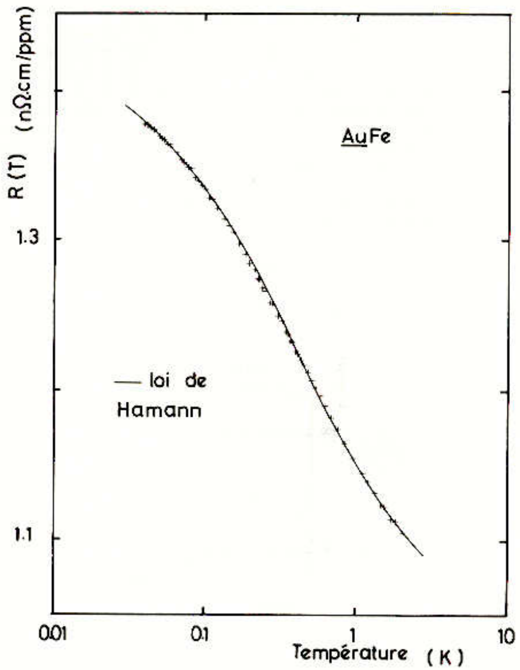
Logarithmic scaling, Abrikosov (1965); log—RG, Anderson (1970):

$$\nu J_0 \rightarrow \nu J(\varepsilon) = \frac{1}{\ln|\varepsilon/T_{\rm K}|}$$

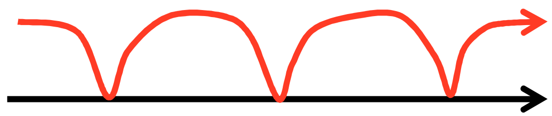
$$T_{\rm K} = D\exp\left(-\frac{1}{J_0 \nu}\right)$$

$$\delta\rho_K(T \gg T_K) \propto \frac{n_s}{\nu}\frac{1}{\ln^2(T/T_K)}$$

$$\sigma(\varepsilon \rightarrow \varepsilon') \sim \lambda_F^2 \left[\frac{1}{\ln|\varepsilon/T_{\rm K}|}\right]^2 \delta(\varepsilon - \varepsilon')$$



Laborde, SSCom. 71'



Kondo singlet

$$H_{\text{Kondo}} = H_0 + J(\mathbf{s} \cdot \mathbf{S})$$

local spin density
of conduction electrons

magnetic impurity

Cartoon: 2 spins-1/2

$$H_{\text{exchange}} = J(\mathbf{S} \cdot \mathbf{S})$$

$S = 1$

$S = 0$

$$E_{\text{triplet}} - E_{\text{singlet}} = J$$

GS is a **singlet** for antiferromagnetic exchange ($J > 0$)

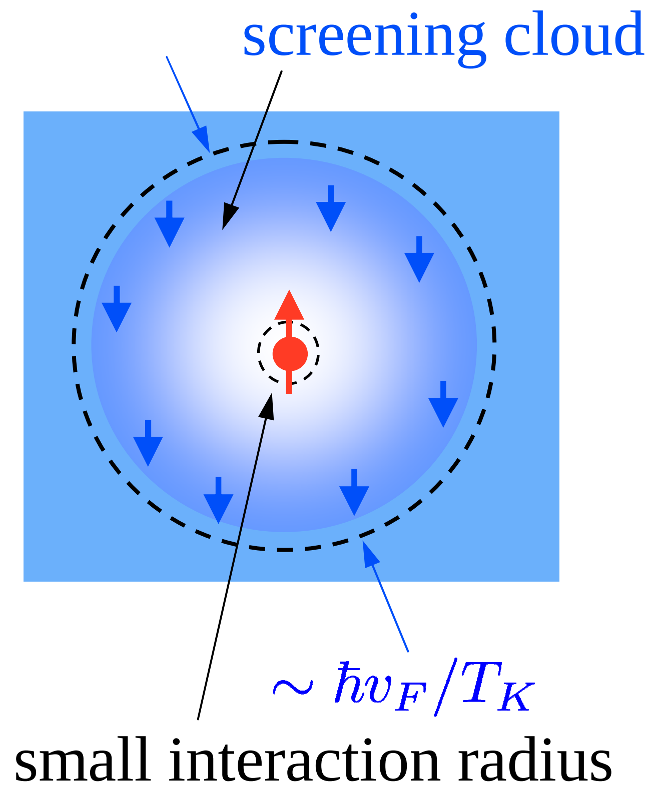
unlike the cartoon, the conduction electrons are delocalized

Kondo singlet

$H_{\mathrm{Kondo}} = H_0 + J(\mathbf{s} \cdot \mathbf{S})$ - favors $\uparrow\downarrow$ for $J > 0$

local spin density of conduction electrons

magnetic impurity



ground state:

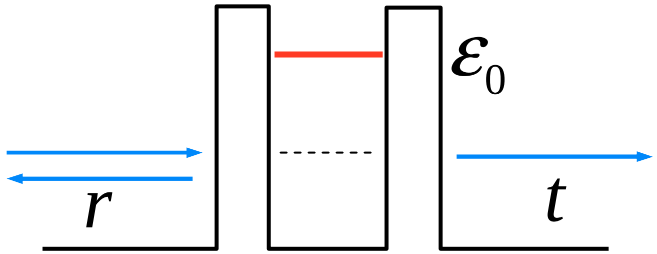
$$\Psi_{\mathrm{Kondo}} = \frac{1}{\sqrt{2}}\left(|\Uparrow\downarrow\rangle - |\Downarrow\uparrow\rangle\right)$$

characteristic energy scale:

Kondo temperature $T_K \sim \epsilon_F e^{-1/J}$

Kondo effect = lifting of the ground state degeneracy

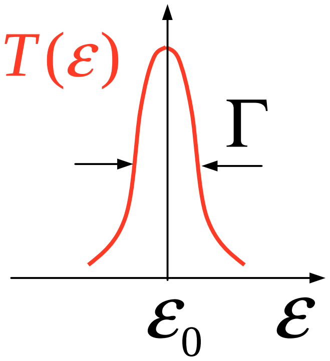
Digression: Resonant tunneling



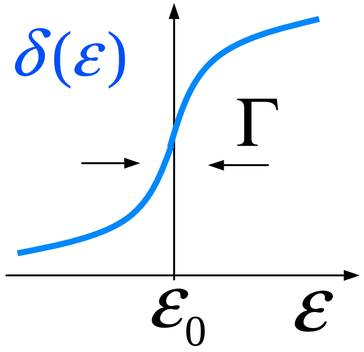
$$T(\varepsilon) = |t|^2 = \frac{\Gamma^2}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2}$$ (Breit - Wigner)

