Low-Dimensional Disordered Electronic Systems (Experimental Aspects)

Lecture II

Electron-Electron Interactions and Dephasing Processes in Disordered Conductors

Michael Gershenson

Dept. of Physics and Astronomy Rutgers, the State University of New Jersey

"Low-Temperature Nano-Physics" Chernogolovka, August 2007

the State University of New Jersey

> Department of Physics and Astronomy

Lecture 2: Electron-electron Interactions and Dephasing Processes in Disordered Conductors

- El.-el. interactions in disordered conductors: anomalous corrections to the DoS and conductivity
- Dephasing induced by interactions
- Dephasing at ultra-low temperatures: dephasing by Kondo impurities and high-frequency electromagnetic noise



Rutgers wants you!

Great opportunities for graduate students and post-docs: from cutting-edge research in organic semiconductors to ultra - low - temperature nanophysics, including quantum computing with ultra-small Josephson junctions



EI.-EI. Interactions in disorder-free conductors

Collisions with large and *T***-independent** momentum transfer $q \sim k_F$



 $\overline{\tau_{ee}}^{-1} \propto \frac{\mathcal{E}^2}{E_F}$ El.-el. interactions do renormalize electron parameters at a large energy scale (~ E_F), this renormalization does not result in anomalous low-T behavior of these parameters.

 T^2 term in the resistivity, no anomalous corrections at low T

Interaction processes in disordered conductors:

Electron scattering by impurities dramatically changes the situation at low energies ($\langle E_{F} \rangle$). Because of the diffusive motion, processes with collisions with a small momentum transfer $q \sim [max(T,\varepsilon)/D]^{1/2} \ll k_F$ becomes important.

Due to the diffusive motion, the renormalization of all electron parameters becomes *T***-dependent**, and all thermodynamic and transport quantities (including DoS, which is not affected by WL) acquire non-trivial T-dependent corrections.

energy transfer

 $\left| \bigstar L_{\varepsilon} = \sqrt{\frac{\hbar D}{\varepsilon}} \rightarrow \right|$

the *thermal dephasing* length - $L_T = \sqrt{\frac{\hbar D}{k_B T}}$

The dimensionality of a conductor: $L \Leftrightarrow L_{\mathrm{T}}(T)$

 $I \ll L_T \leq L_{\omega}$ - a large scale (D=10 cm²/s, T=1K, $L_T \sim 0.1 \mu$ m), *non-locality* and "*universality*" of INT effects, similar to the WL effects.

Because of a large characteristic length scale, the interaction corrections are "*non-local*" and "*universal*" (similar to the WL correction)





With increasing disorder, the ZBA transforms into the Coulomb gap (the SL regime)



Corrections to the Conductivity

The interaction correction to the conductivity - as a result of the interference between wavefunctions of different electrons propagating in a random scattering potential.



Two regimes: **ballistic** ($T_{\tau} > 1$ or $L_{\tau} \le I$) and **diffusive** ($T_{\tau} << 1$ or $L_{\tau} >> I$)

Metals – mostly diffusive regime, semiconductor structures – both regimes (Lecture 3).



2D

Zala, Narozhny, & Aleiner ('01)

 $k_{\rm F} l >> 1$, all orders in interactions, the leading order in T.



Altshuler-Aronov-Lee

Functions f and g are combinations of the "charge" and "spin" terms, the latter depends on F_0^{α} - the Fermi-liquid constant.



"*Elasticity*" of the processes that contribute to $\Delta \sigma_{INT}$ should be emphasized: they preserve the time reversal symmetry and do not cause phase breaking. An illustration: one of the contributions to the interaction corrections to the conductivity in the ballistic regime is due to the interference between the waves backscattered off an impurity (a short-range potential) "dressed" by Friedel oscillations. Other contributions, which involve virtual energy exchange processes at a scale $\varepsilon >> T$, also do not break the time reversal symmetry.

Sign of the corrections: depending of the value of F_0^{α} , the corrections could be either **positive** or **negative**.

 F_0^{α} can be found from independent measurements of g^* , e.g., from the analysis of SdH oscillations in semiconductor structures, or from the ESR with mobile electrons in metals.