transmission coefficient: $T(\varepsilon) = \sin^2 \delta(\varepsilon)$

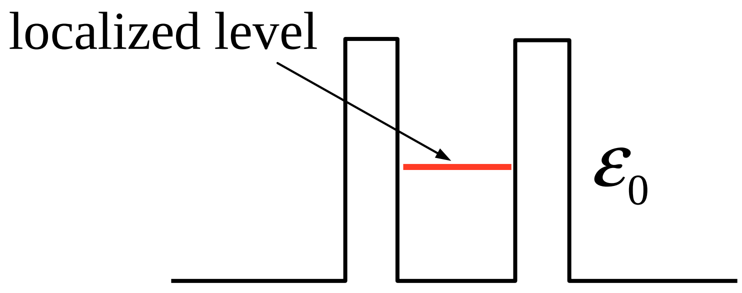
scattering phase shift



$$T(\varepsilon) = 1 \leftrightarrow \delta(\varepsilon) = \pi/2$$

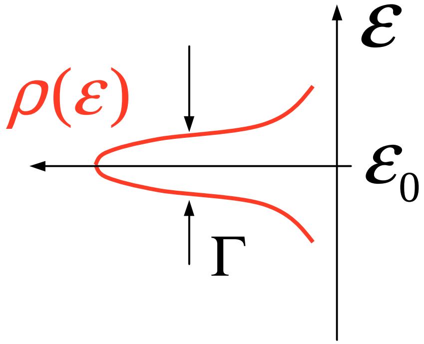


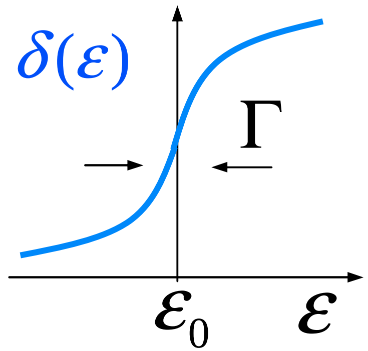
Resonant tunneling



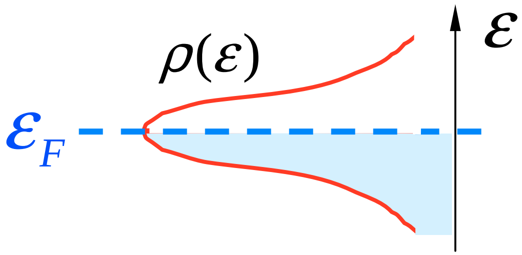
density of states:

$$\rho(\varepsilon) = \frac{1}{\pi}\frac{\partial \delta(\varepsilon)}{\partial \varepsilon}$$





resonance at $\varepsilon = \varepsilon_F$ $\Rightarrow$ localized level is half-occupied



$$N = \int_{-\infty}^{\varepsilon_F} \rho(\omega)\, d\omega = \frac{1}{2}$$

ground state expectation value

$$T(\varepsilon_F) = 1 \;\Leftrightarrow\; \delta(\varepsilon_F) = \frac{\pi}{2} \;\Leftrightarrow\; \rho(\varepsilon_F) = \max \;\Leftrightarrow\; N = 1/2$$

Friedel Sum Rule

N and $\delta(\varepsilon_F)$ are related!

spinless fermions: $$N = \frac{1}{\pi}\delta(\varepsilon_F)$$

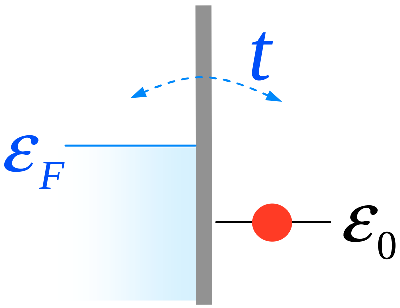
electrons:

$$N = \frac{1}{\pi}\left[\delta_\uparrow(\varepsilon_F) + \delta_\downarrow(\varepsilon_F)\right]$$

$$N = 2 \Rightarrow \delta_\downarrow(\varepsilon_F) = \delta_\uparrow(\varepsilon_F) = \pi$$

no resonance

Anderson impurity: $\varepsilon_0 < \varepsilon_F$ but $\varepsilon_F - \varepsilon_0 < U$



$\Downarrow$

impurity level is singly occupied: $N = 1$

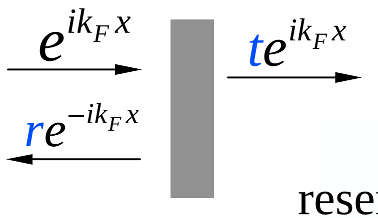
$$\Rightarrow \delta_\uparrow(\varepsilon_F) + \delta_\downarrow(\varepsilon_F) = \pi$$

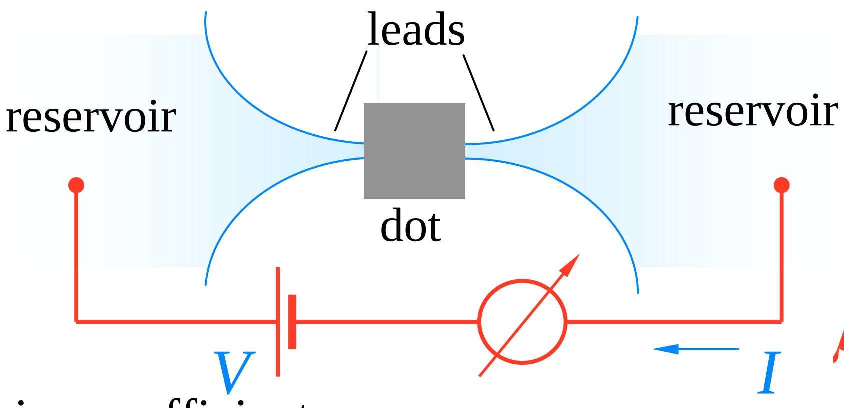
singlet ground state $\Rightarrow \delta_\uparrow(\varepsilon_F) = \delta_\downarrow(\varepsilon_F) = \pi/2$

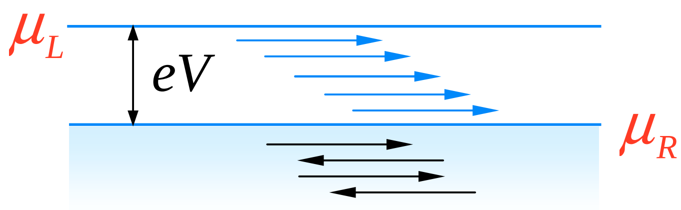
$$T(\varepsilon_F) = \sin^2\delta(\varepsilon_F) = 1$$

resonance!

From Scattering to Transport







transmission coefficient

$$I = e\int \frac{dp}{h}\left(\frac{\partial \varepsilon_p}{\partial p}\right) T(\varepsilon_p) = \frac{e}{h}\int_{\mu_R}^{\mu_L} d\varepsilon\, T(\varepsilon) = \frac{e}{h} T(\varepsilon_F)\, eV$$

$\mu_R < \varepsilon_p < \mu_L$

velocity

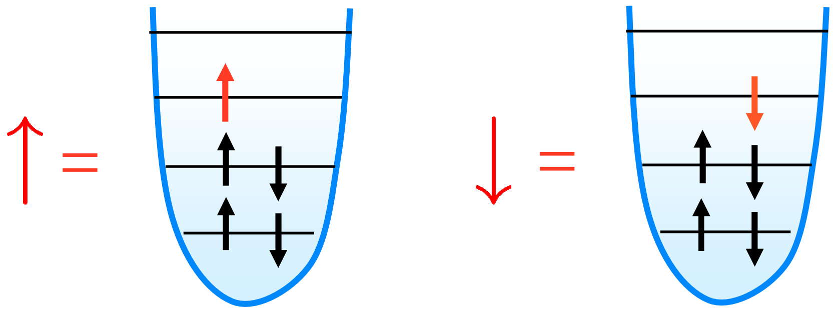
Landauer formula: $G = \frac{e^2}{h}\left(T_\uparrow + T_\downarrow\right)$

$T_\uparrow = T_\downarrow = 1$ (resonance) $\Rightarrow$ $G = 2e^2/h$

conductance quantum: $e^2/h \approx (25\ k\Omega)^{-1}$

Transport in the Kondo regime

Isolated dot:
doubly-degenerate ground state



Dot in contact with leads:
Kondo singlet

$$\Psi_{\text{Kondo}} = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

scattering phase shifts

Conductance: $G = \frac{e^2}{h}\left(T_\uparrow + T_\downarrow\right) = \frac{e^2}{h}\left(\sin^2\delta_\uparrow + \sin^2\delta_\uparrow\right)$

Friedel sum rule: $\delta_\uparrow = \pi N_\uparrow, \quad \delta_\downarrow = \pi N_\downarrow$

ground state expectation values

number of electrons on the dot

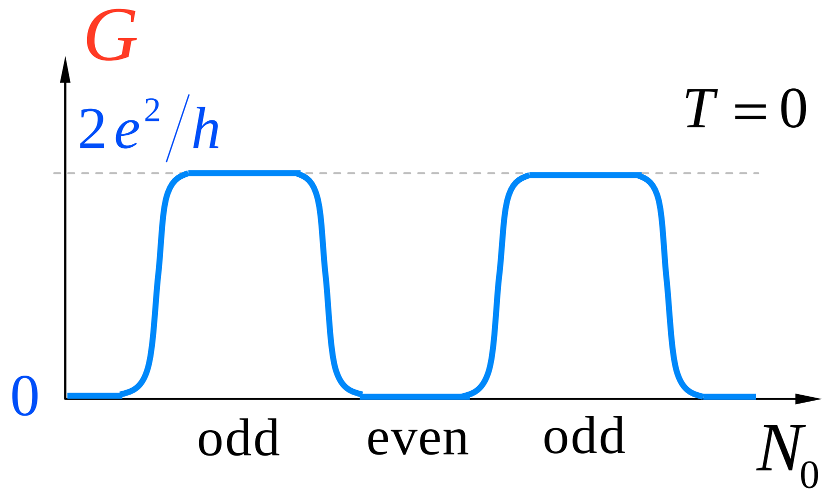
$$N_\uparrow = \left\langle \Psi_{\text{Kondo}} \middle| \hat{N}_\uparrow \middle| \Psi_{\text{Kondo}} \right\rangle = N_\downarrow = N/2$$

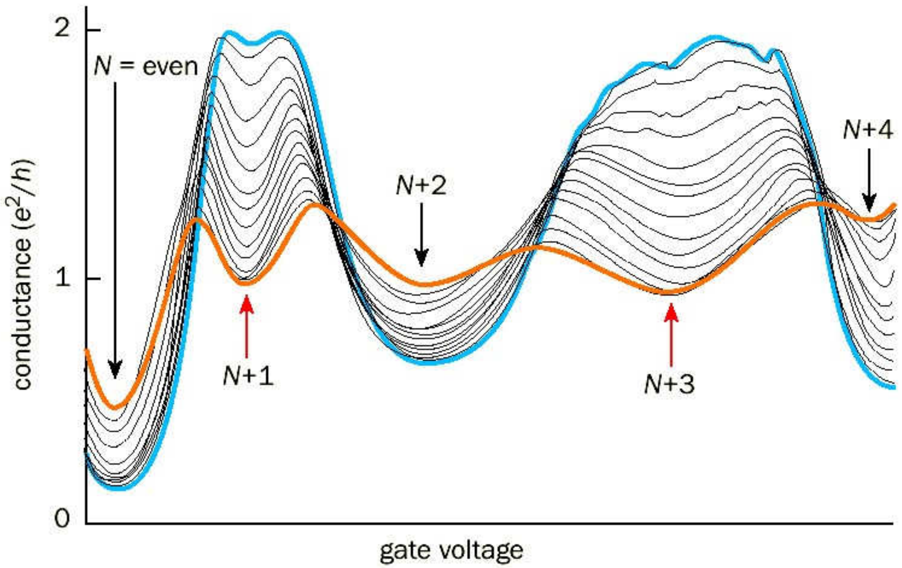
Transport in the Kondo regime

$$G = \frac{e^2}{h}\left(T_\uparrow + T_\downarrow\right) = \frac{e^2}{h}\left(\sin^2\delta_\uparrow + \sin^2\delta_\uparrow\right)$$

$$\delta_\uparrow = \delta_\downarrow = \pi N/2$$

$$\rightarrow \quad G = \frac{2e^2}{h}\sin^2(\pi N/2)$$





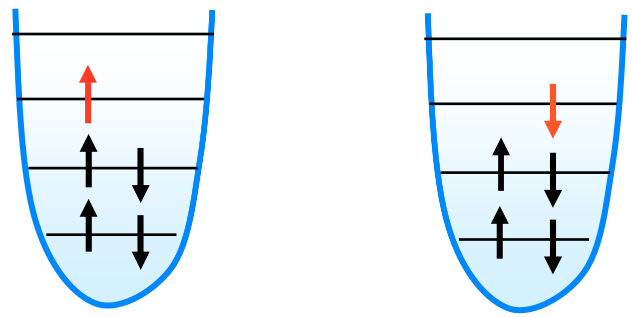
odd N: $G = 2e^2/h$ - perfect transmission

even N: $G = 0$ - perfect blockade

Effect of a Magnetic Field

What is necessary for the Kondo effect to occur?