Typically, $|F_0^{\sigma}| < 0.1$ in metals, and the interaction effects *decrease* the conductivity. In semiconductors, the abs. value of F_0^{σ} increases with the strength of interactions: e.g. $F_0^{\sigma} \approx -0.3$ in Si MOSFETs at low carrier densities (see Lecture 2 by Vladimir Pudalov). In the latter case, the conductivity is *increased* by the interaction effects. For the detailed discussion of the corrections to σ at large $|F_0^{\alpha}|$, see Lecture 3.

Some Comments

Friedel Oscillations



Electron density oscillates as a function of the distance from an impurity.

The period of these oscillations is determined by the Fermi wave length.

The amplitude of the oscillations decays only algebraically.

These oscillations are not screened

$$\Delta \sigma_{INT} = f(F_0^{\ a}) \frac{T\tau}{\hbar} + g(F_0^{\ a}) \ln\left(\frac{\hbar}{T\tau}\right)$$

 $g^* = \frac{g}{1 + F_0^{\alpha}}$

Metals: diffusive approximation, $F_0^{\sigma} \approx 0$



$$\Delta \sigma_{INT} \propto \Delta \sigma_{INT}^{3D} \propto \frac{e^2}{h} \left[\frac{1}{L_T(T)} - const \right]$$

 L_T < all dimensions, ξ







$$\Delta \sigma_{INT}^{1D} \propto -\frac{e^2}{h} L_T(T)$$
$$\delta \sigma_C(T) = -\frac{e^2}{\pi \hbar} \sqrt{\frac{\hbar D}{2\pi T}} \left(\frac{3\zeta(3/2)}{2}\right)$$

 $\varsigma(3/2) \cong 2.612$





WL and INT corrections to the conductivity of quasi-1D conductors

$$\lambda_{_F} << \ell \leq W < L_{_{arphi}}, L_{_T} << \xi_{_{1D}}$$



Echternach, MG et al., PRB 50, 5748 ('94)





How to separate WL and INT corrections?



Magnetic Field Effect

$$\Delta \sigma_{INT} = f(F_0^{\ a}) \frac{T\tau}{\hbar} + g(F_0^{\ a}) \ln\left(\frac{\hbar}{T\tau}\right)$$

ballistic regime diffusive regime

Functions f and g - combinations of the "charge" and "spin" terms.

"Charge" term isn't sensitive to the field at all, "spin" terms are affected by the field if the Zeeman energy $g\mu B$ exceeds T.

diffusive regime

Sloppy analogy:





Note: in a system with interactions, the corrections to the DoS and conductivity are not simply interrelated:

$$\sigma \neq e^2 v D \implies \sigma = e^2 \frac{d\mu}{dn} D$$

When B is applied, two (of three) "triplet" contributions to the DoS are "shifted" in energy by $g\mu B$ with respect to the Fermi energy, and $v(E_F)$ increases.



Experimental separation of WL and INT corrections

Let's apply a magnetic field which, on the one hand, sufficiently strong to suppress the *T*-dependence of WL corrections [$L_{H} << L_{\phi}(T)$], but, on the other hand, too weak to modify el.-el. interactions ($g\mu B << T$).





INT corrections close to the WL-SL crossover

Gated δ -doped heterostructures In_{0.2}Ga_{0.8}As/GaAs, n~1x10¹²cm⁻², μ ~2,000 cm²/Vs Minkov et al., *PRB* **67**, 205306 ('03)



Observation: INT corrections are suppressed well below the theoretical estimate when σ approaches e²/h. (Qualitatively, similar behavior is observed in high- μ Si MOSFETs).



Dephasing in disordered conductors



- e-e term dominates at low T

"Disordered" conductors: static disorder strongly enhances/modifies the *e-e* inelasic processes (while the *e-ph* scattering rate can be even reduced, e.g., in the case of "vibrating" impurities).

In 1D and 2D, the main contribution to the dephasing rate comes from *quasi-elastic collisions* with a *small energy transfer* $\Delta \varepsilon \sim \tau_{\phi}^{-1} << T$. These processes govern the phase relaxation at low temperatures.