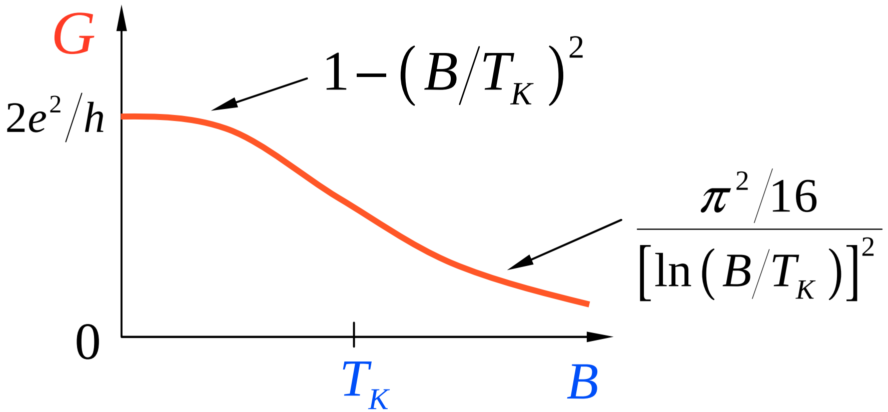
- degeneracy ✶ lifted by a magnetic field
- interaction
- electron gas



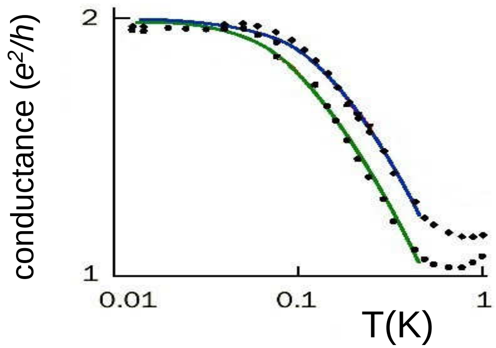
Zeeman energy: $E_{|\uparrow\rangle} - E_{|\downarrow\rangle} = B = g\mu_B H$

$N_\uparrow \neq N_\downarrow$

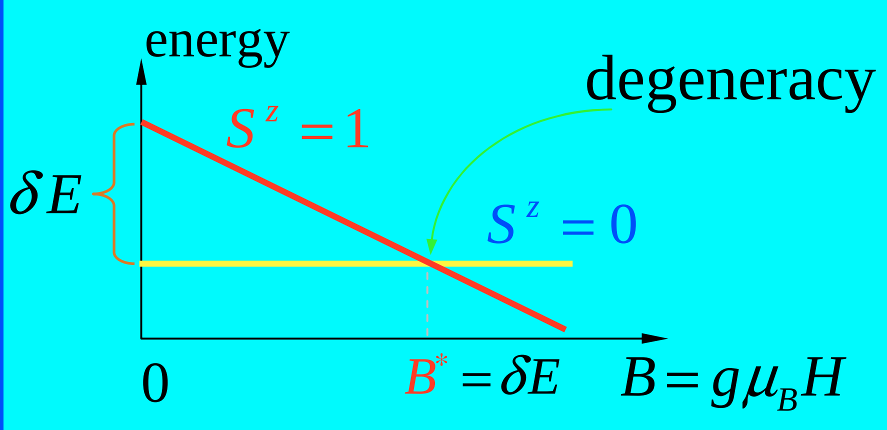
Control parameter: B/T_K



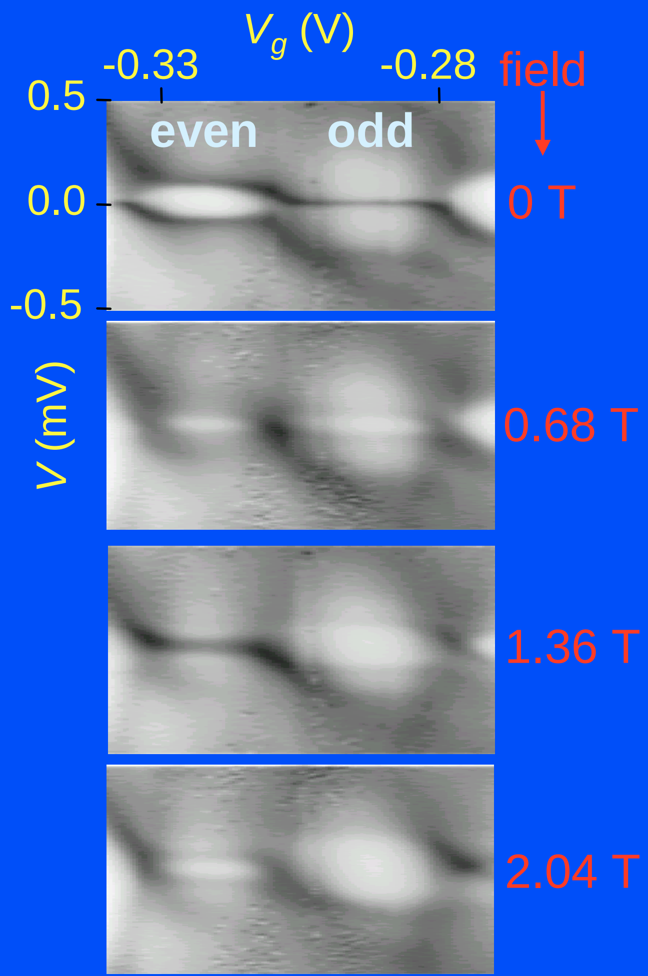
Thermal fluctuations have similar effect:



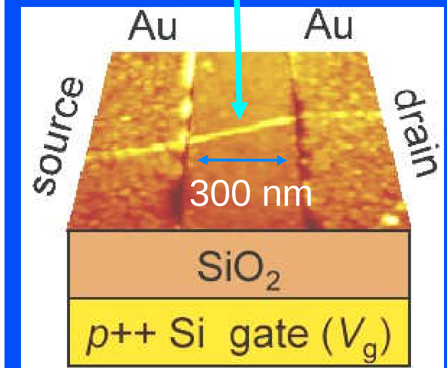
Other local degeneracies—more Kondo effects



Kondo effect recovers at $B = \delta E$
and *even* electron number!



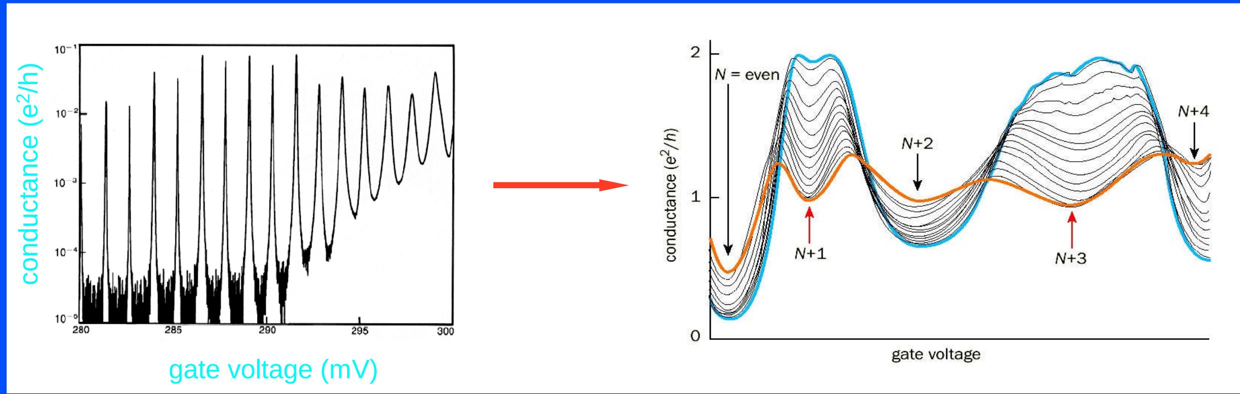
quantum dot:
2 nm-thick
nanotube bundle

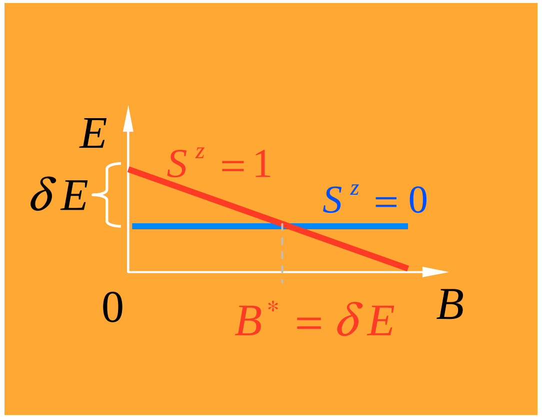


J. Nygård, D.H. Cobden, P.E. Lindelof, Nature **408**, 342 (2000)
review: M. Pustilnik *et. al.* cond-mat/0010336; LNP **579**, 3 (2001)

Summary: Kondo effect in quantum dots

Strong effect:
lifting of the
Coulomb blockade
at low T





Ubiquity in nanostructures:

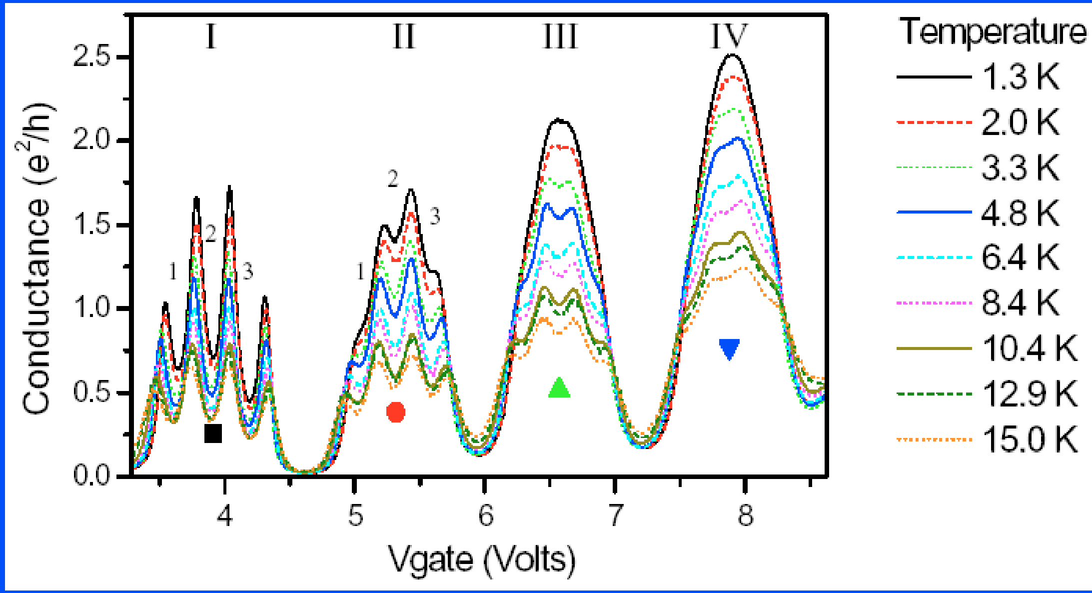
- interaction
- degeneracy
- electron gas

always present

can be tuned

Other stuff: “Quantum Impurity” systems

- Kondo effects: S>1/2, Multi-channel, SU(4), out-of-equilibrium



Kondo in a carbon nanotube quantum dot (G. Finkelstein et al, 2006):
4 states, 2 channels

Other stuff: Evolution of a single electron spin trapped in a dot (GaAs: hyperfine fields)

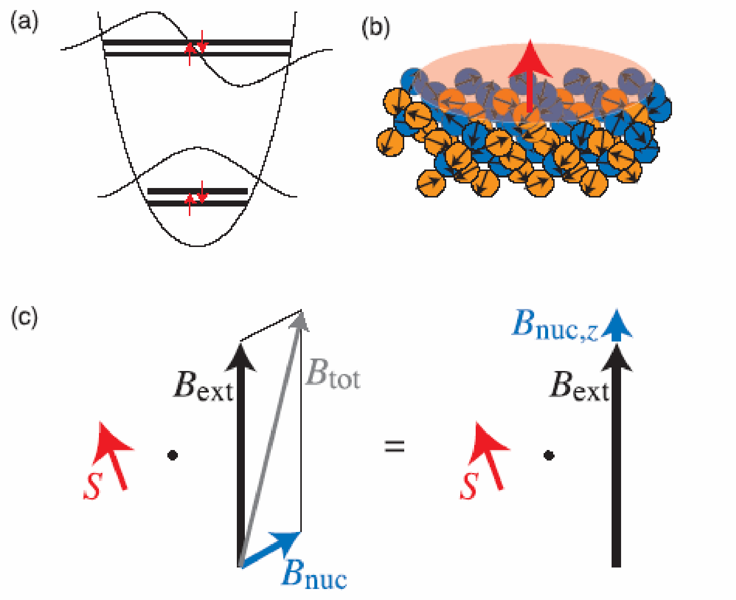


FIG. 1: (a) A schematic potential and energy level diagram for a single quantum dot in which one electron is confined to the low energy spectrum of a three dimensional potential. Only the ground and first-excited states, each a Kramer's doublet, are shown. (b) The lowest orbital state has a spin-1/2 electron interacting with the lattice nuclear spins. (c) Effective magnetic field due to both external field and the nuclear field. When the external field is large, the transverse components of the nuclear field are neglected in a rotating wave approximation.