Interactions-induced Dephasing

3D Collisions with the small momentum transfer $q \sim L_T^{-1}$ and energy transfer $\Delta \varepsilon \sim T$ dominate, *the dephasing and energy relaxation rates are similar*:

Schmid, '74; Altshuler and Aronov, '79 $au_{\varphi}^{-1} \approx \tau_{ee}^{-1} = \frac{\sqrt{2}}{12\pi^2\hbar v} \left(\frac{T}{\hbar D}\right)^{3/2}$

1D **the dephasing and energy relaxation rates are different**. In phase relaxation, the collisions with the energy transfer $\Delta \varepsilon \sim \tau_{\phi}^{-1}$ dominate. These collisions are equivalent to the interaction of an electron with the fluctuating *e.-m*. field produced by all other electrons (dephasing by Nyquist noise).

(Altshuler, Aronov, and Khmelnitskii '82).

2D
$$\frac{1}{\tau_{\varphi}} = \frac{T}{g} \ln g + \frac{\pi}{4} \frac{T^2}{E_F} \ln (E_F \tau)$$
, where $g = \frac{h}{e^2} \sigma_{2D}$

The first (diffusive) term becomes comparable to the second (ballistic) term when $T\tau \sim 1$

1D
$$\frac{1}{\tau_{\varphi}} = \left(\frac{e^2 \sqrt{D}}{\hbar \sigma_1} \frac{k_B T}{\hbar}\right)^{2/3}$$



3D disordered metals



Thick disordered metal films, including metal glasses, and heavily doped semiconductors

- 1µm-thick Cu films ρ=6×10⁻⁵ Ω cm (Aronov, MG, Zhuravlev, '84)
- ▼ Cu_{0.9}Ge_{0.1} ρ=2.8×10⁻⁵ Ω cm (Eschner *et al.*, '84)

solid curve – dephasing due to e-ph collisions in disordered conductors $(D = 10 \text{ cm}^2/\text{s})$

dashed lines – dephasing due to e-e collisions with small momentum transfer

$$\tau_{\varphi}^{-1} = \frac{\sqrt{2}}{12\pi^2 \hbar \nu} \left(\frac{k_B T}{\hbar D}\right)^{3/2}$$



2D metal films



- - Mg film, R_{\Box} = 22.3 Ω (White *et al.*, '84)
- ▲ Al film, R_{\Box} = 112 Ω (MG *et al.*, '83)
- Bi film, R_□= 630 Ω (Komori *et al.*, '83)



• - Au film, R_{\Box} = 32.7 Ω (Aronov *et al.*, '84) • - ultra-thin Ag film, R_{\Box} = 1.5 k Ω



Dephasing in Si MOSFETs



Despite strong interactions, still a Fermi-liquid system

RUTGERS



		d, nm	W, nm	<i>L</i> , μ m	<i>R</i> , Ω
Au	'93	15	90	100	4100
Au	'99	45	90	176	1080
AuPd	'01	7.5	5	1.5	28800

1D wires

1D:
$$au_{\varphi} = \left(\frac{\hbar^2}{e^2 R_1 \sqrt{D} k_B T}\right)^{2/3}$$

Altshuler, Aronov, and Khmelnitskii, '82; Aleiner et al., '99



Validity of the Fermi-liquid approach

Fermi liquid approximation holds (single-particle excitations are well defined)

$$T\frac{\tau_{\varphi}(T)}{\hbar} = g(L_{\varphi}) > 1$$

On the "metallic" side of the WL-SL crossover $(L_{\varphi} << \xi) = g(L_{\varphi}) << g(\xi) \sim 1$

Both quasi-1D and 2D conductors behave as Fermi liquids

At the crossover $g(L_{\varphi}) \sim 1$

The upper limit on τ_{ω} :

$$\tau_{\varphi}(T) < \frac{\xi^2}{D}$$



δ-doped GaAs wires (*W*=50nm, R_{\Box} = 1 kΩ) Khavin, MG, Bogdanov, '98





Puzzle of Low-T Saturation of $\tau_{0}(T)$

Saturation of $\tau_{\phi}(T)$ at low temperatures was observed in many (but not all!) experiments. A trivial cause – overheating by measuring current – has been ruled out.