Precession of a single electron spin in the hyperfine field (C.M. Marcus, A. Yacoby, et al 2007)

Other stuff: Inelastic electron scattering mediated by magnetic impurity

1. Simplest inelastic process in a toy model

$$\mathcal{H} = \mathcal{H}_0 + \hat{V}_{\text{toy}} \qquad \hat{V}_{\text{toy}} = J_0 \sum_{\mathbf{p}_1\mathbf{p}_2} \left(S^+ \sigma^- c^\dagger_{\mathbf{p}_2\downarrow} c_{\mathbf{p}_1\uparrow} + S^- \sigma^+ c^\dagger_{\mathbf{p}_2\uparrow} c_{\mathbf{p}_1\downarrow} \right)$$

$$\mathcal{H}_0 = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma}$$

el.1	el.2	imp

only two electrons in the band, $\Psi_{\text{in}} = |\mathbf{p}'\uparrow, \, \mathbf{p}\downarrow, \, \uparrow\rangle$

$$\Psi_{\text{out}} \equiv \hat{T}\Psi_{\text{in}}$$

T-matrix: $\hat{T} = \hat{V} + \hat{V}\dfrac{1}{\varepsilon - \mathcal{H}_0}\hat{V} + \ldots$

Born

2nd order

$$\Psi^{(2)}_{\text{out}} \propto J_0^2 \sum_{\mathbf{p}_1\mathbf{p}_2} S^+ \sigma^- c^\dagger_{\mathbf{p}_1\downarrow} c_{\mathbf{p}_2\uparrow} \frac{1}{\varepsilon - \mathcal{H}_0} \sum_{\mathbf{p}_3\mathbf{p}_4} S^- \sigma^+ c^\dagger_{\mathbf{p}_3\uparrow} c_{\mathbf{p}_4\downarrow} \, |\mathbf{p}'\uparrow, \, \mathbf{p}\downarrow, \, \uparrow\rangle$$

$$= \frac{J_0^2}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3}} |\mathbf{p}_1\downarrow, \, \mathbf{p}_3\uparrow, \, \uparrow\rangle$$

Inelastic electron scattering

$$A^{(2)}_{\mathbf{p},\mathbf{p}'\to\mathbf{p}_1,\mathbf{p}_3} \sim \frac{J_0^2}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3}}$$

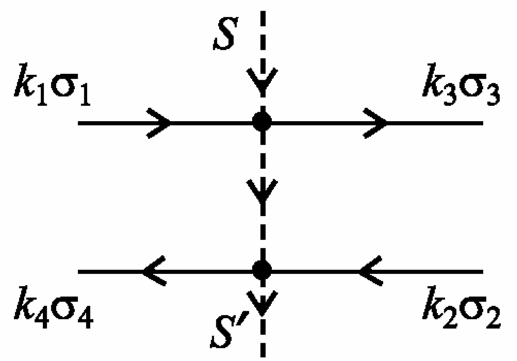
Energy transferred in the collision:

$$\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3} \equiv E$$

Scattering cross-section:

$$\left|A^{(2)}_{\mathbf{p},\mathbf{p}'\to\mathbf{p}_1,\mathbf{p}_3}\right|^2 \sim \frac{J_0^4}{E^2}\delta(\xi_{\mathbf{p}} + \xi_{\mathbf{p}'} - \xi_{\mathbf{p}_1} - \xi_{\mathbf{p}_3})$$

2. Full 2nd order perturbation theory result



Total cross-section $\varepsilon, \varepsilon' \to \varepsilon - E, \varepsilon' + E$

averaged over $\mathbf{S}$:

$$K(E) = \frac{\pi}{2}\frac{n_s}{\nu}(J\nu)^4 S(S+1)\frac{1}{E^2}$$

Experiments on Energy Relaxation: Cu

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Energy Distribution Function of Quasiparticles in Mesoscopic Wires

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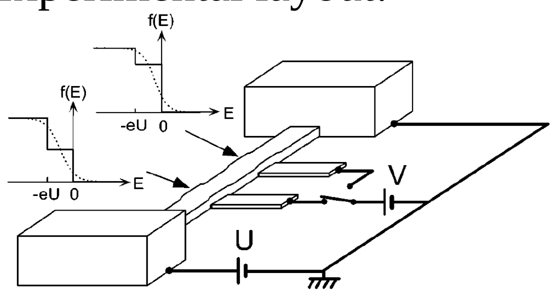
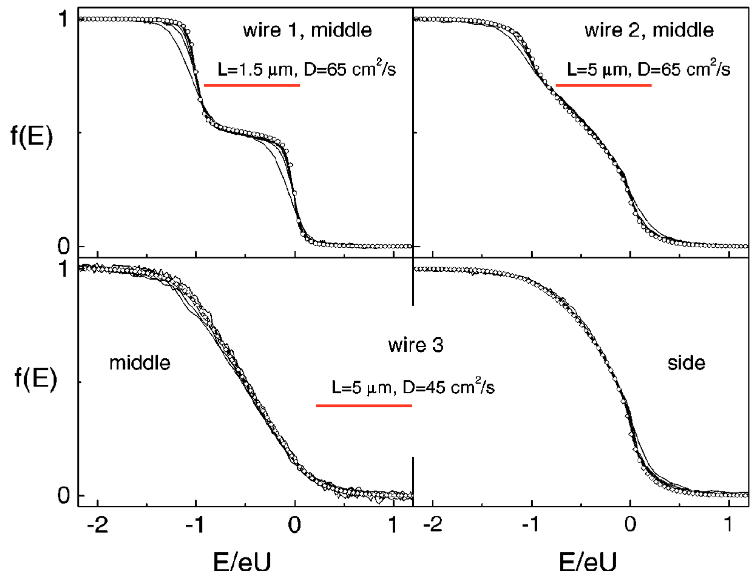
(Received 25 April 1997)

Cu wire

Experimental layout:



Results:



Data for $f(\varepsilon)$ scales with E/eU $\rightarrow$ $K(E,U) \propto \frac{1}{E^2} g\left(\frac{eU}{E}\right)$

If K is U-independent, then

$$K(E) \approx \frac{1}{\tau_0 E^2}$$

with $\tau_0 \approx 1ns$;

$1/\tau_\varepsilon$ does not go to zero at $\varepsilon \rightarrow 0$!

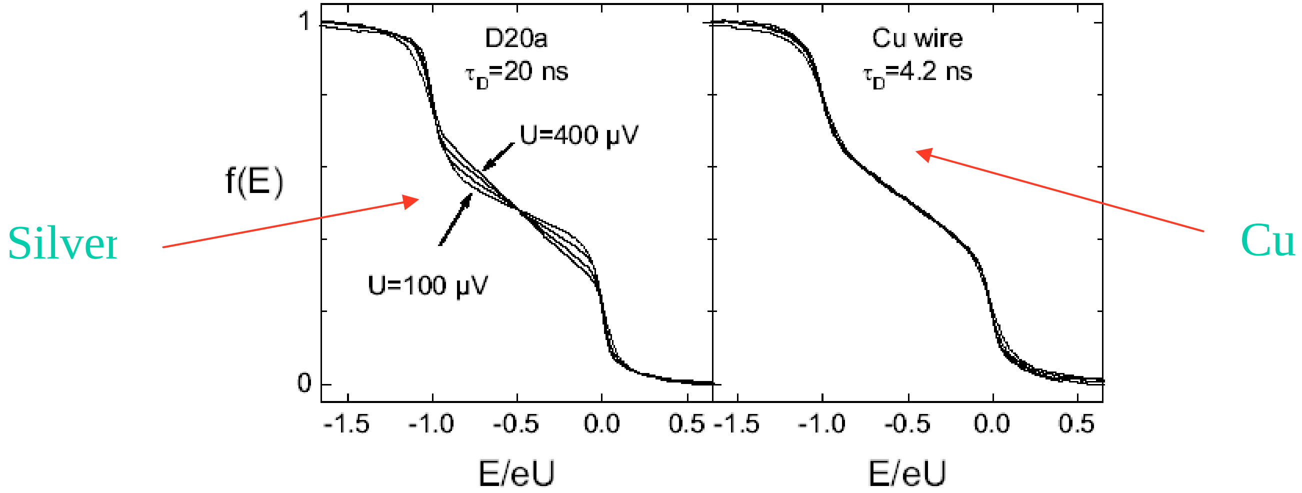
FIG. 3. Continuous lines in all four panels: distribution functions, for U ranging from 0.05 to 0.3 mV by steps of 0.05 mV, plotted as a function of the reduced energy E/eU, for the same positions as in Fig. 2. Open symbols are best fits of the data to the solution of the Boltzmann equation with an interaction kernel $K(x, x', \varepsilon) = \tau_0^{-1}\delta(x - x')/\varepsilon^2$; in top panel, open circles correspond to the calculated distribution function in the middle of wires 1 and 2 ($x = 0.5$), with $\tau_0/\tau_D = 2.5$ and $\tau_0/\tau_D = 0.3$, respectively (both compatible with $\tau_0 \sim 1$ ns). In bottom panels, open diamonds are computed at $x = 0.5$ and $x = 0.25$ with $\tau_0/\tau_D = 0.08$ ($\tau_0 \sim 0.5$ ns).

Experiments on Energy Relaxation: Ag

Energy Redistribution Between Quasiparticles in Mesoscopic Silver Wires
JLTP 118, p. 447 (2000)

F. Pierre, H. Pothier, D. Esteve, and M.H. Devoret

We have measured with a tunnel probe the energy distribution function of quasiparticles in silver diffusive wires connected to two large pads ("reservoirs"), between which a bias voltage was applied. From the dependence in energy and bias voltage of the distribution function we have inferred the energy exchange rate between quasiparticles. In contrast with previously obtained results on copper and gold wires, these data on silver wires can be well interpreted with the theory of diffusive conductors...



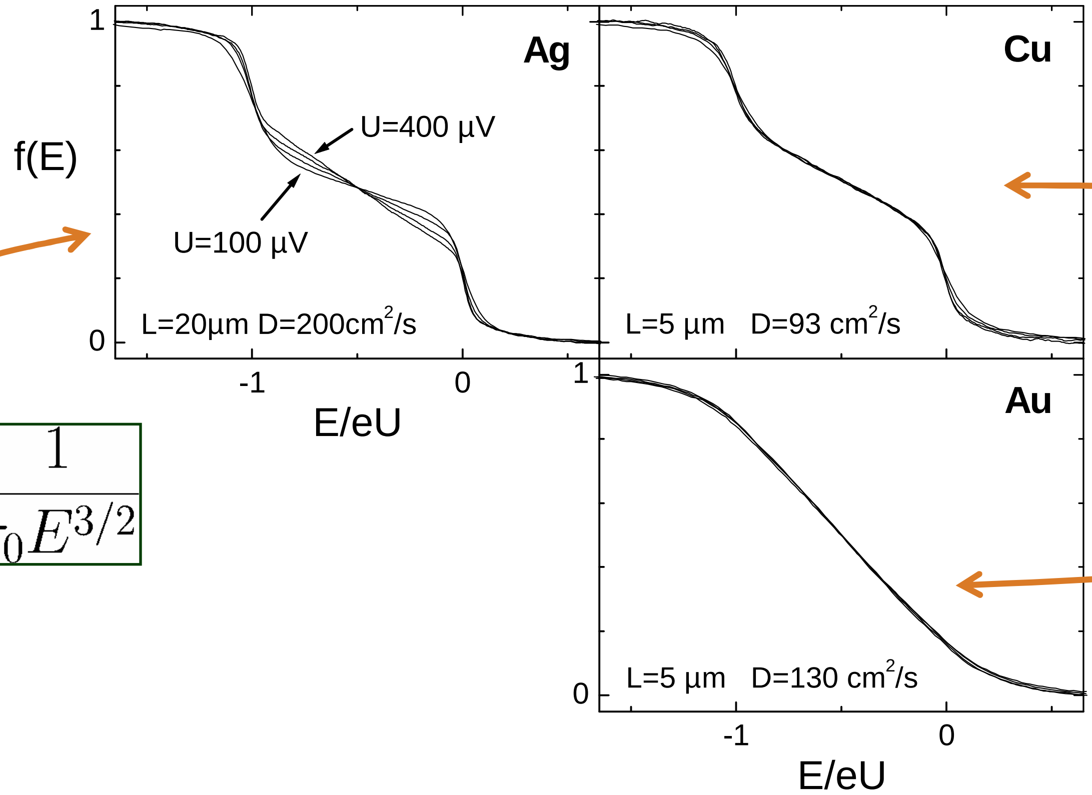
Distribution functions for $U = 0.1, 0.2, 0.3$, and 0.4 mV, plotted as a function of the reduced energy E/eU. Left panel: Ag sample D20a; right panel: Cu sample, $L = 5\ \mu$m.