What causes the apparent low-T saturation of τ_{o} :

• at least in some cases, the saturation can be attributed to the presence of **paramagnetic impurities** in a small concentration undetectable by analytical methods (Michigan + Saclay collaboration, '02-'07, Bauerle *et al.*, '05, etc.):

• dephasing by an *external high-frequency electromagnetic noise*. This effect has not received the deserved attention though it was proposed as an explanation of Webb's results right after the publication of MJW paper [Khavin, MG, Bogdanov, *Phys. Rev. Lett.* 81, 1066 ('98)].



Kondo-related Dephasing



With an increase of the dephasing length, the dephasing rate might be affected by magnetic impurities even if their concentration is very low :

 L_{ϕ} ~10µm \Rightarrow volume $L_{\phi}^{2}d$ contains ~3 impurities at the concentration 1 ppm.

Pierre et al. ('03)



Experimental Test of the NRG Theory for Dephasing by Magnetic Impurities



The theoretical expression for the *T*-dependence of the dephasing rate, calculated on the basis of the Numerical Renormalization Group (NRG) bridges the gap between the low-*T* Fermi liquid theory and the high-*T* Suhl-Nagaoka expansion.

Zarand *et al.*, *PRL* **93**, 107204 ('04) Micklitz *et al.*, *PRL* **96**, 226601 ('06)

C. Bauerle, F. Mallet, F. D. Mailly, G. Eska, and L. Saminadayar, *PRL* **95**, 266805 ('05)

The "quadratically vanishing" dephasing rate appears only well below $T_{\rm K}$.



Experimental Test of the NRG Theory for Dephasing by Magnetic Impurities



Alzoubi and Birge, PRL 97, 266803 ('06)

$$\tau_{\varphi}^{-1} = \tau_{in}^{-1} + \gamma_m$$



Mallet et al., PRL 97, 266804 ('06)



Dephasing by external noise?

Though controlling magnetic impurities at a level of a few *ppm* is a challenge, there were claims that at least in some samples, the observed saturation had nothing to do with magnetic impurities...

One of the suspects – dephasing by **external high**frequency electromagnetic noise without overheating.

Altshuler, Aronov, and Khmelnitsky, SSC 39, 619 ('81)

time-dependent E induces dephasing.

The most efficient dephasing:

 $\omega \approx \tau_{\varphi}^{-1}$

(typically, τ_{ϕ} ~ 1 ns, thus \textit{f}_{MW} in the GHz range)

For an optimal ω , the MW-induced dephasing rate $\tau_{MW}^{-1} \sim \tau_{\phi0}^{-1}$ at

$$e E_{\scriptscriptstyle MW} \sqrt{D \, \tau_{\scriptscriptstyle \varphi}} \sim \frac{\hbar}{\tau_{\scriptscriptstyle \varphi}}$$



Experimental Challenge: MW Dephasing-w/o-Overheating



For observation of the "MW dephasing-without-overheating":

• the lower T, the better : at T > 1K, dephasing in metal films is mostly due to the el.-ph. scattering ($\tau_{\varphi} \sim T^{-3}$) and $P_{MW} \sim T^{-9}$ grows with T much faster than the thermal conductivity ($\sim T^{-5}$).

ID is better than 2D



Prior Experiments on MW-induced Dephasing



Wang and Lindelof, 1987 – Mg films

Vitkalov et al., 1988 – Si MOSFETs



In both experiments, the range of P_{MW} for "dephasing-without-overheating" was very narrow (if any).