"In silver samples we have assumed that the interaction kernel still obeys a power law $K(\varepsilon) = \kappa_\alpha \varepsilon^{-\alpha}$, with κ_α and α taken as fitting parameters... the best fits obtained with the exponent set at its predicted value $\alpha = 3/2$. "

Energy Relaxation in Ag, Cu, and Au wires

F. Pierre et al., JLTP 118, 437 (2000) and NATO Proceedings (cond-mat/0012038)

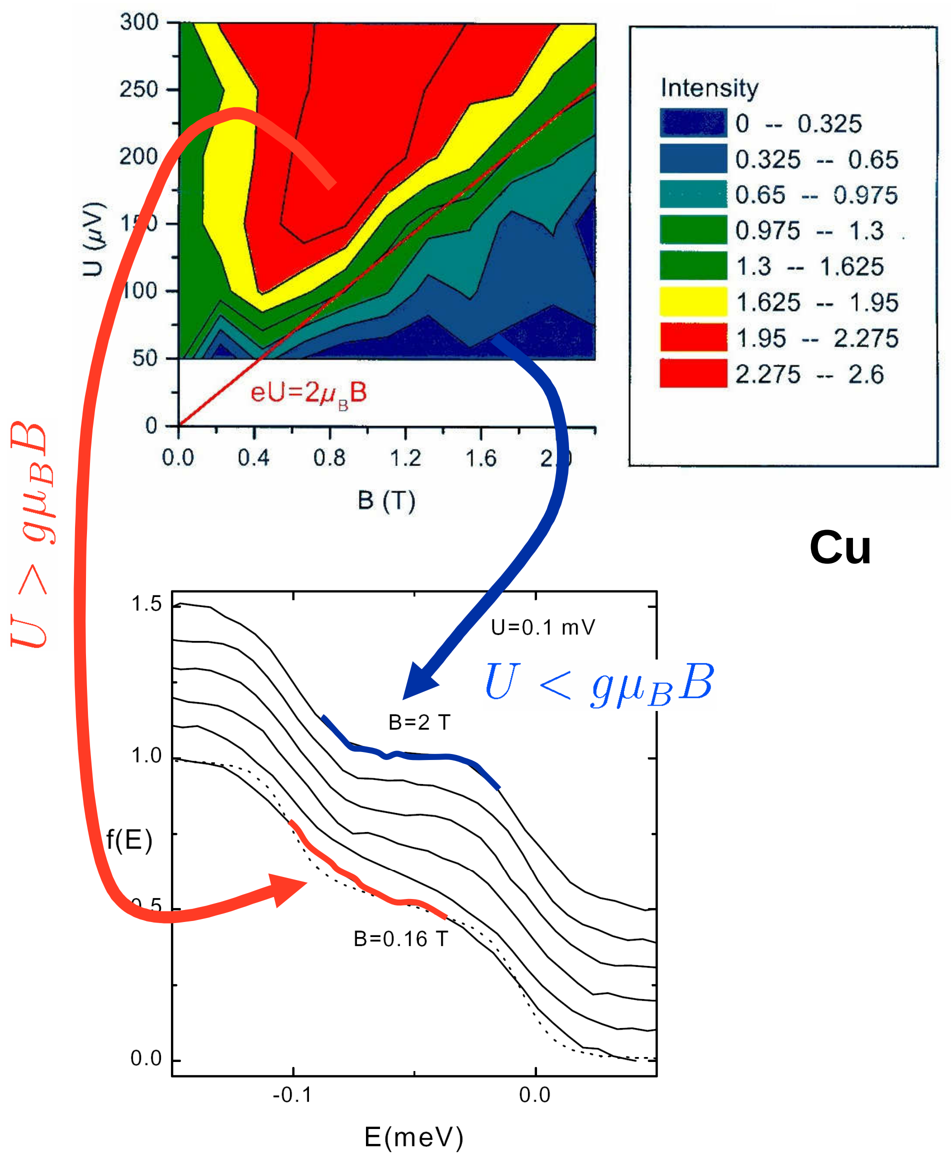
$$K(E) \approx \frac{1}{\tau_0 E^{3/2}}$$

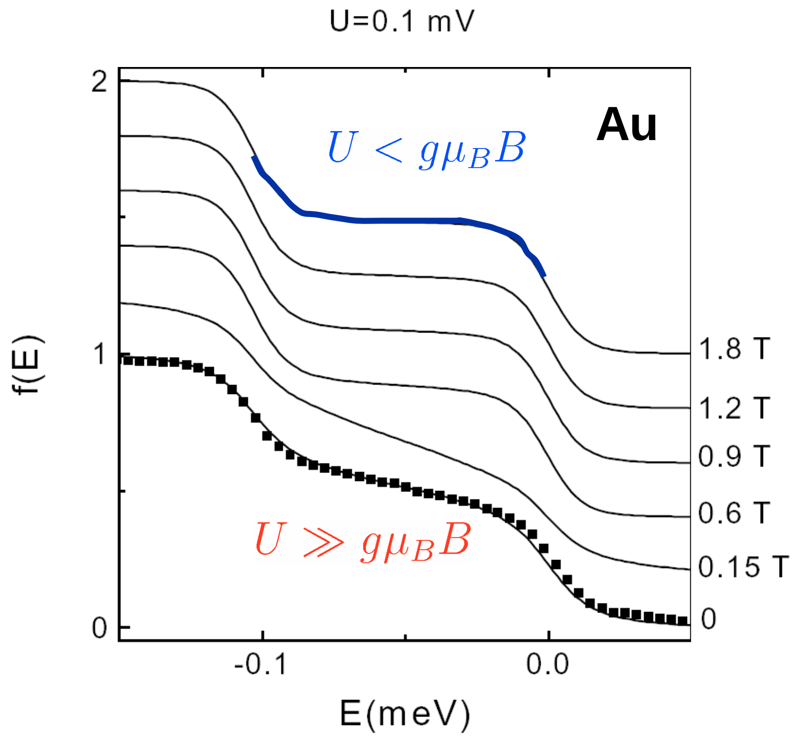


$$K(E) \approx \frac{1}{\tau_0 E^{2}}$$

Energy Relaxation in **Cu** and **Au** Wires: Spins Rule!

A. Anthore, F. Pierre*, H. Pothier, D. Esteve, and M. H. Devoret 2001





Anthore, 2003