"We find that up to 26 GHz this external environment does not cause decoherence without a concomitant increase in the energy relaxation rate"

Webb et al., in "Quantum Coherence and Decoherence" (Elsevier 1999)



Optimization of Sample Geometry

1D wires, ultra-low T

$$P_{MW}^{\phi} \left(\propto \tau_{\phi 0}^{-3} \right) = R \left(\frac{ek_B T}{\hbar} \right)$$

 $\left(\frac{1}{2}\right)^2$ the MW power that results in $\tau_{MW} \sim \tau_{\varphi}$ (at optimal f_{MW})

 $P_{es} \approx \left(\frac{2\pi k_B}{\rho}\right)^2 \frac{T\Delta T}{R}$ cooling due to outdiffusion of hot electrons

 $\frac{P_{es}}{P_{MW}^{\varphi}} \approx \left(\frac{h/e^2}{R}\right)^2 \frac{\Delta T}{T}$

R – the total resistance of a wire

Long wires: "dephasing with - overheating"





ideal samples for probing the intrinsic dephasing mechanisms



Sample Design





- poor UCF averaging, susceptible to the external noise

Long wires:

Short wires:

- better UCF averaging, less susceptible to external noise
 only e-ph cooling, very inefficient at *T* < 1K

Solution: long wires with periodically spaced cooling fins.





Distance between cooling fins $d = 30 \, \mu m$, the total length – 1200 μ m

One can neglect the effect of cooling fins on $\Delta \sigma_{WL}$ if $d > 10 L_{\omega}(T)$.





Effect of Microwave Radiation on the WL MR

T = 300 K



Wei Jian, Pereverzev, MG, PRL. 96, 086801 (2006)



To compare our experiment with the AAK theory:

$$e E_{MW} \sqrt{D \tau_{\varphi}} \sim \frac{\hbar}{\tau_{\varphi}}$$

- $\tau_{\varphi}(T), \tau_{\varphi}(T, P_{MW})$ from the WL magnetoresistance at $P_{MW}=0$ and at $P_{MW}\neq 0$
- E_{MW} (or P_{MW} dissipated in the sample) by comparing the DC and MW heating
- T_e from the interaction corrections in strong magnetic fields $(L_H << L_{\phi})$



Interaction Corrections as a Built-in Thermometer



In strong magnetic fields $(L_H << L_{\phi})$, R(T) is determined solely by the interaction corrections $\Delta \sigma_{\text{EEI}}(T_{e})$.

The measurements of R in strong Bhave been used for the direct measurement of T_e and calibration of the MW power dissipated in the sample, P_{MW} .



Calibration of MW power



At T = 0.1K, $P_{MW} < 1$ pW is sufficient to overheat the electrons in a 1.2 mmlong nanowire with cooling fins. For a typical 1D wire ($L \le 100 \mu$ m), this power is in the *fW* range.





• $\tau_{\phi}(T)$ "saturates" below $T \sim 0.1 \text{ K}$

• $\tau_{\phi}(T)$ depends on the coupling of a sample to its "environment"



THE STATE UNIVERSITY OF NEW JERSEY



The total dephasing rate:

$$\tau_{\varphi}^{-1}(\omega, P_{MW}) = \tau_{\varphi0}^{-1}(P_{MW} = 0) + \tau_{MW}^{-1}(\omega, P_{MW})$$

 τ_{MW}^{-1} – the MW- induced dephasing rate





 $\alpha \equiv \frac{2e^2 D E_{MW}^2}{\hbar^2 \omega^3} - \text{the normalized MW}$ power

Wei Jian, Pereverzev, MG, PRL 96, 086801 ('06)

All experimental results are in good agreement with the AAK theory (no fitting parameters!)





MW-induced Dephasing (cont.)









Summary

Interaction effects in disordered conductors produce quantum corrections to the conductivity, DoS, and other electron parameters.

Dephasing in 1D and 2D conductors at low T is governed by interaction effects

• The observed saturation of $\tau_{\varphi}(T)$ at T < 0.1K - most likely due to scattering by paramagnetic impurities and dephasing by high-frequency electromagnetic noise.



Lecture 3: Quantum Corrections to the Conductivity of High-Mobility Si MOSFETs

- Intro: quantum corrections in Si MOSFETs (the most ubiquitous 2D structure) ⇒ 25-year-old mystery and the work is still in progress
- Ingredients essential for better understanding of interaction effects in Si MOSFETs:
 - interaction parameters in high-mobility Si MOSFETs
 - valley splitting and inter-valley scattering
- Analysis of $\Delta \sigma(T,B)$
- The crossover from "metallic" to "insulating" conductivity: role of inhomogeneity?

